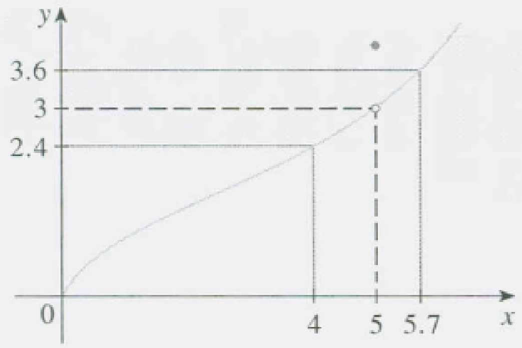
Let's have a look at Section 2.4 #2:

2. Use the given graph of f to find a number δ such that

if $\quad 0 < |x - 5| < \delta \quad$ then $\quad |f(x) - 3| < 0.6$



If you keep x between 4 & 5.7, you'll keep $f(x)$ between 2.4 & 3.6. How close to 5 must x be to keep $f(x)$ within .6 units of 3?

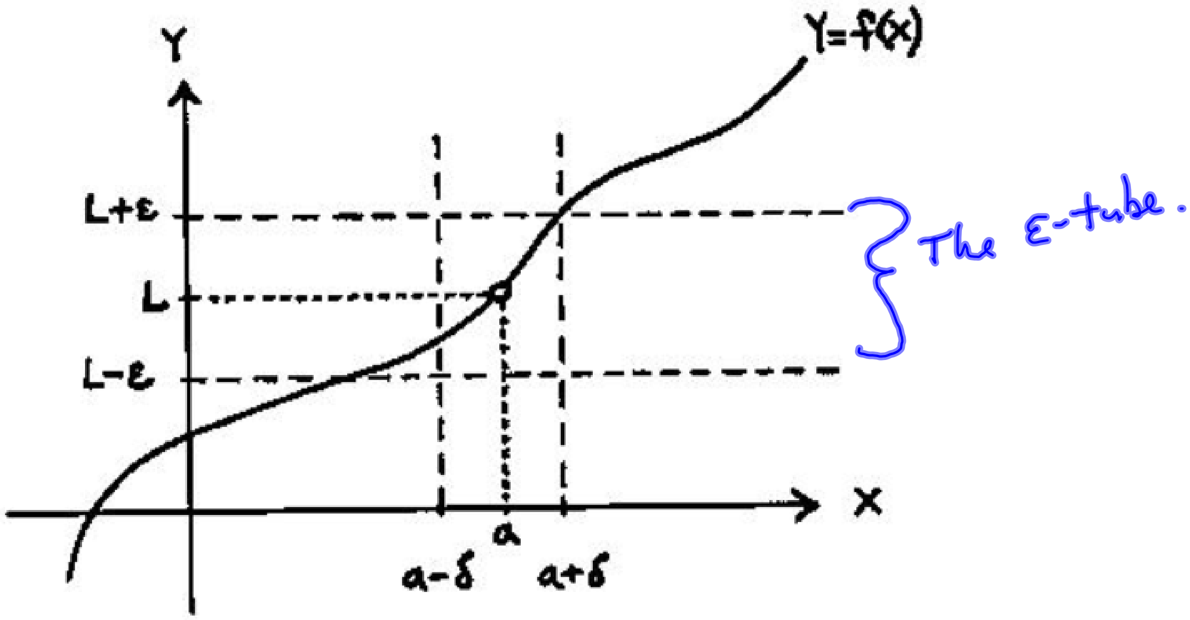
Pick the smaller value between

$5-4=1$

$5.7-5=.7 = \delta$

Now, if $\delta = .7$ Then anytime $0<|x-5|<\delta$, we'll have

$|f(x)-3| < .6 = \varepsilon$


Y=f(x)
Y
L+ε
L
L-ε
The ε-tube.
X
a
a-δ
a+δ

2.4 THE PRECISE DEFINITION OF A LIMIT

> [2] **DEFINITION** Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then we say that the **limit of $f(x)$ as x approaches a is L**, and we write
>
> $$\lim_{x \to a} f(x) = L$$
>
> if $\overbrace{\text{for every}}^{\forall}$ number $\varepsilon > 0$ $\overbrace{\text{there is}}^{\exists}$ a number $\delta > 0$ $\overbrace{\text{such that}}^{\ni}$
>
> $$\text{if} \quad 0 < |x - a| < \delta \qquad \text{then} \qquad |f(x) - L| < \varepsilon$$

You tell me how close $f(x)$ needs to be to L by giving me the tolerance ε, and I'll tell you how close x needs to be to a by giving you the δ.

$\lim_{x \to a} f(x) = L$ means $\forall \varepsilon > 0 \ \exists \delta > 0 \ \ni$
$0 < |x-a| < \delta$
$\Longrightarrow$
$|f(x) - L| < \varepsilon$