§2.4 #29

$\vdots$

$\sqrt{|x-2|^2}$ want $< \sqrt{\varepsilon}$

$|x-2| < \sqrt{\varepsilon}$

Define $\delta = \sqrt{\varepsilon}$

__Claim__

$\lim_{x \to 2} (x^2-4x+5) = 1$

__Proof__

Let $\varepsilon > 0$. Define $\delta = \sqrt{\varepsilon}$.

Then, if $0 < |x-2| < \delta$, we have

$|(x^2-4x+5)-1| = |x^2-4x+4|$

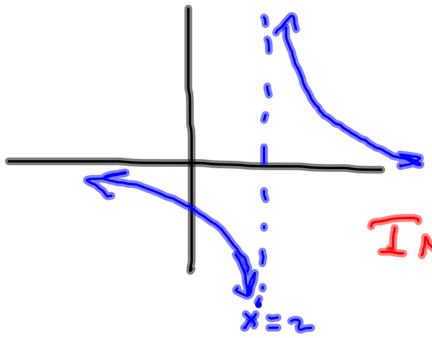
$= |(x-2)^2| = |x-2|^2 < \delta^2 = (\sqrt{\varepsilon})^2$

$= \varepsilon \blacksquare$

§2.5

$$\lim_{x \to c} f(x) = f(c)$$

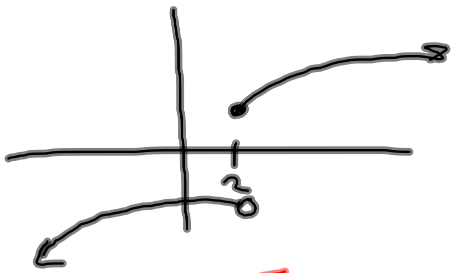
f is cnt$^{s}$ @ $x=c$ if the limit as x approaches c of f(x) exists, and agrees with the value of the function at $x=c$.




$\lim_{x \to 2} f(x)$ $\nexists$

f isn't cnt$^{s}$ @ $x=2$

INFINITE DISCONTINUITY

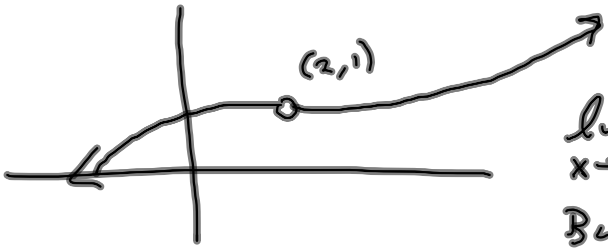



$\lim_{x \to 2} f(x)$ $\nexists$

f isn't cnt$^{s}$ @ $x=2$.

JUMP DISCONTINUITY

Jan 31-12:13 PM

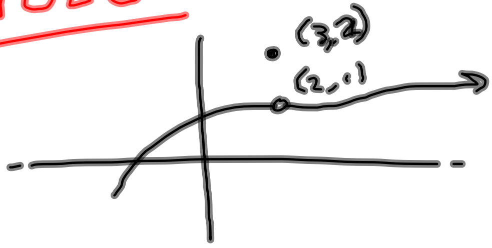



$\lim_{x \to 2} f(x) = 1$

But $f(x)$ isn't even defined.

Not cont.

HOLE




$\lim_{x \to 2} f(x) = 1$

$f(2) = 3$

Not cont. @ $x = 2$.

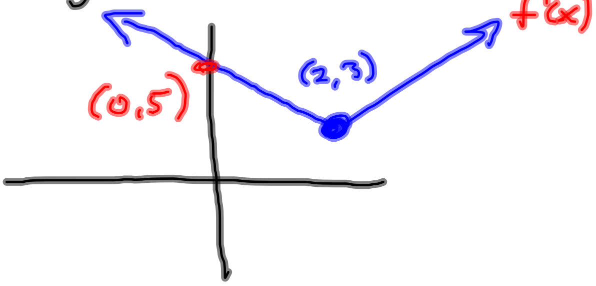
Trig functions, Rational Functions, Radical Functions, Polynomials are ALL continuous on their domains, (absolute value function)

Is $f(x) = \begin{cases} 5-x & \text{for } x \le 2 \\ x+1 & \text{for } x > 2 \end{cases}$ cnt$^{\underline{s}}$ on its domain?

