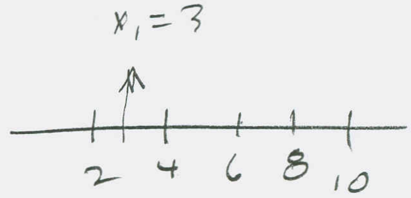
201 §5.2 #s 9, 10, 17, 18, 21, 22, 29, 36, 37

#s 9–12 Midpoint Rule, to 4 places with the given value of n, to approximate the integral. Round to 4 places (in final step).

(9) $\int_2^{10} \sqrt{x^3+1}\, dx$, $n=4$



$\Delta x = \frac{b-a}{n} = \frac{10-2}{4} = \frac{8}{4} = 2$

$x_k = x_{k-1} + \Delta x = x_{k-1} + 2$

$x_1 = 3,\ x_2 = 5,\ x_3 = 7,\ x_4 = 9$

$f(3) \approx 5.291502622$

$f(5) \approx 11.22497216$

$f(7) \approx 18.54723699$

$f(9) \approx 27.01851217$

Area $\approx \sum f(x_k)\Delta x = \Delta x \sum_{k=1}^{n} f(x_k) \approx$

$2[5.291502622 + 11.22497216 +$

$18.54723699 + 27.01851217]$

$= (62.08222395)(2) \approx 124.164479 \approx \boxed{124.1644}$

(10) $\int_0^{\frac{\pi}{2}} \cos^4(x)\, dx$, $n = 4$

$\Delta x = \frac{b-a}{n} = \frac{\frac{\pi}{2} - 0}{4} = \frac{\pi}{8}$, Midpoint: $x_1 = a + \frac{1}{2}\Delta x$

$= 0 + \frac{1}{2}\cdot\frac{\pi}{8}$

$= \boxed{\frac{\pi}{16} = x_1}$

$x_{k+1} = x_k + \Delta x$

$\boxed{x_2 = \frac{\pi}{16} + \frac{\pi}{8} = \frac{3\pi}{16},\ x_3 = \frac{5\pi}{16},\ x_4 = \frac{7\pi}{16},}$

201 § 5.2 #s 10, 17, 18, 21, 22, 29, 36, 37

10 cnt'd

$$\text{Area} \approx \sum_{k=1}^{4} f(x_k)\Delta x = \Delta x \sum_{k=1}^{4} f(x_k)$$

$$= \frac{\pi}{8}\left[\cos^4\left(\frac{\pi}{16}\right) + \cos^4\left(\frac{3\pi}{16}\right) + \cos^4\left(\frac{5\pi}{16}\right) + \cos^4\left(\frac{7\pi}{16}\right)\right]$$

$$\approx \frac{\pi}{8}\left[.7253281139 + .4779533685 + .0952699362 + .0014485814\right]$$

$$= \frac{\pi}{8}\left[1.5\right] \approx .5890486225 \approx \boxed{.5890}$$

#s 17-20 Express the limit as a definite integral on the given interval.

17 $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1-x_k^2}{4+x_k^2}\Delta x$ on $[2,6]$

$$\boxed{\int_2^6 \frac{1-x^2}{4+x^2}\,dx}$$

18 $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{\cos(x_k)}{x_k}\Delta x$ on $[\pi, 2\pi]$

$$\boxed{\int_{\pi}^{2\pi} \frac{\cos(x)}{x}\,dx}$$

201 §5.2 #s 17, 18, 21, 22, 29, 36, 37

#s 17-20 Express the limit as a definite integral on the given interval.

(17) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1-x_k^2}{4+x_k^2} \Delta x$ on $[2,6]$

$$= \boxed{\int_2^6 \frac{1-x^2}{4+x^2}\,dx}$$

(18) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{\cos(x_k)}{x_k} \Delta x$ on $[\pi, 2\pi]$

$$= \boxed{\int_{\pi}^{2\pi} \frac{\cos(x)}{x}\,dx}$$

So nice, I did it twice!

