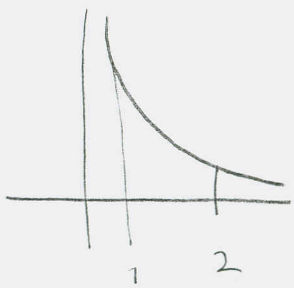
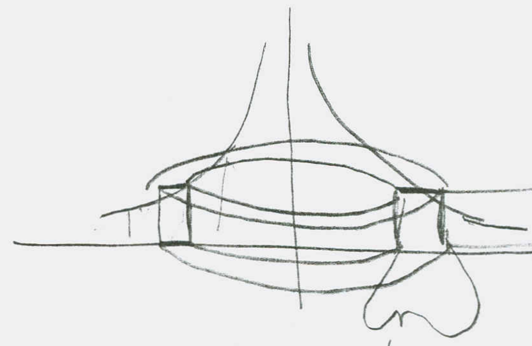
201 §6.3 #s 3, 6, 9, 12, 19, 37, 38, 43

#s 3–7 Use shell method to find the volume generated by rotating the region bdd by the given curves about the y-axis. Sketch the region and a typical shell.

3) $y = \frac{1}{x}$, $y=0$, $x=1$, $x=2$





$2\pi r h \Delta x$

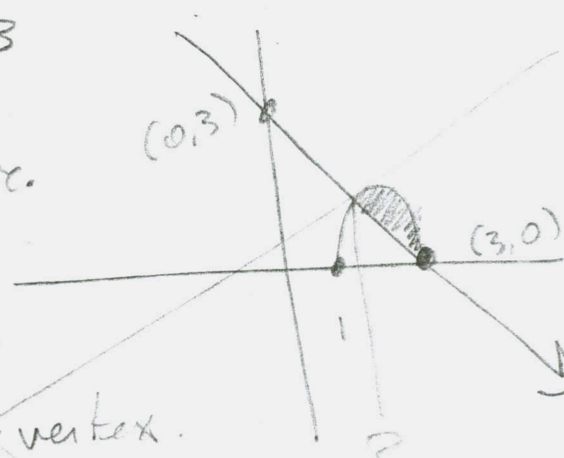
$$2\pi \int_1^2 x \cdot \frac{1}{x}\,dx = 2\pi \int_1^2 dx = 2\pi \Big[x\Big]_1^2 = 2\pi\big[2-1\big] = \boxed{2\pi}$$

6) $y = -x^2+2x+3$, $x+y=3$

$= -(x^2-2x-3)$

$= -(x-3)(x+1)$

oops! wrong pic.



$\frac{3+1}{2} = 2$

$y(2) = (2-3)(2+1)$

$= (-1)(3) = 3$ (2,3) is vertex.

So it's ABOVE $x+y=3$

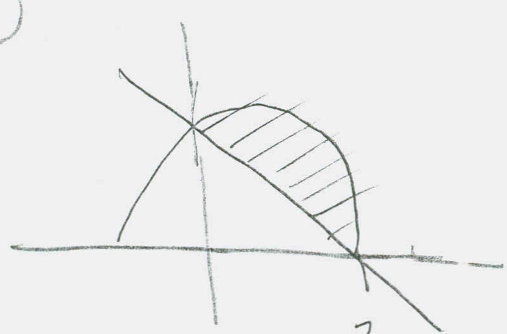
Need to find "?"

parabola $= -x^2+2x+3 = -x+3 =$ line

$-x^2+3x = 0$

201 §6.3 #s 6, 9, 12, 19, 37, 38, 43

6



Vertex: $-(x-3)(x+1)$

zeros: $x=3,-1$

$\frac{3+(-1)}{2} = \frac{2}{2} = 1$

@ $x=1$:

