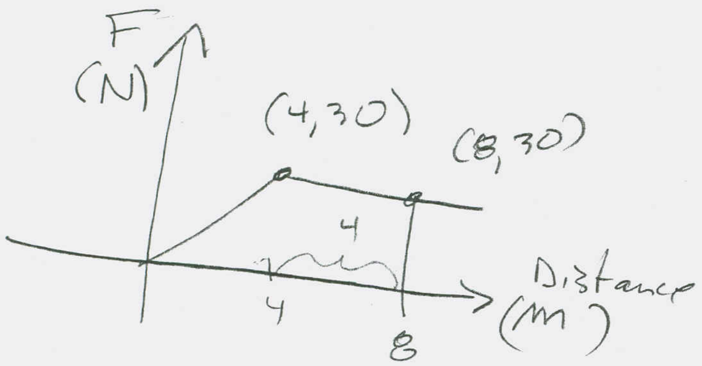
201 §6.4 #s 2, 5, 8, 11, (13), 14, 17, 22, 30*

2 Find the work done if a constant force of 100 lbs is used to pull a cart a distance of 200 ft

$(200)(100) = \boxed{20000 \text{ ft-lbs}}$

5 Shown is the graph of a force function (in N) that increases to its max & then remains constant. How much work is done by moving an object a distance of 8 m?



$(0,0), (4,30)$

$m = \frac{30}{4} = 7.5$

$$\text{work} = \int_0^4 7.5x\,dx + 30(4) =$$

$$\left[\frac{7.5}{2}x^2\right]_0^4 + 120 = \frac{7.5}{2}(4)^2 + 120 = \left(\frac{7.5}{2}\right)(16) + 120$$

$$= (7.5)(8) + 120 = \frac{15}{2}(8) + 120 = 15(4) + 120$$

$$= \boxed{180 \text{ J}} \quad (\text{N-m})$$

201 §6.4 #s 8, 11, (13), 14, 17, 22, 30*

(8) A spring has a natural length of 20 cm. If a 25 N force is required to keep it stretched to 30 cm, how much work is required to stretch it from 20 cm to 25 cm?

$F = kx$ $\qquad$ $30 - 20 = 10$ cm

$= (10 \text{ cm})\left(\frac{1 \text{ m}}{100 \text{ cm}}\right) = .1 \text{ m}$

$F(.1) = .1k = 25$

$\Rightarrow k = \frac{25}{.1} = 250 \frac{\text{kg}}{s^2}$

So $F(x) = 250x$

Now, for the work: From 20 to 25 m is

from $x=0$ to $x=5$:

$$\int_0^5 F(x)\,dx = \int_0^5 250x\,dx = 250\left[\frac{1}{2}x^2\right]_0^5$$

$$= 125(25) = \boxed{3125 \text{ J}}$$

(11) A spring has a natural length of 20 cm. Compare the work, W_1, done in stretching the spring from 20 to 30 cm, with the work, W_2, done in stretching it from 30 to 40 cm. How are W_1 & W_2 related?

201 §6.4 #s 11, (13), 14, 17, 22, 30*

11 cntd Do our work in Dynes $\frac{g\text{-}cm}{s^2}$, if memory serves

20 to 30: $\int_0^{10} kx\,dx = \frac{1}{2}k(10)^2 = 50k = W_1$

30 to 40 $\int_{10}^{20} kx\,dx = \frac{1}{2}k(20)^2 - \frac{1}{2}k(10)^2$

$= 200k - 50k = 150k = W_2$

The 2^{nd} stretch, from 30 to 40 cm, takes 3 times as much work. Same distance, but the force is increasing.

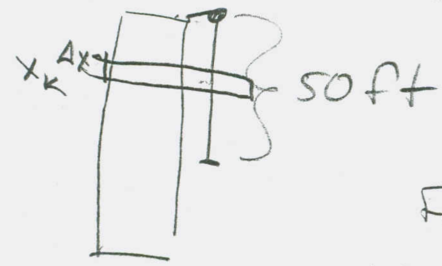
#s 13-20 Approx. the work with a Riemann Sum. Then express it as an integral & evaluate.

13 A heavy rope, 50 ft long, weighs .5 lb/ft and hangs over the edge of a building 120 ft high.

(a) How much work is done lifting the rope to the top of the building?

(b) How much to pull it halfway up?

201 §6.4 #s (13), 14, 17, 22, 30 *



13 cont'd

Force on the rope is

$$(50\text{ ft})\left(.5\ \frac{\text{lb}}{\text{ft}}\right) - .5x_K,$$ where x_K is how far you've drawn up the end of the rope.

(a) $$F = \sum (25 - .5x_K)\Delta x \approx \int_0^{50} (25 - .5x)\,dx$$

$$= \left[25x - \frac{1}{2}\cdot\frac{1}{2}x^2\right]_0^{50} = 25(50) - \frac{1}{4}(50)^2 = 1250 - \frac{2500}{4}$$

$$= 1250 - 625 = 625 \text{ ft-lbs}$$

(b) Halfway up: $$\int_0^{25} \left(25x - \frac{1}{4}x^2\right)dx = 25(25) - \frac{625^2}{4}$$

$$= \frac{2500 - 625}{4} = \frac{1875}{4} = 468.75 \text{ ft-lbs.}$$

Much more than half of part (a), because the rope is heaviest @ the start.

