201 S' 5.1#s 5, 8, 12, 13, 14, 17, 20 Re-check

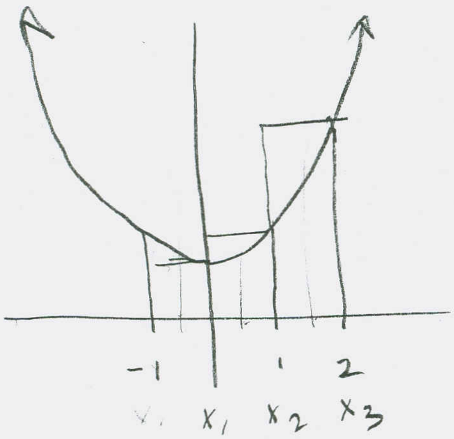
5, 12, 17-20, 22

(5) Estimate the area under the graph of

$f(x) = x^2 + 1$ from $x = -1$ to $x = 2$, using

(a) (i) 3 rectangles, right endpoints.

(ii) 6 rectangles, " "



$$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{3} = \frac{3}{3} = 1 = \Delta x$$

$$x_k = a + k\Delta x$$

$$x_1 = -1 + 1 \cdot 1 = 0$$

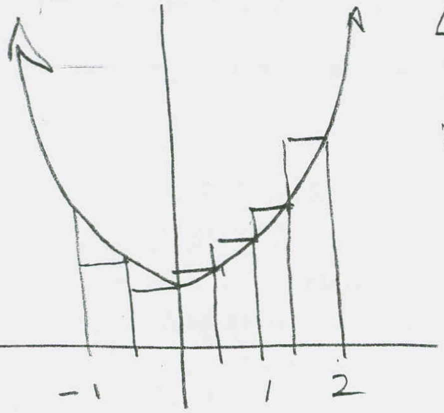
$$x_2 = -1 + 2 \cdot 1 = 1$$

$$x_3 = -1 + 3 \cdot 1 = 2$$

$$\text{Area} \approx \sum_{k=1}^{3} f(x_k)\Delta x = \sum_{k=1}^{3} (x_k^2 + 1) \cdot 1 = \sum_{k=1}^{3} (x_k^2 + 1)$$

$$= 0^2 + 1 + 1^2 + 1 + 2^2 + 1 = \boxed{8}$$

(ii)



$$\Delta x = \frac{b-a}{n} = \frac{2+1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$x_k = a + k\Delta x$$

$$\frac{9}{4} + \frac{4}{4} = \frac{13}{4}$$

$x_1 = -\frac{1}{2}$, $x_2 = 0$, $x_3 = \frac{1}{2}$, $x_4 = 1$, $x_5 = \frac{3}{2}$, $x_6 = 2$

$f(-\frac{1}{2}) = \frac{5}{4}$, $f(0) = 1$, $f(\frac{1}{2}) = \frac{5}{4}$, $f(1) = 2$, $f(\frac{3}{2}) = \frac{13}{4}$, $f(2) = 5$

201 §5.1 #s 5, 8, 12, 13, 14, 17, 20

5 cont'd

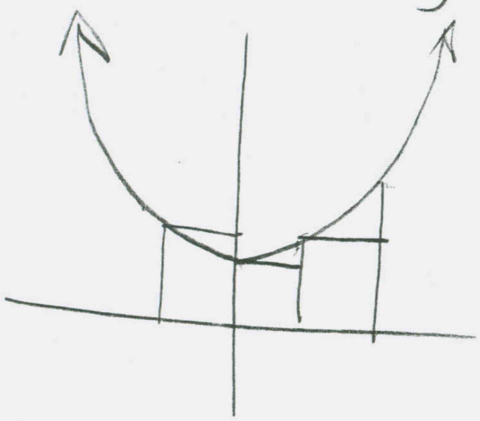
(a) (ii) $\sum_{k=1}^{6} f(x_k)\Delta x = \Delta x \sum_{k=1}^{6} f(x_k)$:

$= \frac{1}{2}\left(\frac{5}{4} + 1 + \frac{5}{4} + 2 + \frac{13}{4} + 5\right)$

$= \frac{1}{2}\left(\frac{23}{4} + 8\right) = \frac{1}{2}\left(\frac{23+32}{4}\right) = \boxed{\frac{55}{8}}$ $\approx$ Area

