201 §5.5 #s 1, 4, 7, 10, 13, 16, 19, 22, 24, 27, 30, 36, 39, 42, 47

#s 1-6 Evaluate the integral by making the given substitution.

(1) $\int \cos(3x)\,dx$, $u = 3x \Rightarrow du = 3dx \Rightarrow dx = \frac{du}{3}$

This gives $\int \cos(u) \cdot \frac{du}{3} = \frac{1}{3}\int \cos(u)\,du = \frac{1}{3}\sin(u) + C$

$= \boxed{\frac{1}{3}\sin(3x) + C}$

(4) $\int \frac{dt}{(1-6t)^2}$, $u = 1-6t \Rightarrow du = -6dt$, and so, $dt = -\frac{du}{6}$

we have $\int \frac{-\frac{du}{6}}{u^2} = -\frac{1}{6}\int u^{-2}\,du = -\frac{1}{6}\left(\frac{1}{-1}u^{-1}\right) + C$

$= \frac{1}{6}u^{-1} + C = \boxed{\frac{1}{6(1-6t)} + C}$

#s 7-30 Evaluate the indefinite integral

(7) $\int x\sin(x^2)\,dx = \int x\sin(u)\,\frac{du}{2x} = \frac{1}{2}\int \sin(u)\,du$

$\left(u = x^2 \Rightarrow du = 2x\,dx \Rightarrow \frac{du}{2x} = dx\right)$

$= \frac{1}{2}(-\cos(u)) + C = \boxed{-\frac{1}{2}\cos(x^2) + C}$

(10) $\int (3t+2)^{2.4}\,dt = \frac{1}{3}\int u^{2.4}\,du = \boxed{\frac{1}{3} \cdot \frac{1}{3.4}(3t+2)^{3.4} + C}$

201 §5.5 #s 13, 16, 19, 22, 24, 27, 30, 36, 39, 42, 47

(13) $\int \sin(\pi t)\,dt$

$\left(u = \pi t \Rightarrow du = \pi\,dt \Rightarrow dt = \frac{du}{\pi}\right)$

$= \int \sin(u)\,\frac{du}{\pi} = \frac{1}{\pi}\int \sin(u)\,du = -\frac{1}{\pi}\cos(u) + C$

$= -\frac{1}{\pi}\cos(\pi t) + C.$

My way:

$u = \pi t \Rightarrow du = \pi\,dt \Rightarrow$

$\int \sin(\pi t)\cdot \pi\,dt$ can be integrated and differs from the original by a factor of π, so we rectify by dividing by π:

$\frac{1}{\pi}\int \sin(\pi t)\,\pi\,dt$ The way I do it, the $\pi\,dt$ dictates the $\frac{1}{\pi}$ out front, and I never go thru the $\frac{dt}{\pi} = du$ step. I BUILD my du and then clean it up, after.

The book way is more procedural than conceptual, so students may prefer it. But as you work many of these, you may eventually "see it" the way I do. In the 1st step, I "see" I need $\pi\,dt = du$ for my u.

201 §5.5 #s 16, 19, 22, 24, 27, 30, 36, 39, 42, 47

(16) $\int \sec(2\theta)\tan(2\theta)\,d\theta$

$u = 2\theta$

$du = 2d\theta$

Building $2d\theta$ requires also dividing by 2.

$= \frac{1}{2}\int \sec(2\theta)\tan(2\theta)\,2d\theta$

$= \boxed{\frac{1}{2}\sec(2\theta) + C}$

(19) $\int \cos(\theta)\sin^6(\theta)\,d\theta$

$\left(u = \sin(\theta) \Longrightarrow du = \cos(\theta)\,d\theta\right)$

$= \int \sin^6(\theta)\cdot\cos(\theta)\,d\theta = \int u^6\,du = \frac{1}{7}u^7 + C$

$= \boxed{\frac{1}{7}\sin^7(\theta) + C}$

(22) $\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\,dx = \int \cos(\pi x^{-1})\cdot x^{-2}\,dx$

$\left(u = \frac{\pi}{x} = \pi x^{-1} \Longrightarrow du = -1\pi x^{-2}\,dx\right.$

$\left.\Longrightarrow dx = \frac{du}{-\pi x^{-2}} = -\frac{x^2\,du}{\pi}\right)$

BOOK WAY

$= \int \cos(u)\cdot x^{-2}\left(-\frac{x^2\,du}{\pi}\right) = -\frac{1}{\pi}\int \cos(u)\,du$

$= -\frac{1}{\pi}\sin(u) + C = \boxed{-\frac{1}{\pi}\sin\left(\frac{\pi}{x}\right) + C}$

201 §5.5 #s 24, 27, 30, 36, 39, 42, 47

(27) $\displaystyle\int \frac{\cos(x)}{\sin^2(x)}\,dx = \int \sin^{-2}(x)\cdot\cos(x)\,dx$

$\left(u = \sin(x) \Longrightarrow du = \cos(x)\,dx \Longrightarrow dx = \dfrac{du}{\cos(x)} \right)$