(14) A chain lying on the ground is 10 m long and its mass is 80 kg. How much work does it take to lift one end of the chain 5 m above the ground?

201 §6.4 #s 14, 17, 22, 30

14 cnt'd

x_k = distance it's been lifted. $\frac{80 \text{ Kg}}{10 \text{ m}} = 8 \frac{\text{kg}}{\text{m}}$

$$\text{Mass} = (8 \text{ kg/m})(x_k \text{ m}) = 8x_k \text{ Kg}$$

$$\text{Force} = (8x_k \text{ kg})\left(9.8 \frac{\text{m}}{\text{s}^2}\right)$$

$$\text{Work} = \left((8)(9.8)\, x_k \frac{\text{kg-m}}{\text{s}^2}\right)(\Delta x \text{ m})$$

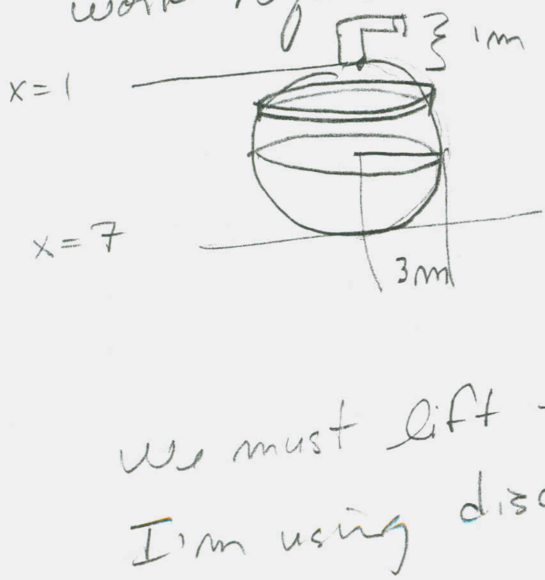
$$\approx \sum (8)(9.8)\, x_k \Delta x \approx \int_0^5 8(9.8)\, x\, dx = 8(9.8)\left[\tfrac{1}{2}x^2\right]_0^5$$

$$= 4(9.8)[25] = 100(9.8) = \boxed{980 \text{ J}}$$

17 Done in class

22 A tank is full of water. Find the work required to pump the tank dry. Water's density is

$$\frac{1000 \text{ Kg}}{\text{m}^3}$$



x = distance water is pumped. starting at the bottom, x = 6+1 = 7m.

We must lift the water out of the tank. I'm using discs that are dx in thickness

201 §6.4 #s 22, 30*

22 cont'd

Volume of one "disc of water" is

$\pi\,(\text{radius of the disk})^2 \Delta x$ and we lift it x m

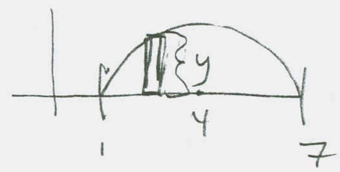
x runs from 1 m to 7 m.

@ $x = 1$ m, the radius is zero.

@ $x = 7$ m, " " " "

Circle of radius 3, with center @ $x = 4$

$(x-4)^2 + y^2 = r^2$, where y is going to give us our radius for a given x.



$$(x-4)^2 + y^2 = 3^2$$

$$y^2 = 3^2 - (x-4)^2$$

$$y = \pm\sqrt{9 - (x-4)^2}.$$

$$\pi \int_1^7 \left(\sqrt{9-(x-4)^2}\right) x\,dx$$ No. Units?

$$\frac{1000\,\text{kg}}{\text{m}^3} \cdot \pi \int_1^7 \underbrace{\left(\sqrt{9-(x-4)^2}\right)^2}_{\text{m}^2} \underbrace{x}_{\text{m}}\,\underbrace{dx}_{\text{m}} \cdot 9.8\,\frac{\text{m}}{\text{s}^2}$$ will be in

$$\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2}\cdot\text{m} = \text{Joules}$$ ✓

$$9800\pi \int_1^7 \left(9-(x-4)^2\right) x\,dx = 141{,}200\pi \text{ J}$$

$$\approx 4.43415553 \times 10^6 \text{ J}$$

(over 4 million J)

201 §6.4 #30*

(30) How much work is involved in lifting a 1000-kg satellite to 1000 km? Assume Earth's mass is 5.98×10^{24} kg, concentrated @ its center. Take the radius of the Earth to be 6.37×10^{6} m, and

$$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$$

$$F = G \frac{m_1 m_2}{r^2}$$, and r varies between

6.37×10^6 m and 7.37×10^6 m

(1000 km $= 1 \times 10^6$ m)

$$\int_{6.37 \times 10^6}^{7.37 \times 10^6} \left(6.67 \times 10^{-11} \frac{kg \cdot m^3}{kg^2}\right)\left(\frac{(1000)(5.98 \times 10^{24})}{x^2}\right) dx$$

$$\approx 8.496109434 \times 10^{9} \text{ J}$$

($8\frac{1}{2}$ billion Joules)

This ain't quite rocket science, since we're not worried about maintaining orbit, which involves ΔV.