RIGHT
n=6

(b) Same, using left endpoints



(i) $x_1 = -1$, $x_2 = 0$, $x_3 = 1$

$f(x_1) = 2$, $f(0) = 1$, $f(1) = 2$

$\sum_{k=1}^{3} f(x_k)\Delta x = \Delta x \sum_{k=1}^{3} f(x_k) = 1\,(2+1+2)$

$= \boxed{5 \approx \text{Area}}$

Left
n=3

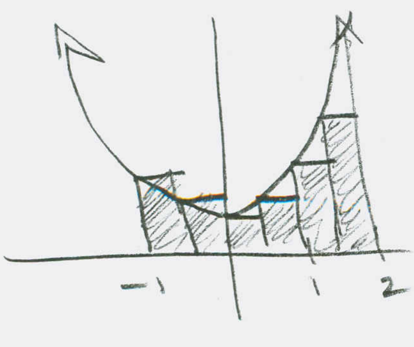
(ii) $x_1 = -1$, $x_2 = -\frac{1}{2}$, $x_3 = 0$, $x_4 = \frac{1}{2}$, $x_5 = 1$, $x_6 = \frac{3}{2}$

$f(-1) = 2$, $f(-\frac{1}{2}) = \frac{5}{4}$, $f(0) = 1$, $f(\frac{1}{2}) = \frac{5}{4}$, $f(1) = 2$, $f(\frac{3}{2}) = \frac{13}{4}$

Area $\approx$ $\Delta x \sum_{k=1}^{6} f(x_k) = \frac{1}{2}\left(2 + \frac{5}{4} + 1 + \frac{5}{4} + 2 + \frac{13}{4}\right)$

$= \frac{1}{2}\left(5 + \frac{23}{4}\right) = \frac{1}{2}\left(\frac{43}{4}\right) = \boxed{\frac{43}{8} \approx \text{Area}}$

Left
n=6



201 §5.1 #s 5, 8, 12, 13, 14, 17, 20

$\boxed{5(c)}$ Same, with MIDPOINTS

(i) $x_1 = -\frac{1}{2}$, $x_2 = \frac{1}{2}$, $x_3 = \frac{3}{2}$ $\quad n = 3$, $\Delta x = 1$

(ii) $n = 6$; $x_1 = -1 + \frac{1}{4} = -\frac{3}{4}$ $\quad f(-3/4) = \frac{25}{16}$

$n = 6$
$\Delta x = \frac{1}{2}$

$x_2 = -\frac{3}{4} + \frac{1}{2} = -\frac{1}{4}$ $\quad f(-\frac{1}{4}) = \frac{17}{16}$

$x_3 = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$ $\quad f(\frac{1}{4}) = \frac{17}{16}$

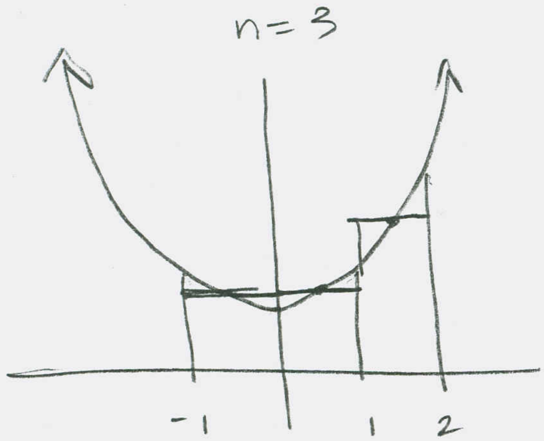
$x_4 = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$ $\quad f(\frac{3}{4}) = \frac{25}{16}$

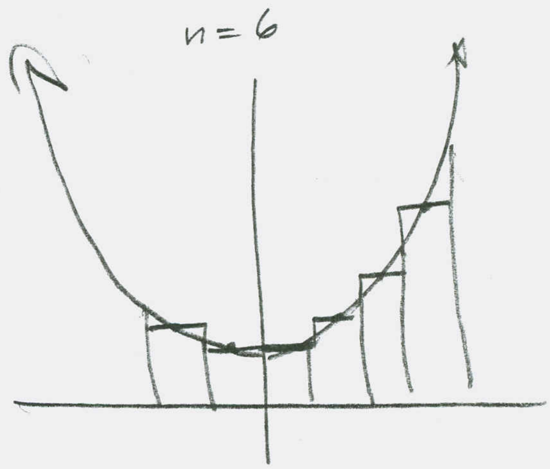
$x_5 = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$ $\quad f(\frac{5}{4}) = \frac{41}{16}$

$x_6 = \frac{5}{4} + \frac{1}{2} = \frac{7}{4}$ $\quad f(\frac{7}{4}) = \frac{65}{16}$

$$\text{Area} \approx \Delta x \sum f(x_k) = \frac{1}{2}\left(\frac{25+17+17+25+41+65}{16}\right)$$

$$= \frac{1}{2}\left(\frac{190}{16}\right) = \frac{95}{16} = \boxed{5.9375 \approx \text{Area}}$$





