Algebra Skills?

See Diagnostic Test

Start plugging through the questions & Review Discussion.

Ask questions.

Class average was sickeningly HIGH.

Test 1 #7 "$x = 4$"

a. $\dfrac{\sqrt{x} - 2}{x - 4}$

b. $\dfrac{\sqrt{x} - 2}{x - 4} \cdot \dfrac{\sqrt{x} + 2}{\sqrt{x} + 2} = \dfrac{\cancel{(x-4)}^{1}}{\cancel{(x-4)}_{1}(\sqrt{x}+2)} = \dfrac{1}{\sqrt{x}+2}$

$(x \neq 4)$

$\xrightarrow{x \to 4} \dfrac{1}{\sqrt{4}+2} = \dfrac{1}{2+2} = \dfrac{1}{4}$

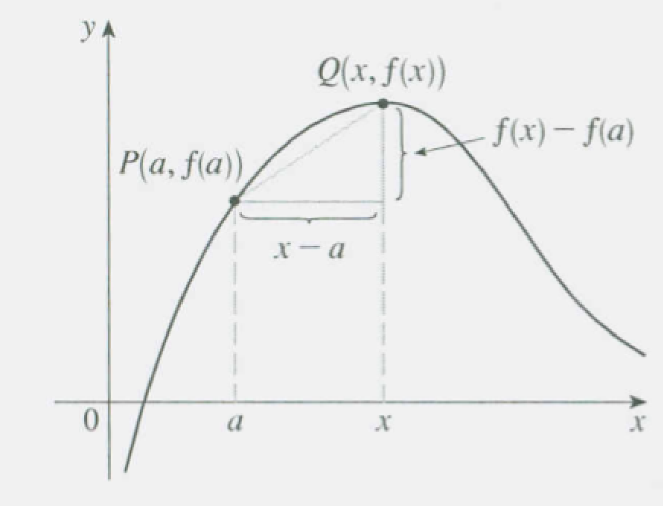
c. $y = m(x - x_1) + y_1$

$y = \frac{1}{4}(x-4) + 2$

3.1 DERIVATIVES AND RATES OF CHANGE

Recall, from Section 2.1 and Test 1:

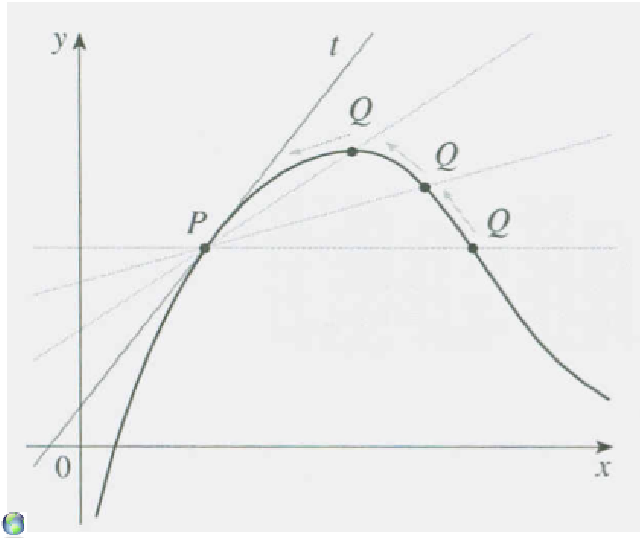
The slope of the secant line between P and Q: $m_{PQ} = \dfrac{f(x) - f(a)}{x - a}$



Tangent Zoom Demo: Smooth functions are locally linear.

The world is flat! Because we are so small.

As we take the point Q closer and closer to the point P, the resulting line approaches the tangent line to f at P:



FOLLOW LINK AT BOTTOM LEFT OF DIAGRAM TO SEE THE DAY 1 SECANT-LINE APPROXIMATION TO THE TANGENT LINE VISUAL.