$-(1-3)(1+1) = -(-2)(2)$
$= 4$

$y = -x^2+2x+3 = -x+3 = y$

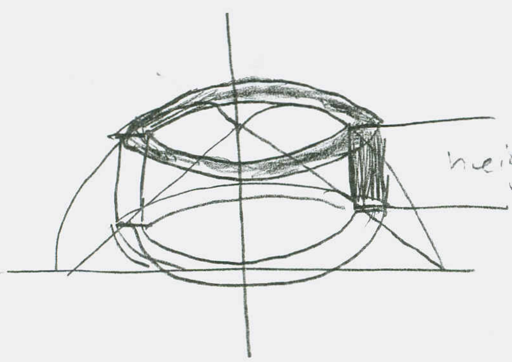
$-x^2+3x = 0$

$-x(x-3) = 0$

$x = 0, 3$ are x-coords of intersection.

Much better.

$2\pi r h \Delta x$



height $= -x^2+2x+3-(-x+3) = h$

$= -x^2+3x$

$2\pi \int_0^3 x(-x^2+3x)\,dx$

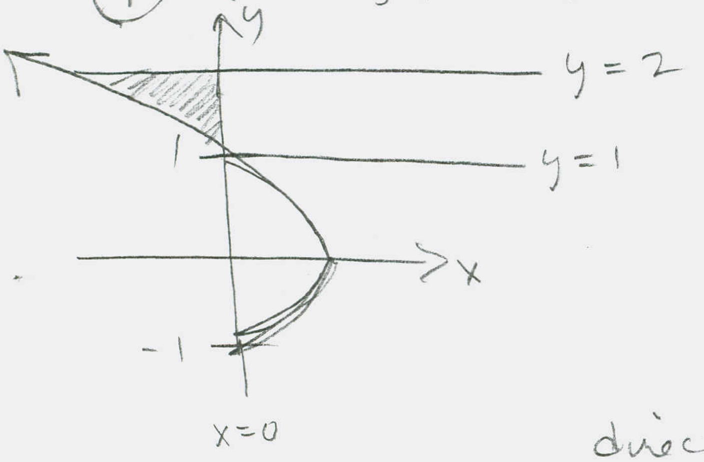
$= 2\pi \int_0^3 (-x^3+3x^2)\,dx = 2\pi \left[-\frac{1}{4}x^4+x^3\right]_0^3 = 2\pi\left[-\frac{1}{4}(3)^4+3^3\right]$

$= \left[-\frac{81}{4}+27\right](2\pi) = 2\pi\left[\frac{108-81}{4}\right] = \pi\left[\frac{27}{2}\right] = \boxed{\frac{27\pi}{2}}$

201 §6.3 #s 9, 12, 19, 37, 38, 43

#s 9-14 Use shell method to find the volume of the solid of revolution obtained by rotating the region bdd by the given curves about the x-axis. Sketch the region and a typical shell.