$\displaystyle = \int u^{-2}\cdot\cos(x)\cdot\frac{du}{\cos(x)} = \int u^{-2}\,du = \frac{1}{-1}u^{-1} + C$

$\displaystyle = -\frac{1}{u} + C = -\frac{1}{\sin(x)} + C = \boxed{-\csc(x) + C}$

ALTERNATE:

$\displaystyle\int \frac{\cos(x)}{\sin^2(x)}\,dx = \int \frac{1}{\sin(x)}\cdot\frac{\cos(x)}{\sin(x)}\,dx = \int \csc(x)\cot(x)\,dx$

$= \boxed{-\csc(x) + C}$, since $\frac{d}{dx}[\csc(x)\cot(x)] = -\csc(x)$

(30) $\displaystyle\int x^3\sqrt{x^2+1}\,dx$

(Let $u = x^2+1$. Then $du = 2x\,dx$ & $dx = \dfrac{du}{2x}$)

$\displaystyle = \int \sqrt{u}\cdot x^3\cdot\frac{du}{2x} = \frac{1}{2}\int \sqrt{u}\;x^2\,du$ (Still an x^2 in there. Can we get rid of it?

Hmmmm... $u = x^2 + 1 \Longrightarrow u - 1 = x^2$. Yes!)

$\displaystyle = \frac{1}{2}\int \sqrt{u}\,(u-1)\,du = \frac{1}{2}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du = \frac{1}{2}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + C\right]$

$= \boxed{\frac{1}{5}(x^2+1)^{\frac{5}{2}} - \frac{2}{3}(x^2+1)^{\frac{3}{2}} + C}$

201 §5.5 #s 36, 39, 42, 47

#s 35-50 Evaluate the definite integral.

TWO METHODS:

(m1) $\displaystyle\int_{x=a}^{x=b} f(x)\,dx = \int_{u=u(a)}^{u=u(b)} g(u)\,du$ Change limits to expression in terms of u.

(m2) $\displaystyle\int f(x)\,dx = \int g(u)\,du = G(u) + C$ Indefinite integral. Convert back to $F(x)+C$, use $x=a$, $x=b$.

$= F(x) + C \longrightarrow$

$\displaystyle\int_a^b f(x)\,dx = F(b) - F(a)$

(36) $\displaystyle\int_0^7 \sqrt{4+3x}\,dx$

$u = 4+3x \Rightarrow du = 3\,dx \Rightarrow \dfrac{du}{3} = dx$

(m1) $x = 0 \Rightarrow u = 4+3(0) = 4$, This gives
$x = 7 \Rightarrow u = 4+3(7) = 25$

$\displaystyle\int_4^{25} u^{\frac{1}{2}} \cdot \frac{du}{3} = \frac{1}{3}\int_4^{25} u^{\frac{1}{2}}\,du = \frac{1}{3}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_4^{25} = \frac{2}{9}\left[25^{\frac{3}{2}} - 4^{\frac{3}{2}}\right]$

$= \dfrac{2}{9}\left[5^3 - 2^3\right] = \dfrac{2}{9}\left[125 - 8\right] = \dfrac{2}{9}\left[117\right] = 2\left[13\right] = \boxed{26}$

201 § 5.5 #s 36, 39, 42, 47

36 cnt'd

M2 $u = 4+3x$, $du = 3dx$, $dx = \frac{du}{3}$. This gives

$$\int u^{\frac{1}{2}} \cdot \frac{du}{3} = \frac{1}{3}\int u^{\frac{1}{2}} du = \frac{1}{3}\cdot\frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{9}u^{\frac{3}{2}} + C$$

$$= \frac{2}{9}(4+3x)^{\frac{3}{2}} + C, \text{ so}$$

$$\int_0^7 \sqrt{4+3x}\, dx = \left[\frac{2}{9}(4+3x)^{\frac{3}{2}}\right]_0^7 = \frac{2}{9}\left[(4+3(7))^{\frac{3}{2}} - (4+3(0))^{\frac{3}{2}}\right]$$

$$= \frac{2}{9}\left[(4+21)^{\frac{3}{2}} - (4)^{\frac{3}{2}}\right] = \frac{2}{9}\left[25^{\frac{3}{2}} - 4^{\frac{3}{2}}\right] = \frac{2}{9}\left[5^3 - 2^3\right]$$

$$= \frac{2}{9}[125 - 8] = \frac{2}{9}[117] = 2[13] = \boxed{26}$$

39 $\displaystyle\int_0^{\pi} \sec^2\left(\frac{t}{4}\right) dt$

M1 $u = \frac{1}{4}t \Rightarrow du = \frac{1}{4}dt \longrightarrow dt = 4du$

$x = 0 \Rightarrow u = 0$

$x = \pi \Rightarrow u = \frac{1}{4}\pi = \frac{\pi}{4}$

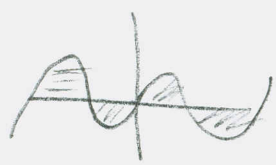
$$= \int_0^{\frac{\pi}{4}} \sec^2(u)\cdot 4du = 4\int_0^{\frac{\pi}{4}} \sec^2(u) du = 4\left[\tan(u)\right]_0^{\frac{\pi}{4}}$$

$$= 4\left(\tan\left(\frac{\pi}{4}\right) - \tan(0)\right) = \boxed{4}$$

201 §5.5 #s 42, 47

42 $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \sin(x)}{1+x^6}\,dx$ Looks hard. Take a step back.

