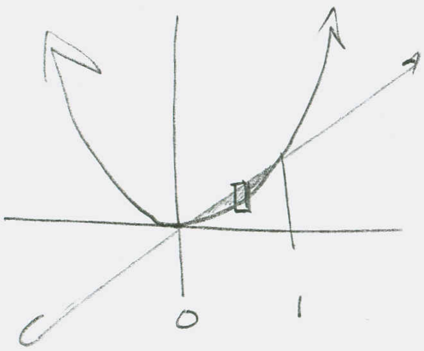
201 §6.1 #S 7, 10, 14, 17, 25, 32, 35, 49

7 $y = x \quad y = x^2$



$x^2 = x \Rightarrow$

$x^2 - x = 0 \Rightarrow$

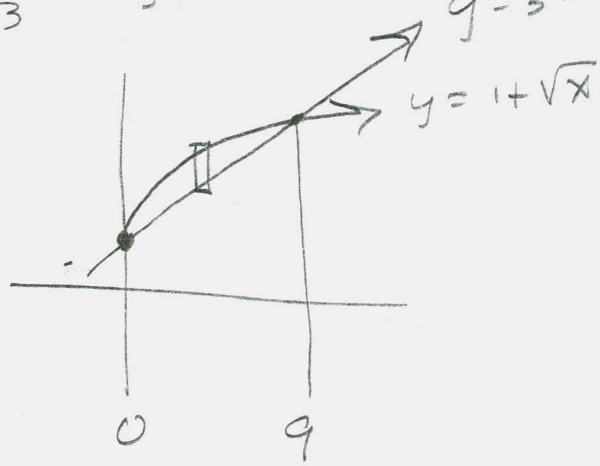
$x(x-1) = 0 \Rightarrow$

$x = 0$ OR $x = 1$

$$\text{Area} = \int_0^1 (x - x^2)\,dx = \left[\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right]_0^1$$

$$= \tfrac{1}{2} - \tfrac{1}{3} - [0 - 0] = \frac{3-2}{6} = \boxed{\tfrac{1}{6} = \text{Area}}$$

10 $y = 1 + \sqrt{x}, \quad y = \dfrac{3+x}{3} = \tfrac{1}{3}x + 1$




$1 + \sqrt{x} = \dfrac{3+x}{3}$

$3 + 3\sqrt{x} = 3 + x$

$3\sqrt{x} = x$

$9x = x^2$

$x^2 - 9x = 0$

$x(x-9) = 0$

$x = 0$ OR $x = 9$

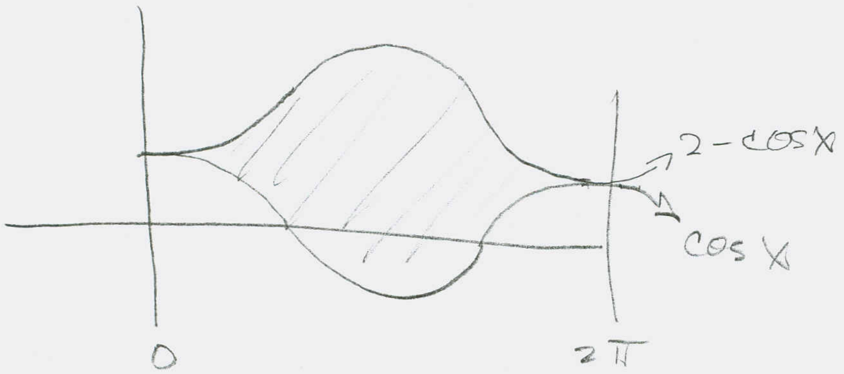
$$\text{Area} = \int_0^9 \left(1 + \sqrt{x} - \left(\tfrac{1}{3}x + 1\right)\right)dx$$

$$= \int_0^9 \left(\sqrt{x} - \tfrac{1}{3}x\right)dx = \left[\tfrac{2}{3}x^{\frac{3}{2}} - \tfrac{1}{6}x^2\right]_0^9 = \tfrac{2}{3}(9)^{\frac{3}{2}} - \tfrac{1}{6}(9)^2 - 0$$

$$= \tfrac{2}{3}(3)^3 - \frac{81}{6} = 2(3)^2 - \frac{27}{2} = 18 - \frac{27}{2} = \frac{36-27}{2} = \boxed{\tfrac{9}{2}}$$