(9) $x = 1+y^2$, $x=0$, $y=1$, $y=2$

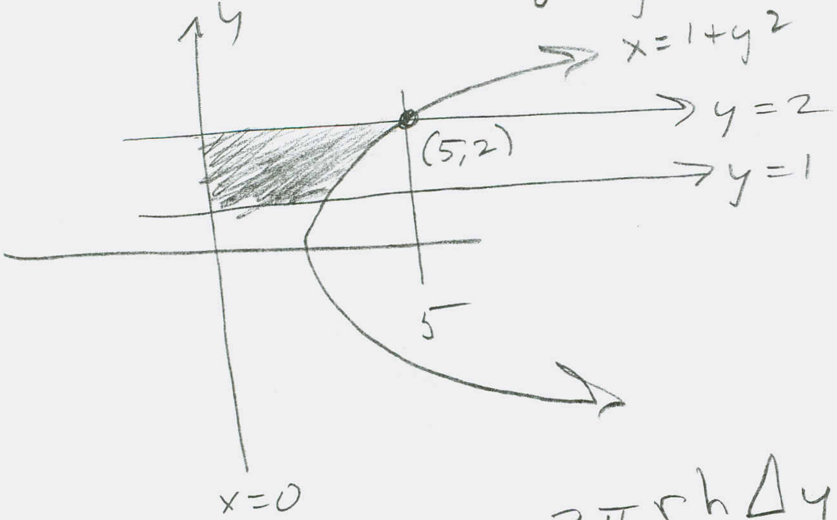


$x = 1+y^2$

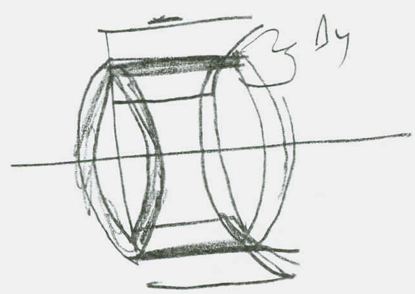
let $y = 2$:

$x = 1+2^2 = 5$

I drew the parabola opening in the wrong direction, and didn't notice until I found $x=5$ when solving for the intersection of $y=2$ & $x=1+y^2$!



$$2\pi \int_1^2 y(1+y^2)\,dy$$



$2\pi r h \Delta y$

$2\pi y(1+y^2)\Delta y$

201 §6.3 #s 9, 12, 19, 37, 38, 43

9 cnt'd

$$= 2\pi \int_1^2 (y+y^3)\,dy = 2\pi\left[\frac{1}{2}y^2 + \frac{1}{4}y^4\right]_1^2$$

$$= 2\pi\left[\frac{1}{2}(2)^2 + \frac{1}{4}(2)^4 - \left(\frac{1}{2}(1)^2 + \frac{1}{4}(1)^4\right)\right]$$

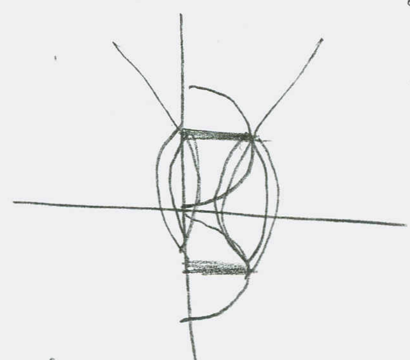
$$= 2\pi\left[2 + 4 - \left(\frac{1}{2} + \frac{1}{4}\right)\right] = 2\pi\left[6 - \frac{3}{4}\right] = \boxed{\frac{21\pi}{2}}$$

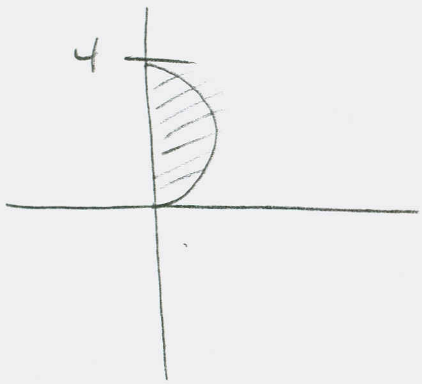
12 $x = 4y^2 - y^3$, $x = 0$

$= y^2(4-y)$

$2\pi r h \Delta y$

$2\pi y(4y^2 - y^3)\Delta y$





$$V = 2\pi \int_0^4 (4y^3 - y^4)\,dy = 2\pi\left[y^4 - \frac{1}{5}y^5\right]_0^4$$

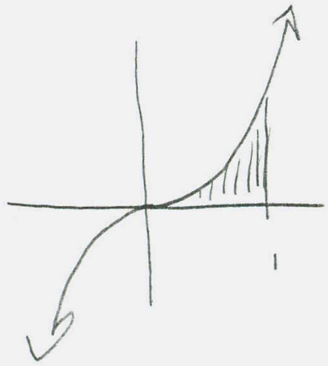
$$= 2\pi\left[4^4 - \frac{1}{5}(4)^5 - 0\right] = 2\pi\left[256 - \frac{1024}{5}\right] =$$