$f(x) = \dfrac{x^2 \sin(x)}{1+x^6}$ is an odd function, and the upper & lower limits are symmetric about the y-axis



$\boxed{\text{Area} = 0\,!}$

47 Ken asked about this one and we looked at a couple different substitutions. The 1st method I showed in class is the standard way, but there's more 'n one way to skin a cat!

$$\int_1^2 x\sqrt{x-1}\,dx$$

Let $u = x-1$. Then $du = dx$ and $x = u+1$.
Also, $x=1 \Longrightarrow u=0$ and $x=2 \Longrightarrow u=1$.

This gives

$$\int_0^1 (u+1)\,u^{\frac{1}{2}}\,du = \int_0^1 \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)du = \left[\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{5} + \frac{2}{3} = \frac{6+10}{15} = \boxed{\frac{16}{15}}$$

201 § 5.5 #47.

Another approach:

$$\int_1^2 x\sqrt{x-1}\,dx$$

$u=\sqrt{x-1} \Rightarrow du=\frac{1}{2}(x-1)^{-\frac{1}{2}}dx \Rightarrow$

$\underline{dx = 2\sqrt{x-1}\,du}$, $x=1 \Rightarrow u=\sqrt{1-1}=0$

$x=2 \Rightarrow u=\sqrt{2-1}=1$. This gives

$$\int_0^1 x\cdot u\cdot 2\sqrt{x-1}\,du = 2\int_0^1 x\cdot u\cdot u\,du.$$ Still

an x to get rid of.

$u=\sqrt{x-1} \Rightarrow u^2=x-1 \Rightarrow x=u^2+1$, so

we have

$$2\int_0^1 (u^2+1)(u^2du) = 2\int_0^1 (u^4+u^2)\,du = 2\left[\frac{1}{5}u^4+\frac{1}{3}u^3\right]_0^1$$

$$= 2\left[\frac{1}{5}+\frac{1}{3}\right] = 2\left[\frac{3+5}{15}\right] = \boxed{\frac{16}{15}}$$

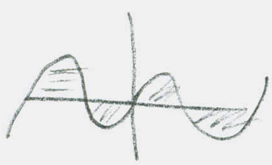
Note: $\int_0^1 (u^{\frac{3}{2}}+u^{\frac{1}{2}})\,du = 2\int_0^1 (u^4+u^2)\,du$!

Who'd've thought?

201 §5.5 #s 42, 47

(42) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \sin(x)}{1+x^6}\,dx$ Looks hard. Take a step back.

$f(x) = \dfrac{x^2 \sin(x)}{1+x^6}$ is an odd function, and the upper & lower limits are symmetric about the y-axis



$\boxed{\text{Area} = 0 \,!}$

(47) Ken asked about this one and we looked at a couple different substitutions. The 1st method I showed in class is the standard way, but there's more in one way to skin a cat!

$\displaystyle\int_1^2 x\sqrt{x-1}\,dx$

Let $u = x-1$. Then $du = dx$ and $x = u+1$.

Also, $x=1 \Longrightarrow u=0$ and $x=2 \Longrightarrow u=1$.

This gives

$$\int_0^1 (u+1)\,u^{\frac{1}{2}}\,du = \int_0^1 \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)du = \left[\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{5} + \frac{2}{3} = \frac{6+10}{15} = \boxed{\frac{16}{15}}$$

201 § 5.5 #47.

Another approach:

$$\int_1^2 x\sqrt{x-1}\,dx$$

$u=\sqrt{x-1} \Longrightarrow du = \frac{1}{2}(x-1)^{-\frac{1}{2}}dx \Longrightarrow$

$\underline{dx = 2\sqrt{x-1}\,du}$, $x=1 \Rightarrow u=\sqrt{1-1}=0$

$x=2 \Rightarrow u=\sqrt{2-1}=1$. This gives

$$\int_0^1 x\cdot u\cdot 2\sqrt{x-1}\,du = 2\int_0^1 x\cdot u\cdot u\,du.$$ Still

an x to get rid of.

$u=\sqrt{x-1} \Longrightarrow u^2 = x-1 \Longrightarrow x=u^2+1$, so

we have

$$2\int_0^1 (u^2+1)(u^2\,du) = 2\int_0^1 (u^4+u^2)\,du = 2\left[\frac{1}{5}u^4+\frac{1}{3}u^3\right]_0^1$$

$$= 2\left[\frac{1}{5}+\frac{1}{3}\right] = 2\left[\frac{3+5}{15}\right] = \boxed{\frac{16}{15}}$$

Note: $\displaystyle\int_0^1 (u^{\frac{3}{2}}+u^{\frac{1}{2}})\,du = 2\int_0^1 (u^4+u^2)\,du$!

Who'd've thought?