$\lim\limits_{x \to 2^-} f(x) = 5-2 = 3$ (The 5-x piece)

$\lim\limits_{x \to 2^+} f(x) = 2+1 = 3$ (The x+1 piece)

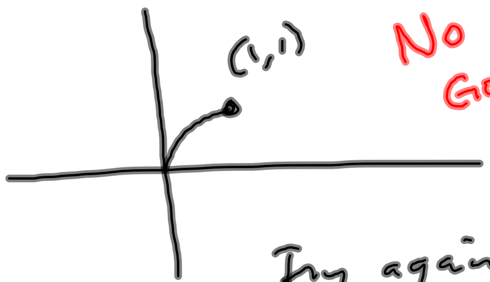
They meet @ the suture point: $x=2$




$y = 5-x$
$= -x+5$ for $x \le 2$

$y = x+1$ for $x > 2$

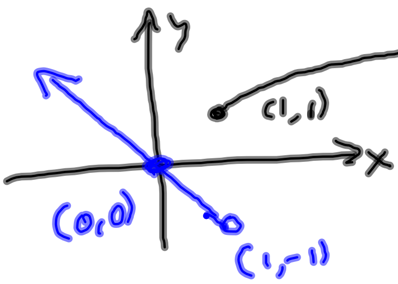
$$f(x) = \begin{cases} \sqrt{x} & \text{for } x \geq 1 \\ -x & \text{for } x < 1 \end{cases}$$




No. Got the pieces wrong / background.

continuous

Try again




Not cnt's @ $x = 1$.

For what values of $x$ is $f(x)$ cnt's?

$\mathbb{R} \setminus \{1\}$

$= (-\infty, 1) \cup (1, \infty)$

$= \{x \mid x \neq 1\}$

For what value(s) of c is

$f(x) = \begin{cases} cx+3 & \text{for } x \le 1 \\ x^2 & \text{for } x > 1 \end{cases}$ cnt$^{\Sigma}$?

Want $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$c(1)+3 = c+3 = 1^2 = 1$

$\Rightarrow c = -2$

$\lim_{x \to 1^-} f(x) = -2(1)+3 = 1$ ✓

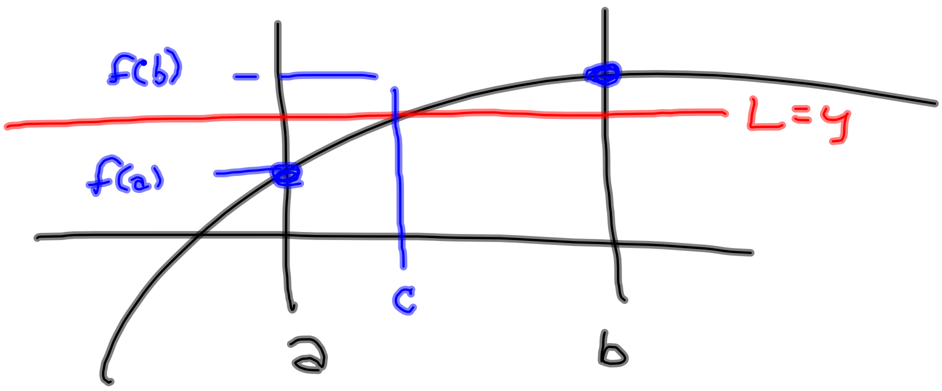
$\lim_{x \to 1^+} f(x) = 1^2 = 1$

#s 41, 42 like this one.

Don't panic on #60. I'll explain.

Intermediate Value Theorem –

If $f$ is continuous on $[a,b]$ and $f(a) \neq f(b)$, then, if $L$ is a number between $f(a)$ & $f(b)$, then there is a $c \in (a,b)$ such that $f(c) = L$




$f$ cont$^s$ on $[a,b]$ and $f(a) < f(b)$ and $f(a) < M < f(b)$

$\Rightarrow \exists\, c \in (a,b) \ni f(c) = M$

Show that

$f(x) = x^3 + x - 2$ has a zero in $(0,2)$, using IVT

$f(0) = -2$

$f(2) = 8+2-2 = 8$

Say something about $f(x)$ that lets you apply IVT to it.

$-2 < 0 < 8$

$\therefore$ By <u>IVT</u>, $\exists$ $c \in (0,2)$ such that $f(c) = 0$.

$f$ is a polynomial & they're cnt$^{\underline{s}}$.

§2.5 I ~~tomorrow~~ Wednesday

2.5 II Wednesday