2.5 quickies.

$$\lim_{x \to c} f(x) = f(c)$$

$$\lim_{x \to 3} \left[2f(x) - g(x)\right] = 2f(3) - g(3) = 4$$

1. The point $P = (-1, 2)$ lies on the graph of $f(x) = x^2 + x + 2$. Let $Q = (x, \underline{x^2 + x + 2})$ be another point on the graph of f.

   a. (5 pts) Find the slope $\boxed{m_{PQ}}$ to 4 decimal places for the following values of x:

   Let $x = -1.001$, then

   - $x = -1.001$

   $$m_{PQ} = \frac{x^2+x+2-2}{x-(-1)} = \frac{f(x)-f(-1)}{x-(-1)} =$$

   $$\frac{(-1.001)^2+(-1.001)}{-1.001+1} = \frac{.001001}{-.001} \approx -1.0010$$

   - $x = -0.999$

   $$\frac{(-.999)^2+(-.999)+2-2}{-.999-(-1)} \approx -.9990$$

   b. (5 pts) Based on your work for part a., estimate the slope of the tangent line $m_{\tan}$ at $x = -1$. to 4 places.

   $$m_{\tan} \approx \frac{-1.0010 + -.9990}{2} \approx -1.0000$$

   c. (5 pts) Based on your work for part b., construct an equation for the tangent line to $f(x) = x^2 + x + 2$. (Point-slope form is just fine: $y = m(x - x_1) + y_1$.

   $$y = -1.0000(x-(-1)) + 2$$

2. (10 pts) Sketch the graph of a function that satisfies all of the following requirements:

O.P.

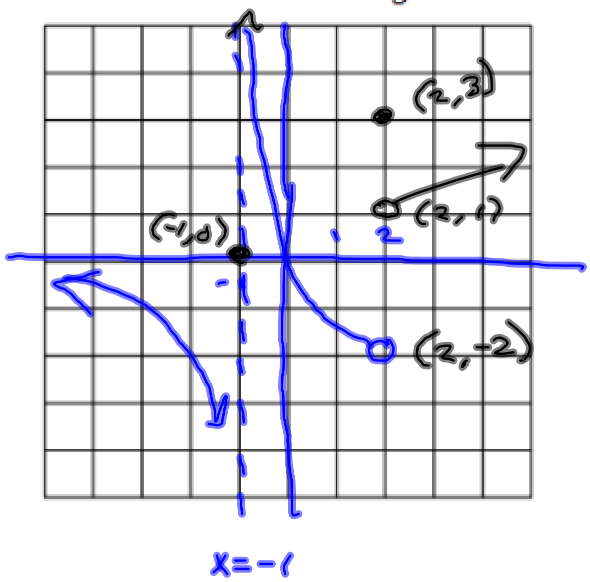
a. $\lim_{x \to -1^-} f(x) = -\infty$

b. $\lim_{x \to -1^+} f(x) = \infty$

c. $\lim_{x \to 2^-} f(x) = -2$

d. $\lim_{x \to 2^+} f(x) = 1$

e. $f(2) = 3$



5. Find the limit, if it exists. If it doesn't, say so (in writing, of course).

a. (5 pts) $\lim_{x\to 3}\frac{x^2-2x+1}{x^2+3}$

$$= \frac{3^2-2(3)+1}{3^2+3} = \frac{9-6+1}{9+3} = \frac{4}{12} = \frac{1}{3}$$

b. (5 pts) $\lim_{x\to 3}\frac{x^2-2x-15}{x^2-9}$

$$= \lim_{x\to 3}\frac{(x-5)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}} = \lim_{x\to 3}\frac{x-5}{x-3} \quad \nexists$$

$$\frac{x^2-2x-15}{x^2-9} = \frac{(x-5)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}} = \frac{x-5}{x-3} \qquad x\to 3 \Rightarrow \text{DNE.}$$

$x \neq -3$

$$\boxed{\begin{gathered} x-3 \geq 0 \\ x \geq 3 \end{gathered}}$$

c. (5 pts) $\lim_{x\to 3}\frac{x^2-2x+1}{x^2-9}$

$=$

$$\lim_{x\to 3}\left(\frac{x^2-2x+1}{x^2-9}\right) \qquad \nexists$$

$$\frac{|x-3|}{x^2-9} = \begin{cases} \dfrac{x-3}{x^2-9}, & \text{if } x \geq 3 \\ \dfrac{-(x-3)}{x^2-9} & \text{if } x < 3 \end{cases}$$

d. (5 pts) $\lim_{x\to 3^-}\frac{|x-3|}{x^2-9}$

$$= \lim_{x\to 3^-}\frac{-\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} = \lim_{x\to 3^-}\left(\frac{-1}{x+3}\right) = \boxed{\frac{-1}{6}}$$

$$|x-3| = \begin{cases} x-3 & \text{if } x \geq 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$$

$\therefore$ so, $\lim_{x\to 3^-}\frac{|x-3|}{x^2-9} = \lim_{x\to 3^-}\frac{-\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}$

4. (10 pts) Use the precise definition of a limit to prove that $\lim_{x \to 2} (3x - 5) = 1$.