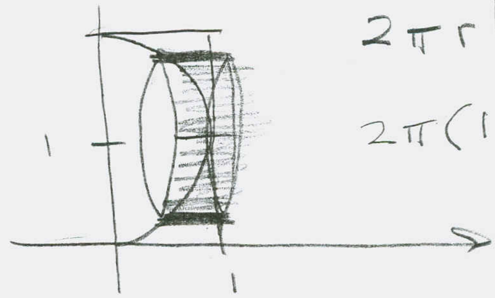
$$= 2\pi\left[\frac{1280 - 1024}{5}\right] = 2\pi\left[\frac{256}{5}\right] = \boxed{\frac{512\pi}{5}}$$

#s 15-20 Use cylindrical shells to find the volume generated by revolving the region bdd by the given curves about the specified axis. Sketch the region and a representative shell.

201 §6.3 #s 19, 37, 38, 43

19 $y = x^3$, $y = 0$, $x = 1$; about $y = 1$





$2\pi r h \Delta y$

$2\pi(1-y)(1-x)\Delta y$

But $x = \sqrt[3]{y} = y^{\frac{1}{3}}$

$$V = 2\pi \int_0^1 (1-y)(1-y^{\frac{1}{3}})\,dy = 2\pi \int_0^1 \left(1 - y^{\frac{1}{3}} - y + y^{\frac{4}{3}}\right)dy$$

$$= 2\pi \left[y - \frac{3}{4} y^{\frac{4}{3}} - \frac{1}{2} y^2 + \frac{3}{7} y^{\frac{7}{3}} \right]_0^1$$

$$= 2\pi \left[1 - \frac{3}{4} - \frac{1}{2} + \frac{3}{7} \right] = 2\pi \left[\frac{28 - 21 - 14 + 12}{28} \right]$$

$$= 2\pi \left[\frac{5}{28} \right] = \boxed{\frac{5\pi}{14}}$$

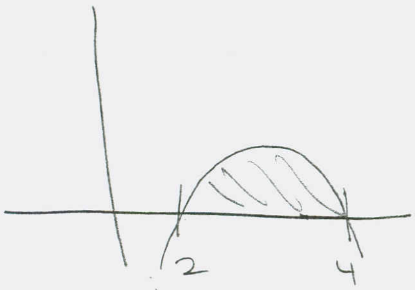
#s 37-42 The region bdd by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

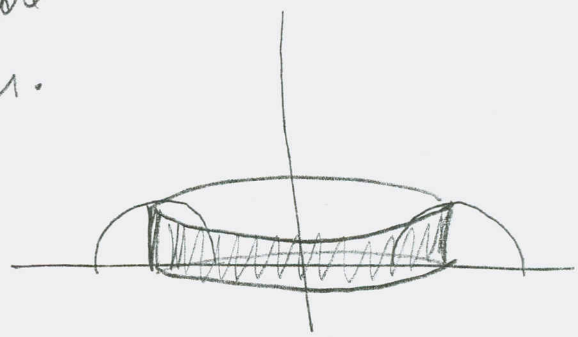
37 $y = -x^2 + 6x - 8$, $y = 0$, about the y-axis

201 §6.3 #s 37, 38, 43

37 cont'd

Shells are easier.





$-x^2+6x-8=0$

$x^2-6x+8=0$

$(x-2)(x-4)=0$

$V=2\pi r h \Delta x$

$=2\pi x(-x^2+6x-8)\Delta x$

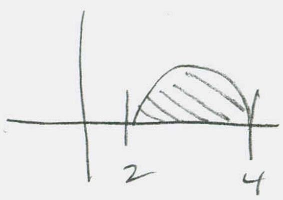
$$V=-2\pi\int_2^4 (x^3-6x^2+8x)\,dx = -2\pi\left[\frac{1}{4}x^4-2x^3+4x^2\right]_2^4$$

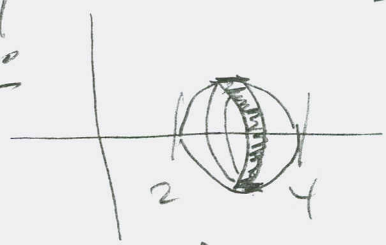
$$=-2\pi\left[\frac{256}{4}-2(64)+4(16)-\left(\frac{16}{4}-2(8)+4(4)\right)\right]$$

$$=-2\pi\left[64-128+64-(4-16+16)\right] = -2\pi[-4] = \boxed{8\pi}$$

38 $y=-x^2+6x-8$, $y=0$, about the x-axis

Discs!





