201 §5.4 #s 2, 4, 7, 10, 14, 20, 27, 38, 42, (45), 46, 52

#s 1-4 verify by differentiation that the formula is correct.

(2) $\int x\cos(x)\,dx = x\sin(x) + \cos(x) + C$

$\frac{d}{dx}\left[x\sin(x) + \cos(x) + C\right] = \sin(x) + x\cos(x) - \sin x$

$= x\cos(x)$ ✓

(4) $\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2}{3b^2}(bx-2a)\sqrt{a+bx} + C$

$\frac{d}{dx}\left[\frac{2}{3b^2}(bx-2a)\sqrt{a+bx} + C\right]$

$= \frac{2}{3b^2}\left[b\sqrt{a+bx} + (bx-2a)\left(\frac{1}{2}(a+bx)^{-\frac{1}{2}}(b)\right)\right]$

$= \frac{2b}{3b^2}\sqrt{a+bx} + \frac{2b}{6b^2}(bx-2a)\frac{1}{\sqrt{a+bx}}$

$= \frac{2}{3b}\sqrt{a+bx} + \frac{1}{3b}\cdot\frac{bx}{\sqrt{a+bx}} - \frac{1}{3b}\cdot\frac{2a}{\sqrt{a+bx}}$

$= \left(\frac{2}{3b}\sqrt{a+bx}\right)\left(\frac{\sqrt{a+bx}}{\sqrt{a+bx}}\right) + \frac{bx-2a}{3b\sqrt{a+bx}}$

$= \frac{2(a+bx) + bx - 2a}{3b\sqrt{a+bx}} = \frac{2a+2bx+bx-2a}{3b\sqrt{a+bx}}$

$= \frac{3bx}{3b\sqrt{a+bx}} = \frac{x}{\sqrt{a+bx}}$ ✓

201 §5.4 #s 7, 10, 14, 20, 27, 38, 42, (45), 46, 52

#s 5–16 Find the general indefinite integral

7 $\int \left(x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2\right) dx$

$$= \boxed{\frac{1}{5}x^5 - 2x^4 + \frac{1}{8}x^2 - 2x + C}$$

10 $\int v(v^2+2)^2 \, dv = \int v(v^4 + 4v^2 + 4)\, dv$

$= \int (v^5 + 4v^3 + 4v)\, dx \quad = \frac{1}{6}v^6 + v^4 + 2v^2 + C$

§5.5 way: $u = v^2 + 2 \Rightarrow du = 2v\,dv$

$\Rightarrow dv = \frac{du}{2v}$, so that

$\int v(v^2+2)^2\, dv = \int v \cdot u^2 \cdot \frac{du}{2v} = \frac{1}{2}\int u^2 \, du$

$= \frac{1}{2}\left(\frac{u^3}{3}\right) + C = \frac{u^3}{6} = \frac{1}{6}u^3 + C = \frac{1}{6}(v^2+2)^3 + C$

1
1 1
1 2 1
1 3 3 1

$= \frac{1}{6}\left[(v^2)^3 + 3(v^2)^2(2) + 3(v^2)(2)^2 + 2^3\right] + C$

$= \frac{1}{6}v^6 + \frac{1}{6}(6v^4) + \frac{1}{6}(12v^2) + \frac{1}{6}(8) + C$

$= \frac{1}{6}v^6 + v^4 + 2v^2 + \boxed{\frac{4}{3} + C}$

Again, just a constant.

So, they're the same.

Advantage, §5.5, since you'd stop @ $\frac{1}{6}(v^2+2)^3 + C$

201 §5.4 #s 14, 20, 27, 38, 42, (45), 46, 52

(14) $\int \sec(t)(\sec(t) + \tan(t))\,dt$

$= \int (\sec^2(t) + \sec(t)\tan(t))\,dt = \boxed{\tan(t) + \sec(t) + C}$

#s 19-42 Evaluate the integral

(20) $\int_1^3 (1 + 2x - 4x^3)\,dx = \left[x + x^2 - x^4\right]_1^3$

$= (3 + 3^2 - 3^4) - (1 + 1^2 - 1^4)$

$= 3 + 9 - 81 - (1) = 12 - 82 = \boxed{-70}$

(27) $\int_{-2}^{-1} (4y^3 + 2y^{-3})\,dy = \left[y^4 - y^{-2}\right]_{-2}^{-1}$

$= (-1)^4 - (-1)^{-2} - \left((-2)^4 - (-2)^{-2}\right)$

$= 1 - 1 - \left(16 - \frac{1}{4}\right) = -16 + \frac{1}{4} = \frac{-64+1}{4} = \boxed{-\frac{63}{4}}$