$|x-2| < \delta$

Scratch

want $|3x-5-1| < \varepsilon$

$|3x-6| < \varepsilon$

$3\underbrace{|x-2|}_{<\delta} < \varepsilon$

$3\delta \equiv \varepsilon$

$\delta = \frac{\varepsilon}{3}$

Proof:

Let $\varepsilon > 0$ be given

Define $\delta = \frac{\varepsilon}{3}$. Then if $0 < |x-2| < \delta$, we have

$|3x-5-1| = |3x-6| = 3|x-2|$

$< 3\delta = 3 \cdot \frac{\varepsilon}{3} = \varepsilon$ ■

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5. (5 pts) Use the precise definition of a limit to prove that $\lim_{x\to 3}(x^2-2x+1)=4$.

Scratch

$|x^2-2x+1-4|<\varepsilon$

$|x^2-2x-3|=|x-3||x+1|$

$<\delta$

Need a bound on this.

$x\to 3$, so if $\delta\le 1$,

$|x-3|<1$

$-1<x-3<1$

$2<x<4$

$3<x+1<\underline{5}$, so $|x+1|<5$

$|x+1||x-3|<5\delta\le\varepsilon$

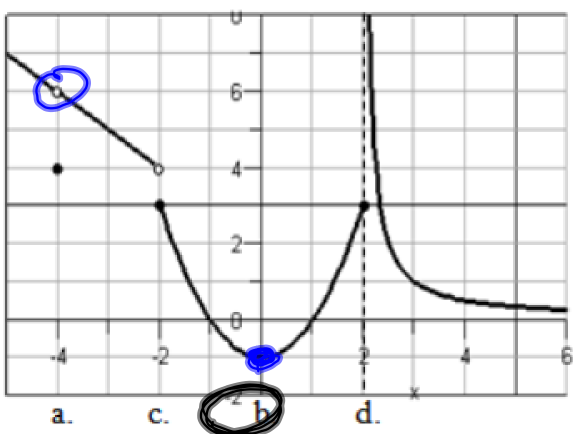
Proof

Let $\varepsilon>0$ be given.

Define $\delta=\min\{1,\frac{\varepsilon}{5}\}$

etc.

6. The graph of a function f is given on the right. Evaluate each limit, if it exists. If the limit does *not* exist, explain why. Employ the language (shorthand) of limits in your explanation(s), as needed.



a. (5 pts) $\lim_{x \to -4} f(x)$ $= 6$

b. (5 pts) $\lim_{x \to 0} f(x)$ $= -1$

c. (5 pts) $\lim_{x \to -2} f(x)$ $\nexists$, because

$\lim_{x \to -2^-} f(x) = 4 \neq 3 = \lim_{x \to -2^+} f(x)$

d. (5 pts) $\lim_{x \to 2} f(x)$ $\nexists$,

$\lim_{x \to 2^-} f(x) = 3 \neq \infty = \lim_{x \to 2^+} f(x)$

e. (5 pts) At which of a, b, c., and/or d. is f continuous? (Circle one (or more).)

f. (5 pts) Use the definition of continuity to explain why f is *not* continuous at $x = -2$. Oops! Gave you a little help on part e., there.

1st of all $\lim_{x \to 2} f(x)$ $\nexists$, let alone

$\lim_{x \to 2} f(x) = f(2)$

7. This problem is a lead-in to Chapter 3. You should have all the tools necessary to answer these questions, even though you might not have thought of the work we've been doing in quite this way, just yet. Let $f(x) = x^2 + x + 2$. Then the slope of the secant line between two points on the graph of f $(x, f(x))$ and $(3, f(3))$ is given by the difference quotient:

$$m_{sec} = \frac{f(x) - f(3)}{x-3}.$$

$f(3) = 3^2 + 3 + 2 = 14$

a. (5 pts) Simplify the difference quotient, i.e., evaluate $m_{sec} = \dfrac{f(x)-f(3)}{x-3}$.

$$\frac{f(x)-f(3)}{x-3} = \frac{x^2+x+2-14}{x-3} = \frac{x^2+x-12}{x-3} = \frac{(x+4)\cancel{(x-3)}}{\cancel{x-3}}$$

$$= \boxed{x+4 \quad (x \neq 3)}$$

b. (5 pts bonus) Based on your work in part a., compute the slope of the tangent line to f at $x = 3$, by computing the limit $m_{tan} = \lim\limits_{x \to 3} \dfrac{f(x)-f(3)}{x-3}$.

$$x+4 \xrightarrow{x \to 3} 7 = m_{tan}$$

c. (5 pts) Based on your work in a. and b., construct an equation of the tangent line to f at $x = 3$. If you didn't *get* b., then *make up an answer and use it*!

$(x_1, y_1) = (3, f(3)) = (3, 14)$, $m = 7$

$$\boxed{y = 7(x-3) + 14}$$

$y = m(x - x_1) + y_1$

~~$y - y_1 = m(x - x_1)$~~