$\pi(-x^2+6x-8)^2\Delta x$

$=\pi(x^2-6x+8)^2\Delta x$

$=\pi$

$(x^2-6x+8)(x^2-6x+8)$

$= x^4-6x^3+8x^2$

$-6x^3+36x^2-48x$

$+8x^2-48x+64$

$x^4-12x^3+52x^2-96x+64$

$$V=\pi\int_2^4 (x^4-12x^3+52x^2-96x+64)\,dx$$

201 $\S 6.3$ #s 38, 43

38 cnt'd

$$= \pi \int_2^4 (x^4 - 12x^3 + 52x^2 - 96x + 64)\, dx$$

$$= \pi \left[\frac{1}{5}x^5 - 3x^4 + \frac{52}{3}x^3 - 48x^2 + 64x \right]_2^4$$

$\frac{5600}{2} = 2800$

2800
+ 108

56
52

720
−128
592

$$= \pi \left[\frac{1}{5}(4)^5 - 3(4)^4 + \frac{52}{3}(4)^3 - 48(4)^2 + 64(4) - \left(\frac{1}{5}(2)^5 - 3(2)^4 + \frac{52}{3}(2)^3 - 48(2)^2 + 64(2) \right) \right]$$

$$= \pi \left[\frac{1}{5}(1024 - 32) - 3(256 - 16) + \frac{52}{3}(64 - 8) - 48(16 - 4) + 64(4 - 2) \right]$$

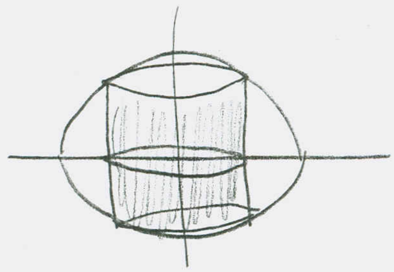
$$= \pi \left[\frac{992}{5} - 720 + \frac{2912}{3} - 576 + 128 \right]$$

$$= \pi \left[\frac{992}{5} + \frac{2912}{3} - 1168 \right] = \pi \left[\frac{2976 + 14560 - 17520}{15} \right]$$

$$= \boxed{\frac{16\pi}{15}}$$

201 S'6.3 #43

(43) Use shells to find volume of a sphere of radius r.



We can take the circle & rotate it around the y-axis.

Top half of the circle is

$y = \sqrt{r^2-x^2}$

BOTTOM $\frac{1}{2}$ is $y = -\sqrt{r^2-x^2}$

$2\pi r h \Delta x$

$$2\pi\int_0^r x\left(\sqrt{r^2-x^2} - \left(-\sqrt{r^2-x^2}\right)\right)dx$$

Use symmetry: Just double the top half:

$$4\pi\int_0^r x\sqrt{r^2-x^2}\,dx$$

$$4\pi\int_0^r x\sqrt{r^2-x^2}\,dx = -2\pi\int_0^r \sqrt{r^2-x^2}\,(-2x\,dx)$$

$u = r^2-x^2 \qquad du = -2x\,dx$

$x=0 \Rightarrow u = r^2, \quad x=r \Rightarrow u = r^2-r^2 = 0$

$$= -2\pi\int_{r^2}^0 u^{\frac{1}{2}}du = 2\pi\int_0^{r^2} u^{\frac{1}{2}}du = 2\pi\left[\frac{2}{3}u^{\frac{3}{2}}\right]_0^{r^2}$$

$$= \frac{4\pi}{3}\left[\left(r^2\right)^{\frac{3}{2}}\right] = \boxed{\frac{4}{3}\pi r^3}$$