(38) $\int_0^1 (1 + x^2)^3\,dx = \int_0^1 \left(1^3 + 3(1)^2(x^2) + 3(1)(x^2)^2 + (x^2)^3\right)dx$

$= \int_0^1 (1 + 3x^2 + 3x^4 + x^6)\,dx = \left[x + x^3 + \frac{3}{5}x^5 + \frac{1}{7}x^7\right]_0^1$

$= 1 + 1^3 + \frac{3}{5}(1)^5 + \frac{1}{7}(1)^7 = 2 + \frac{3}{5} + \frac{1}{7} = \frac{70 + 21 + 5}{35} = \boxed{\frac{96}{35}}$

201 S. 5.4 #s 42(45), 46, 52

42 $$\int_0^{\frac{3\pi}{2}} |\sin(x)|\,dx = \int_0^{\pi} \sin(x)\,dx - \int_{\pi}^{\frac{3\pi}{2}} \sin(x)\,dx,$$

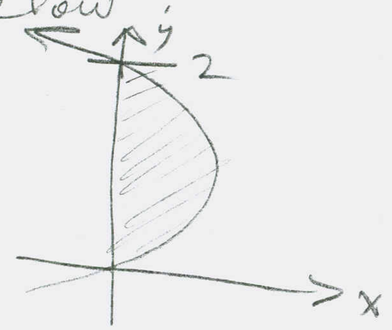
(since $|\sin(x)| = \begin{cases} \sin(x) & \text{if } 0 \le x \le \pi \\ -\sin(x) & \text{if } \pi < x \le \frac{3\pi}{2} \end{cases}$)

$$= \Big[-\cos(x)\Big]_0^{\pi} - \Big[-\cos(x)\Big]_{\pi}^{\frac{3\pi}{2}} = -\cos(\pi) - (-\cos(0))$$

$$- \Big[-\cos\left(\tfrac{3\pi}{2}\right) - (-\cos(\pi))\Big] = -(-1) + 1 - \Big[-0 - (-(-1))\Big]$$

$$= 1 + 1 - [-1] = \boxed{3}$$

45 The area between $x = 2y - y^2$ and the y-axis is given by $\int_0^2 (2y - y^2)\,dy$ and shown below:



Find the area:

$$\int_0^2 (2y - y^2)\,dy = \Big[y^2 - \tfrac{1}{3}y^3\Big]_0^2$$

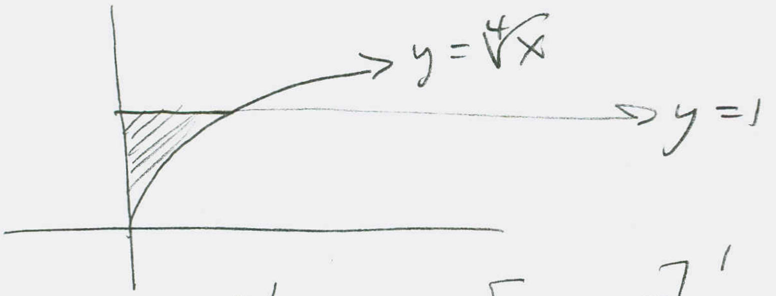
$$= 4 - \frac{8}{3} = \frac{12 - 8}{3} = \boxed{\frac{4}{3}}$$

46 The boundaries of the shaded region are the y-axis, the line $y = 1$, and the curve $y = \sqrt[4]{x}$. Find the area by writing $x = f(y)$

201 §5.4 #s 46, 52

46 cnt'd

and integrating wrt y.



$y = \sqrt[4]{x} \Rightarrow$

$y^4 = x = f(y)$

This is $\int_0^1 y^4 \, dy = \left[\frac{1}{5} y^5 \right]_0^1 = \boxed{\frac{1}{5}}$

52 If $f(x)$ is the slope of a trail at a distance of x miles from the start, what does $\int_3^5 f(x)\,dx$ represent?

It represents the net change in altitude.