§2.1

#1 : a compute the m_{sec}'s 1pt

b $m_{tan} \approx -33.3$ 1pt

#3 b $m_{tan} = .25$ @ $x = 1$ 1pt

c $y = .25(x-1) + \frac{1}{2}$ is fine 1pt

Note 1/4x and $\frac{1}{4x}$ and anything else unclear is going to cost you

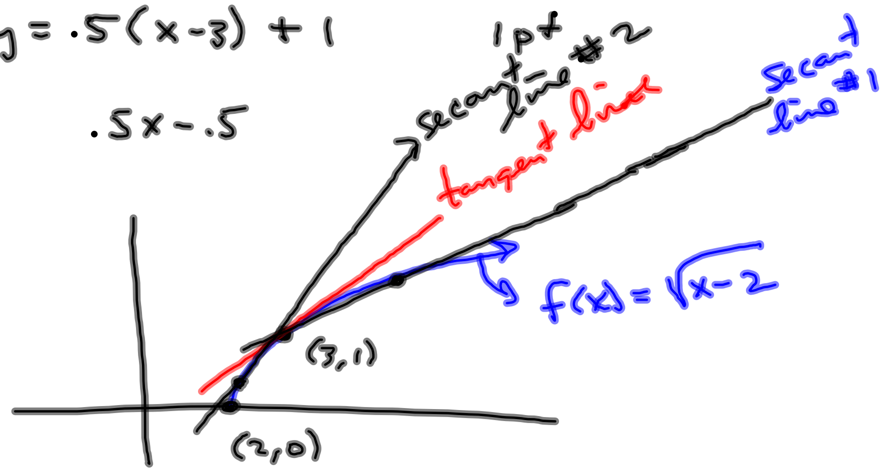
$\frac{1}{4}x + \frac{1}{4}$ was OK answer.

$\frac{1}{4}x$ $\frac{x}{4} + \frac{1}{4}$ $.25x + .25$

4c $y = .5(x-3) + 1$ 1pt

$.5x - .5$

d. 1pt



§2.1 Solns posted.

9. a They don't appear to be approaching a limit 1pt

8pts Plus 2pts for context.

$\frac{8}{10} + \frac{2}{10} = \frac{10}{10}$

c. $m \approx -31.42 \approx m_{tan}$, using $x = 1.00001$ was my 2^{nd} x-value.

Last time, we handed back 2.1 and the first page of yesterday's notes gives the points breakdown, for those who are interested.

Today, a couple announcements:

Section 2.3 II: OMIT #55

Section 2.4: Change the Assignment to the following:

#s 1, 11, 15, 16*, 29, 30, 41, 42 is bonus

* No picture for #16 is needed, but draw one for #15.

Our focus in 2.4 is getting the main idea, and seeing how it might come up in the real world, *and*, mechanically speaking, be able to do epsilon-delta proofs for LINEAR and QUADRATIC functions.

Standard limits that will come up repeatedly are of the form

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

for example #s 17, 23, 28, 30 (even though it might not look like it).

1
1 1
1 2 1
1 3 3 1

20. $\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$:

$$\frac{(2+h)^3 - 8}{h} =$$

$$\frac{1(2)^3(h)^0 + 3(2)^2(h) + 3(2)^1(h^2) + 1(2)^0(h)^3 - 8}{h}$$

$$= \frac{2^3 + 12h + 6h^2 + h^3 - 8}{h} = \frac{12h + 6h^2 + h^3}{h}$$

$$= \frac{\cancel{h}(12 + 6h + h^2)}{\cancel{h}} = 12 + 6h + h^2 \xrightarrow{h \to 0} \boxed{12}$$

22. $\lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h}$

$\boxed{2.3\,\mathrm{I}}$ Thursday

2 **THEOREM** If $f(x) \leq g(x)$ when x is near a (except possibly at a) and the limits of f and g both exist as x approaches a, then

$$\boxed{\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)}$$

The above is sometimes referred to as a "bounded convergence" result. One provides a ceiling for the other. This can be handy if you don't have a handle on f, but you *do* know that it's bounded by g and that g has a limit. This doesn't mean $f(x)$ we know what the limit *is* - only that its limit is no bigger than the limit of $g(x)$ as x approaches a.