201 §5.1 #s 8, 12-14, 17, 20

8 Shouldn't have assigned.

12 Speedometer readings were taken @ 12-second intervals. (See Table)

(a) Estimate distance using velocities @ the beginning of the time intervals (LEFT)

(b) Same thing, but with the END of the intervals (RIGHT)

(c) Are (a) & (b) upper & lower estimates? Explain.

t (s)	0	12	24	36	48	60
v (ft/s)	30	28	25	22	24	27

(a) $D \approx \sum f(t)\Delta t = \Delta t \sum f(t) = 12 \sum f(t)$

$= 12\left[30 + 28 + 25 + 22 + 24\right] = 2748 \text{ ft}$

(b) $D \approx 12\left[28 + 25 + 22 + 24 + 27\right] = 2712 \text{ ft}.$

(c) I don't think they're either upper or lower estimates. Monotone functions would be better suited for upper/lower estimates.

201 §5.1 #s ~~17,20~~ 17–20, 22

17 Use Defin 2 to find an expression for the area under the graph of f as a limit. Do not evaluate the limit

$$f(x) = \sqrt[4]{x}, \quad 1 \le x \le 16$$

$$\Delta x = \frac{16-1}{n} = \frac{15}{n}$$

$$f(x_k) = 1 + k\cdot\frac{15}{n} = a + k\Delta x = 1 + \frac{15k}{n} = \frac{n+15k}{n}$$

$$\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x = \lim_{n\to\infty}\sum_{k=1}^{n}\sqrt[4]{1+\frac{15k}{n}}\cdot\frac{15}{n}$$

$$= \lim_{n\to\infty}\frac{15}{n}\sum_{k=1}^{n}\sqrt[4]{1+\frac{15k}{n}} = \lim_{n\to\infty}\frac{15}{n}\sum_{k=1}^{n}\sqrt[4]{\frac{n+15k}{n}}$$

Out of sequence. #s 18,19 to follow

20 Determine a region whose area is equal to the given limit.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left(5+\frac{2i}{n}\right)^{10} \qquad a=5$$

$$\Delta x = \frac{2}{n} = \frac{b-a}{n} = \frac{b-5}{n} \Rightarrow b = 7$$

$$\boxed{f(x) = x^{10}, \text{ on } [5,7]} \quad a=5,\ b=7$$

18 $f(x) = 1 + x^4, \quad 2 \le x \le 5 \Longrightarrow \Delta x = \frac{5-2}{n} = \frac{3}{n}$

$$x_k = a + k\Delta x = 2 + \frac{3}{n}k$$

$$\text{Area} = \boxed{\lim_{n\to\infty}\sum_{k=1}^{n}\left[\left(1+\left(2+\frac{3}{n}k\right)^4\right)\left(\frac{3}{n}\right)\right]}$$

201 §5.1 #s 19, 20, 22

(19) $f(x) = x \cos x \qquad 0 \le x \le \frac{\pi}{2}$

$\Delta x = \frac{\frac{\pi}{2}}{n} = \frac{\pi}{2n}$, $x_k = 0 + \frac{\pi}{2n} k = \frac{\pi}{2n} k$

$$\text{Area} = \lim_{n \to \infty} \sum_{k=1}^{n} \left[\left(\frac{\pi}{2n} k \cos\left(\frac{\pi}{2n} k\right)\right)\left(\frac{\pi}{2n}\right) \right]$$

(20) on previous page

(22) (a) Use Defn 2 to find an expression for the area under $y = x^3$ from 0 to 1 as a limit

(b) Use $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^4 + \text{smaller degree}}{4}$

(a) $\text{Area} = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{k}{n}\right)^3 \left(\frac{1}{n}\right)$

since $\Delta x = \frac{1-0}{n} = \frac{1}{n}$ & at $k\Delta x = 0 + \frac{k}{n} = \frac{k}{n}$

(b) From part (a), we have

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^3}{n^3} \cdot \frac{1}{n} = \lim_{n\to\infty} \left[\frac{1}{n^4} \sum_{k=1}^{n} k^3 \right]$$

$$= \lim_{n\to\infty} \left[\frac{1}{n^4} \left(\frac{n^4 + \text{smaller}}{4} \right) \right] = \lim_{n\to\infty} \left[\frac{n^4}{4n^4} + \frac{\text{smaller}}{4n^4} \right]$$

$$= \boxed{\frac{1}{4}}$$