201 §6.1 #s 14, 17, 25, 32, 35, 49

(14) $y = \cos x$, $y = 2 - \cos x$, $0 \le x \le 2\pi$




( $\cos x \to -\cos x$
$\downarrow$
$-\cos x + 2$

$$\text{Area} = \int_0^{2\pi} (2 - \cos x - \cos x)\,dx = \int_0^{2\pi} (2 - 2\cos x)\,dx$$

$$= 2\int_0^{2\pi} (1 - \cos x)\,dx = 2\Big[x - \sin x\Big]_0^{2\pi}$$

$$= 2\Big[2\pi - \sin(2\pi) - (0 - \sin(0))\Big] = \boxed{4\pi}$$

(17) $y = \sqrt{x}$, $y = \frac{1}{2}x$, $x = 9$

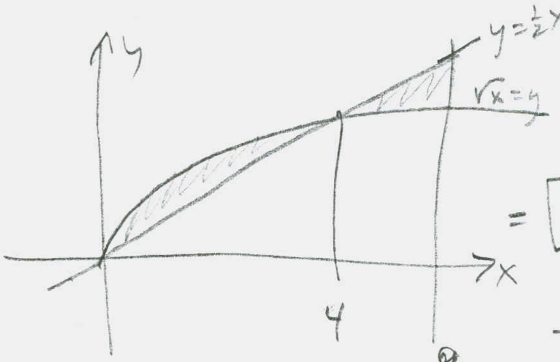
$\frac{1}{2}x = \sqrt{x}$

$\frac{1}{4}x^2 = x$

$\frac{1}{4}x^2 - x = 0$

$x^2 - 4x = 0$

$x(x-4) = 0$




$$\text{Area} = \int_0^4 \left(\sqrt{x} - \tfrac{1}{2}x\right)dx + \int_4^9 \left(\tfrac{1}{2}x - \sqrt{x}\right)dx$$

$$= \left[\tfrac{2}{3}x^{\frac{3}{2}} - \tfrac{1}{4}x^2\right]_0^4 + \left[\tfrac{1}{4}x^2 - \tfrac{2}{3}x^{\frac{3}{2}}\right]_4^9$$

$$= \left[\tfrac{2}{3}(4)^{\frac{3}{2}} - \tfrac{1}{4}(4)^2 - \left(\tfrac{2}{3}(0)^{\frac{3}{2}} - \tfrac{1}{4}(0)^2\right)\right]$$

$$+ \left[\tfrac{1}{4}(9)^2 - \tfrac{2}{3}(9)^{\frac{3}{2}} - \left(\tfrac{1}{4}(4)^2 - \tfrac{2}{3}(4)^{\frac{3}{2}}\right)\right] =$$

201 §6.1 #s 17, 25, 32, 35, 49

17 cnt'd $= \left[\frac{2}{3}(2)^3 - \frac{1}{4}(16)\right] + \left[\frac{1}{4}(81) - \frac{2}{3}(3)^3 - \left(\frac{1}{4}(16) - \frac{2}{3}(2)^3\right)\right]$