3 **THE SQUEEZE THEOREM** If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

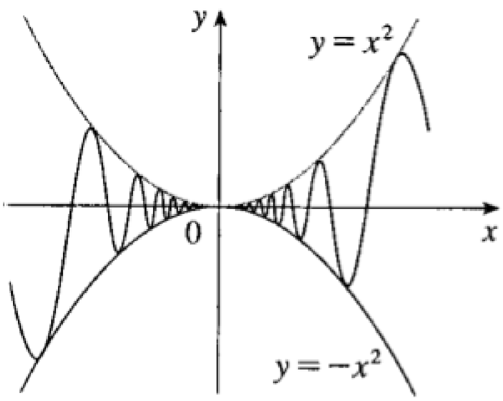
then

$$\lim_{x \to a} g(x) = L$$

Bounded above and below REALLY lets you SAY something about the limit as x approaches a of g.

What can be said about $\lim_{x\to 0} x^2 \sin\frac{1}{x}$

#33 is very similar to this example.



$-1 \le \sin(\frac{1}{x}) \le 1$

$-x^2 \le x^2 \sin(\frac{1}{x}) \le x^2$

So, $x^2 \sin(\frac{1}{x})$ lives between $-x^2$ & $+x^2$

Even though $x=0$ is bad, the following is true everywhere else!

$-x^2 \le x^2 \sin(\frac{1}{x}) \le x^2$

$\therefore \lim_{x\to 0}(-x^2) \le \lim_{x\to 0}(x^2 \sin(\frac{1}{x})) \le \lim_{x\to 0}(x^2)$

$0 \le \lim_{x\to 0}(x^2 \sin(\frac{1}{x})) \le 0$

$\therefore \boxed{\lim_{x\to 0}(x^2 \sin(\frac{1}{x})) = 0}$

We needed the limits to exist and to agree for this to work.

36. If $2x \leqslant g(x) \leqslant x^4 - x^2 + 2$ for all x, evaluate $\lim_{x \to 1} g(x)$.

$\lim_{x \to 1} (2x) = 2$

$\lim_{x \to 1} (x^4 - x^2 + 2) = 2$

$\therefore \lim_{x \to 1} g(x) = 2$, by the Squeeze Theorem.

39–44 Find the limit, if it exists. If the limit does not exist, explain why.

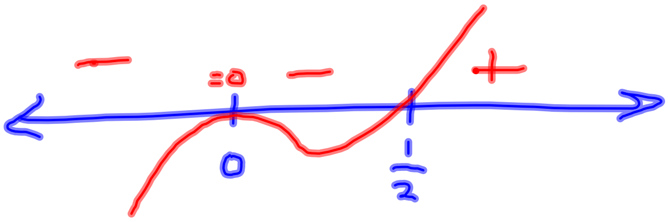
41. $\lim_{x \to 0.5^-} \dfrac{2x - 1}{|2x^3 - x^2|}$

The issue is with the | | thing

$$|2x^3 - x^2| = \begin{cases} 2x^3 - x^2 & \text{if } 2x^3 - x^2 \geq 0 \\ -(2x^3 - x^2) & \text{if } 2x^3 - x^2 < 0 \end{cases}$$

$2x^3 - x^2 \geq 0$

$x^2(2x-1) \geq 0$



$x^2 = 0$ $2x - 1 = 0$

$x = 0$ $x = \frac{1}{2}$

$$|2x^3 - x^2| = \begin{cases} 2x^3 - x^2 & \text{if } x \geq \frac{1}{2} \text{ OR } x = 0 \\ -(2x^3 - x^2) & \text{if } x < \frac{1}{2} \end{cases}$$

$$\therefore \lim_{x \to .5^-} \frac{2x-1}{|2x^3 - x^2|} = \lim_{x \to .5^-} \left(\frac{2x-1}{-(2x^3 - x^2)} \right)$$

$$= -\lim_{x \to .5^-} \left(\frac{\cancel{(2x-1)}}{x^2\cancel{(2x-1)}} \right) = -\lim_{x \to .5^-} \left(\frac{1}{x^2} \right)$$

$$= -\frac{1}{(.5)^2} = -\frac{1}{.25} = \boxed{-4}$$

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

See §2.3 #30.

Rationalize the Numerator.

$(a-b)(a+b) = a^2 - b^2$

$$\left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$

$$= \frac{\overset{a^2 \; - \; b^2}{x+h-x}}{h(\sqrt{x+h} + \sqrt{x})} = \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$\xrightarrow{h \to 0} \frac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$