Claim: $\lim_{x \to 2} (3x-5) = 1$ want to PROVE IT.

<u>Scratch</u> want

$|3x-5-1| < \varepsilon$

$|3x-6| < \varepsilon$

$3\underbrace{|x-2|}_{\delta} < 3\delta = \varepsilon$

$\delta = \frac{\varepsilon}{3}$

<u>PROOF</u>

Let $\varepsilon > 0$ be given. Define $\boxed{\delta = \frac{\varepsilon}{3}}$. Then if $0 < |x-2| < \delta$, we have

$|3x-5-1| = |3x-6|$

$= 3|x-2| < 3\delta = \cancel{3} \cdot \frac{\varepsilon}{\cancel{3}} = \varepsilon.$

Let $\varepsilon = 2$

$\delta = \frac{2}{3}$

want $|3x-5-1| < \varepsilon \iff$

$|3x-6| < 2 \iff$

$3|x-2| < 2 \iff$

$|x-2| < \frac{2}{3} = \delta \iff$

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$$\lim_{x \to 3} (x^2 + 2x - 4) = 11$$

$|x-3| < \delta$

Scratch:

Want $|x^2+2x-4-11| = |x^2+2x-15| < \varepsilon$

$|x+5||x-3| < \varepsilon$

$\delta$

Need a bound on $|x+5|$

Key assumption: $\boxed{\text{Assume } \delta \le 1}$

$|x-3| < \delta \le 1$

$|x-3| < 1$

$|x-3| < 1$

$-1 < x-3 < 1$

Tweak it to get $x+5$ out

$+3 \quad = \quad +3 = 3$

$2 < x < 4$

$2+5 < x+5 < 4+5$

$7 < x+5 < 9 \Longrightarrow$

$|x+5| < 9$

$\therefore \quad |x+5||x-3| < 9|x-3| < 9\delta \leq \varepsilon$

$\Longrightarrow \delta \leq \dfrac{\varepsilon}{9}$

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Proof Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{9}\}$. Then if $0 < |x-3| < \delta$, we have

$$|x^2+2x-4-11| = |x^2+2x-15|$$

$$= \underbrace{|x+5|}_{\leq 9}\underbrace{|x-3|}_{< \delta} < 9\delta \leq 9 \cdot \frac{\varepsilon}{9} = \varepsilon \quad \blacksquare$$

$\delta \leq \frac{\varepsilon}{9}$

$$\lim_{x \to c} f(x) = L \iff$$

$$\forall \varepsilon > 0 \ \exists \delta > 0 \ \ni$$

$$0 < |x-c| < \delta \implies |f(x)-L| < \varepsilon$$

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