1 DEFINITION The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided that this limit exists.

Its equation is given by: $y - y_1 = m(x - x_1)$

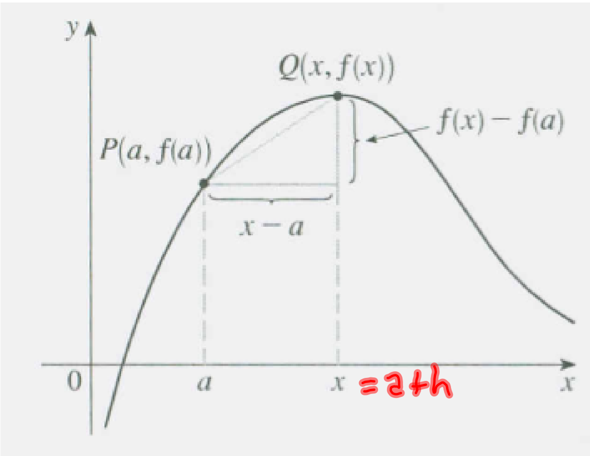
$$y = m(x - x_1) + y_1$$

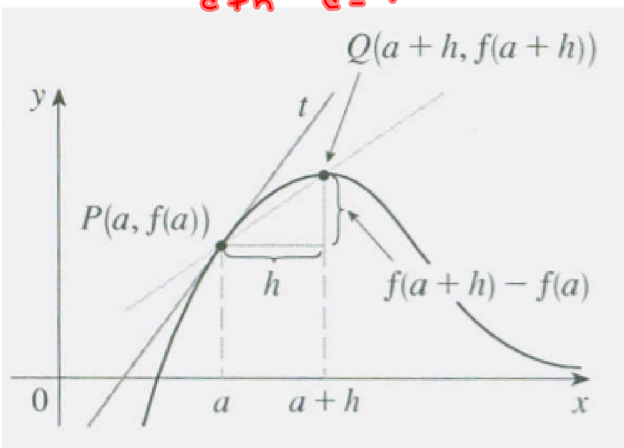
Equation 2 for the slope of the tangent line is equivalent to Equation 1.

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$a+h-a=$





Smooth curves are "locally linear." The more you zoom in, the flatter things look. A man in space *knows* the Earth is round, but (some) people on the ground thought it was flat for thousands of years.

Demo a quadratic function (parabola). Follow the link to see Local Linearity with the "TANGENT ZOOM."

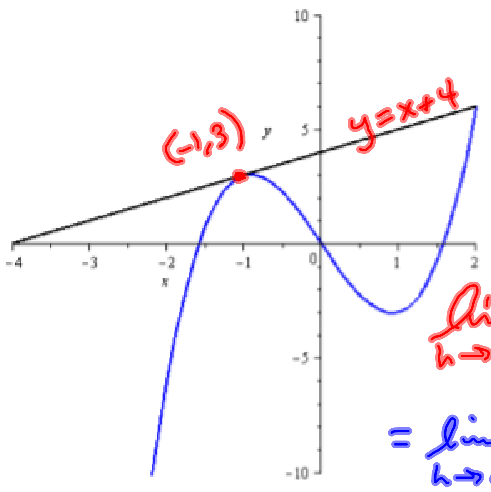
Sequencing, Steve.

Tangent line questions:

5–8 Find an equation of the tangent line to the curve at the given point.

6. $y = 2x^3 - 5x, \quad (-1, 3)$

Find the slope of the tangent line (the slope of the curve) at $x = -1$ by the definition.
Find the equation of the tangent line. The book does some ugly things with these. Point-slope or slope-intercept is the way to go.



Here's a picture of what we're doing. The tangent line is in black.

@ $(-1, 3) = (x, f(x))$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h\to 0} \frac{f(-1+h) - 3}{h}$$

Scratch

$f(-1+h) = 2(-1+h)^3 - 5(-1+h)$

Scratch 4 Scratch

$(-1+h)^3 = 1(-1)^3(h)^0 + 3(-1)^2(h)^1 + 3(-1)^1(h)^2 + 1(-1)^0(h)^3$

$= -1 + 3h - 3h^2 + h^3$

1
1 1
1 2 1
1 3 3 1

$= 2(-1+3h-3h^2+h^3) + 5 - 5h$

$= -2 + 6h - 6h^2 + 2h^3 + 5 - 5h$

$= 3 + h - 6h^2 + 2h^3$

$= \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h\to 0} \frac{f(-1+h) - 3}{h}$

$= \lim_{h\to 0} \frac{3 + h - 6h^2 + 2h^3 - 3}{h} = \lim_{h\to 0} \frac{h - 6h^2 + 2h^3}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(1 - 6h + 2h^2)}{\cancel{h}} = \lim_{h\to 0} (1 - 6h + 2h^2) = 1 = m_{tan}$ @ $x = -1$

Using $(-1, 3) = (x_1, y_1)$, $m = 1$, we have

$\boxed{y = 1(x+1) + 3} = x + 4$

$y = x + 4$

6. $y = 2x^3 - 5x, \quad (-1, 3)$

Better way, for me, is to just simplify $\frac{f(x+h)-f(x)}{h}$, pass to the limit, and THEN let $x = -1$.

↳ Gives $f'(x)$ = the Derivative function

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^3 - 5(x+h) - (2x^3 - 5x)}{h}$$

$$= \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - 5x - 5h - 2x^3 + 5x}{h}$$

$$= \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - 5x - 5h - 2x^3 + 5x}{h}$$

$$= \frac{6x^2h + 6xh^2 + 2h^3 - 5h}{h} = \frac{\cancel{h}(6x^2 + 6xh + 2h^2 - 5)}{\cancel{h}}$$

$$= 6x^2 + 6xh + 2h^2 - 5 \xrightarrow{h \to 0} \boxed{6x^2 - 5 = f'(x)}$$

$(h \neq 0)$

$f'(-1) = 6 - 5 = 1 = m_{tan}$ @ $x = -1$

I like this way better. I can use $f'(x)$ to find m_{tan} @ any x I want.