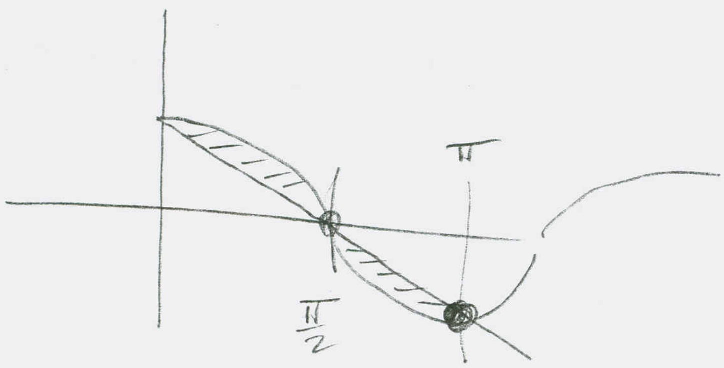
$= \frac{2}{3}(8) - 4 + \frac{81}{4} - \frac{2}{3}(27) - \left(4 - \frac{2}{3}(8)\right)$

$= \frac{16}{3} - 4 + \frac{81}{4} - 18 - \left(4 - \frac{16}{3}\right)$

$= \frac{16}{3} - 22 + \frac{81}{4} - 4 + \frac{16}{3} = \frac{32}{3} - 26 + \frac{81}{4} = \frac{128 - 312 + 243}{12}$

$= \frac{371 - 312}{12} = \boxed{\frac{59}{12} = 4.91\overline{6}}$

25 $y = \cos x \qquad y = 1 - \frac{2x}{\pi} = -\frac{2}{\pi}x + 1$




$\cos x = -\frac{2}{\pi}x + 1$

$\cos x + \frac{2}{\pi}x - 1 = 0$

This is one you can do by hand, by happy coincidence!

Same as cos(x) $\begin{cases} -\frac{2}{\pi}x + 1 = -1 \\ -\frac{2}{\pi}x = -2 \\ x = \pi \end{cases}$

$-\frac{2}{\pi}x + 1 = 0$

$\frac{2}{\pi}x = 1$

$x = \frac{\pi}{2}$ → Same as cos(x).

$\text{Area} = \int_0^{\frac{\pi}{2}} \left(\cos(x) - \left(1 - \frac{2}{\pi}x\right)\right)dx$

$+ \int_{\frac{\pi}{2}}^{\pi} \left(1 - \frac{2}{\pi}x - \cos(x)\right)dx = \left[\sin(x) - x + \frac{2}{\pi}\left(\frac{x^2}{2}\right)\right]_0^{\frac{\pi}{2}}$

$+ \left[x - \frac{2}{\pi}\left(\frac{x^2}{2}\right) - \sin(x)\right]_{\frac{\pi}{2}}^{\pi} = \left[\sin(x) - x + \frac{1}{\pi}x^2\right]_0^{\frac{\pi}{2}} +$

201 §6.1 #s 25, 35, 49

25 cnt'd $+\left[x - \frac{1}{\pi}x^2 - \sin(x)\right]_{\frac{\pi}{2}}^{\pi}$

$$= \left[\sin\left(\tfrac{\pi}{2}\right) - \tfrac{\pi}{2} + \tfrac{1}{\pi}\left(\tfrac{\pi}{2}\right)^2 - \left(\sin(0) - 0 + \tfrac{1}{\pi}(0)^2\right)\right]$$

$$+ \left[\pi - \tfrac{1}{\pi}(\pi)^2 - \sin(\pi) - \left(\tfrac{\pi}{2} - \tfrac{1}{\pi}\left(\tfrac{\pi}{2}\right)^2 - \sin\left(\tfrac{\pi}{2}\right)\right)\right]$$

$$= \left[1 - \tfrac{\pi}{2} + \tfrac{1}{\pi}\cdot\tfrac{\pi^2}{4} - (0 - 0 + 0)\right]$$

$$+ \left[\pi - \tfrac{1}{\pi}\cdot\tfrac{\pi^2}{1} - 0 - \left(\tfrac{\pi}{2} - \tfrac{1}{\pi}\cdot\tfrac{\pi^2}{4} - 1\right)\right]$$

$$= 1 - \tfrac{\pi}{2} + \tfrac{\pi}{4} + \pi - \pi - \tfrac{\pi}{2} + \tfrac{\pi}{4} + 1$$