#s 21-25 ~~Find~~ Evaluate the integral, using Theorem 4

(21) $\int_{-1}^{5} (1+3x)\,dx$

$$\Delta x = \frac{5+1}{n} = \frac{6}{n}$$

$$x_k = -1 + k\Delta x = -1 + \frac{6}{n}k$$

This gives $\lim_{n\to\infty} \sum_{k=1}^{n} \left(1+3\left(-1+\frac{6k}{n}\right)\right)\cdot\frac{6}{n}$

$$= \lim_{n\to\infty}\left[\frac{6}{n}\sum_{k=1}^{n}\left(-2+\frac{18}{n}k\right)\right] = \lim_{n\to\infty}\left[\frac{6}{n}\left(\sum_{k=1}^{n}(-2) + \sum_{k=1}^{n}\frac{18}{n}k\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{6}{n}\sum_{k=1}^{n}(-2) + \frac{6}{n}\sum_{k=1}^{n}\frac{18}{n}k\right] =$$

201 §5.2 #s 21, 22, 24, 26, ...

$\boxed{\text{21 cnt'd}}$

$$= \lim_{n\to\infty} \left[\frac{6}{n}(-2n) + \frac{6}{n}\cdot\frac{18}{n}\sum_{k=1}^{n} k \right]$$

$$= \lim_{n\to\infty} \left[-12 + \frac{108}{n^2}\cdot \frac{n^2+\text{smaller}}{2} \right]$$

$$= -12 + \lim_{n\to\infty} \frac{108n^2 + 108\,\text{smaller}}{2n^2} = -12 + \frac{108}{2}$$

$$= -12 + 54 = \boxed{42}$$

22 $\displaystyle\int_1^4 (x^2+2x-5)\,dx$

$$\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$$

$$x_k = a + k\Delta x = 1 + \frac{3}{n}k$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(1+\frac{3}{n}k\right)^2 + 2\left(1+\frac{3}{n}k\right) - 5 \right]\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left[1 + \frac{6}{n}k + \frac{9}{n^2}k^2 + 2 + \frac{6}{n}k - 5 \right]\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left[-2 + \frac{12}{n}k + \frac{9}{n^2}k^2 \right]\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty} \left[\frac{3}{n}\left(-2n + \frac{12}{n}\cdot\frac{n^2}{2} + \frac{9}{n^2}\cdot\frac{n^3}{3} \right) \right]$$

The n's all cancel. Ignoring "smaller"

$$= (3)(-2) + \frac{(3)(12)}{2} + \frac{3(9)}{3} = -6 + 18 + 9 = \boxed{21}$$

201 §5.2 #s 29, 36, 37

#s 29, 30 Express the integral as a limit of Riemann Sums.

(29) $\displaystyle\int_2^6 \frac{x}{1+x^5}\,dx$

$\Delta x = \frac{b-a}{n} = \frac{6-2}{n} = \frac{4}{n}$

$x_k = a + k\Delta x = 2 + \frac{4}{n}k$

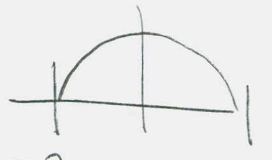
$$= \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{2+\frac{4}{n}k}{1+\left(2+\frac{4}{n}k\right)^5} \cdot \frac{4}{n} \right]$$

#s 35–40 Evaluate the integral by interpreting it in terms of areas.

(36) $\displaystyle\int_{-2}^{2} \sqrt{4-x^2}\,dx$

Ah. $y = \sqrt{4-x^2}$ is the top half of a circle of radius 2.

$x^2 + y^2 = 4$



$\text{Area} = \frac{1}{2}\text{ circle} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2^2)$

$= \frac{1}{2}\cdot 4\pi = \boxed{2\pi}$

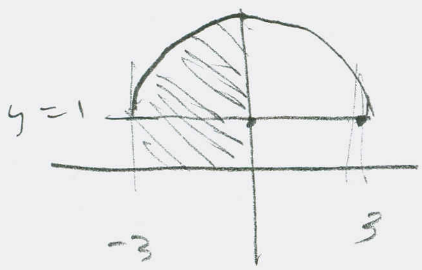
(37) Almost the same $\displaystyle\int_{-3}^{0} \left(1+\sqrt{9-x^2}\right)dx$

$y = 1+\sqrt{9-x^2} \implies y-1 = \sqrt{9-x^2} \implies$

$(y-1)^2 = 9-x^2 \longrightarrow x^2+(y-1)^2 = 9$

201 §5.2 #37

This is a circle of radius 3, centered at $(h,k) = (0,1)$



So the area under the curve from -3 to 0 is $\frac{1}{4}$ circle + rectangle

$$= \frac{1}{4}\pi(3^2) + (3)(1) = \boxed{\frac{9}{4}\pi + 3}!$$