10. (a) Find the slope of the tangent to the curve $y = 1/\sqrt{x}$ at the point where $x = a$.

(b) Find equations of the tangent lines at the points $(1, 1)$ and $(4, \frac{1}{2})$.

(c) Graph the curve and both tangents on a common screen.

PART C IS ON THE NEXT PAGE.

$$\frac{f(x+h)-f(x)}{h} \quad \text{They want} \quad \frac{f(a+h)-f(a)}{h}$$

$$\frac{\frac{1}{\sqrt{a+h}} - \frac{1}{\sqrt{a}}}{h} = \frac{1}{h}\left[\frac{1}{\sqrt{a+h}} - \frac{1}{\sqrt{a}}\right]$$

LCD= $\sqrt{a}\sqrt{a+h}$, etc.

$$= \frac{1}{h}\left[\frac{\sqrt{a}-\sqrt{a+h}}{\sqrt{a+h}\sqrt{a}}\right]\left[\frac{\sqrt{a}+\sqrt{a+h}}{\sqrt{a}+\sqrt{a+h}}\right]$$

$$= \frac{1}{h}\left[\frac{a-(a+h)}{\sqrt{a+h}\sqrt{a}(\sqrt{a}+\sqrt{a+h})}\right]$$

$$= \frac{1}{\cancel{h}_{1}}\left[\frac{-\cancel{h}^{1}}{\sqrt{a+h}\sqrt{a}(\sqrt{a}+\sqrt{a+h})}\right]$$

$$= \frac{-1}{\sqrt{a+h}\sqrt{a}\left(\sqrt{a}+\sqrt{a+h}\right)} \xrightarrow{h \to 0} \frac{-1}{\sqrt{a}\sqrt{a}\left(\sqrt{a}+\sqrt{a}\right)}$$

$(h \neq 0)$

$$= \frac{-1}{a(2\sqrt{a})} = \frac{-1}{2a\sqrt{a}} = f'(a)$$

$(1,1)$ & $(4, \frac{1}{2})$.

@ $(1,1)$, $m_{tan} = \frac{-1}{2(1)\sqrt{1}} = \frac{-1}{2}$

$$\boxed{y = -\frac{1}{2}(x-1) + 1}$$

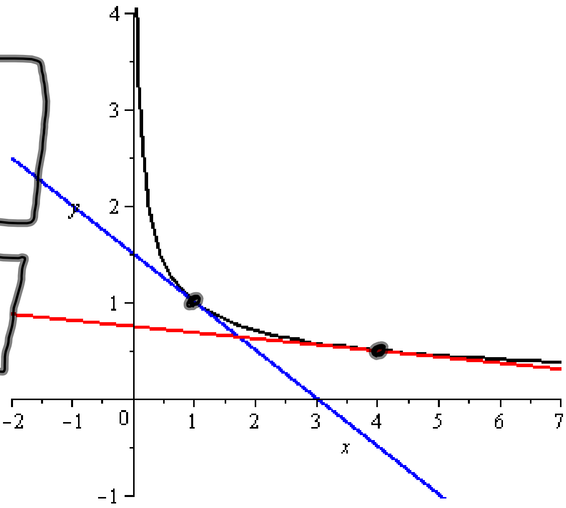
@ $(4, \frac{1}{2})$, $m_{tan} = \frac{-1}{2(4)\sqrt{4}} = -\frac{1}{16}$

$$\boxed{y = -\frac{1}{16}(x-4) + \frac{1}{2}}$$

$$f'(a) = -\frac{1}{2\sqrt{a^3}}$$

$$\boxed{-\frac{1}{2}x + \frac{3}{2}}$$

$$\boxed{-\frac{1}{16}x + \frac{3}{4}}$$



More of the same, just different words:

4 DEFINITION The **derivative of a function f at a number a**, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

if this limit exists.

Other names for this:
Velocity is the derivative of distance. It's all about the *instantaneous rate of change of $f(x)$ with respect to x when $x = a$*.

Other ways of expressing it:

$$\boxed{6} \qquad \text{instantaneous rate of change} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

25–30 Find $f'(a)$.

28. $f(x) = \dfrac{x^2 + 1}{x - 2}$

29. $f(x) = \dfrac{1}{\sqrt{x + 2}}$

51–52 Determine whether $f'(0)$ exists.

$\boxed{51.}$ $f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ Doesn't

52. $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ Does.

This is an advanced calculus question that is very difficult to answer formally, but isn't *too* bad, if you approach it numerically.

The way I think of it, the x dampens the sin($1/x$) enough to make it converge to zero at $x = 0$ (in the limit), so #51 is CONTINUOUS, with this definition.

#52 passes a higher standard. It's not only continuous at $x = 0$, but it's *smooth* at $x = 0$. This is very cool.