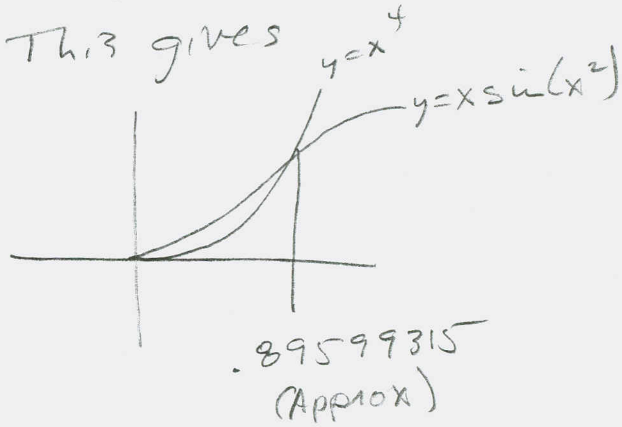
$$= 2 - \tfrac{2\pi}{2} + \tfrac{2\pi}{4} = 2 - \pi + \tfrac{\pi}{2} = \boxed{2 - \tfrac{\pi}{2} \approx .4292036732}$$

35 $y = x\sin(x^2)$, $y = x^4$

I got this wrong in class

$x$ is odd. $\sin(x)$ is odd, but $\sin(x^2)$ is even, because $x^2$ is even. This gives

$x\sin(x^2)$ is ODD




$$\text{Area} \approx \int_0^{.89599315} \left(x\sin(x^2) - x^4\right) dx$$

$$= \frac{1}{2}\int_0^{.89599315} \sin(x^2)\cdot 2x\,dx - \int_0^{.89599315} x^4\,dx$$

$$= \frac{1}{2}\left[-\cos(x^2)\right]_0^{.89599315} - \left[\frac{1}{5}x^5\right]_0^{.89599315}$$

201 §6.1 #s 35, 49

35 cont'd

$$= \frac{1}{2}\left[-\cos\left((.89599315)^2\right) - \left(-\cos(0)\right)\right]$$

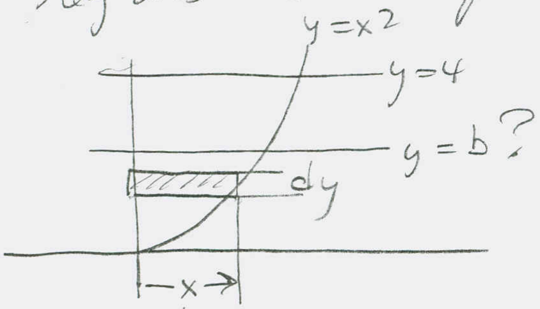
$$- \left[\frac{1}{5}(.89599315)^5 - \frac{1}{5}(0)\right]$$

$$\approx \frac{1}{2}\left[-.6946927045 + 1\right] - \left[\frac{1}{5}(.5774620485)\right]$$

$$\approx .1526536478 - .1154924097 \approx .0371612381$$

$$\approx \boxed{.037161}$$

49 Find the # b $\ni$ $y=b$ divides the region bdd by $y=x^2$ & $y=4$ into two regions with equal area




we integrate vertically, that is, with respect to y, and try to find b.

$x = \sqrt{y}$, since $y=x^2$, So, one way to find b is to solve the following for b:

$$\int_0^4 \sqrt{y}\,dy = 2\int_0^b \sqrt{y}\,dy$$

$$\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^4 = 2\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^b \Longrightarrow \frac{2}{3}\left[4^{\frac{3}{2}}\right] = 2\left(\frac{2}{3}\left(b^{\frac{3}{2}}\right)\right)$$

$$\Longrightarrow 8 = 2b^{\frac{3}{2}} \Longrightarrow 4 = b^{\frac{3}{2}} \Longrightarrow \boxed{b = 4^{\frac{2}{3}}}$$