4.9 I Solutions

1.
$$f(x) = x - 3 = x^1 - 3 \implies F(x) = \frac{x^{1+1}}{1+1} - 3x + C = \frac{1}{2}x^2 - 3x + C$$

Check: $F'(x) = \frac{1}{2}(2x) - 3 + 0 = x - 3 = f(x)$

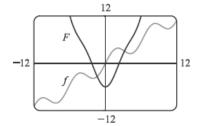
6.
$$f(x) = x(2-x)^2 = x(4-4x+x^2) = 4x-4x^2+x^3 \Rightarrow$$

$$F(x) = 4\left(\frac{1}{2}x^2\right) - 4\left(\frac{1}{3}x^3\right) + \frac{1}{4}x^4 + C = 2x^2 - \frac{4}{3}x^3 + \frac{1}{4}x^4 + C$$

13.
$$f(u) = \frac{u^4 + 3\sqrt{u}}{u^2} = \frac{u^4}{u^2} + \frac{3u^{1/2}}{u^2} = u^2 + 3u^{-3/2} \implies$$

$$F(u) = \frac{u^3}{3} + 3\frac{u^{-3/2+1}}{-3/2+1} + C = \frac{1}{3}u^3 + 3\frac{u^{-1/2}}{-1/2} + C = \frac{1}{3}u^3 - \frac{6}{\sqrt{u}} + C$$

20.
$$f(x) = x + 2\sin x \implies F(x) = \frac{1}{2}x^2 - 2\cos x + C$$
.
 $F(0) = -6 \implies 0 - 2 + C = -6 \implies C = -4$, so $F(x) = \frac{1}{2}x^2 - 2\cos x - 4$.



The graph confirms our answer since f(x) = 0 when F has a local minimum, f is positive when F is increasing, and f is negative when F is decreasing.

21.
$$f''(x) = 6x + 12x^2 \implies f'(x) = 6 \cdot \frac{x^2}{2} + 12 \cdot \frac{x^3}{3} + C = 3x^2 + 4x^3 + C \implies$$

$$f(x) = 3 \cdot \frac{x^3}{3} + 4 \cdot \frac{x^4}{4} + Cx + D = x^3 + x^4 + Cx + D \qquad [C \text{ and } D \text{ are just arbitrary constants}]$$

26.
$$f'''(t) = t - \sqrt{t} \implies f''(t) = \frac{1}{2}t^2 - \frac{2}{3}t^{3/2} + C \implies f'(t) = \frac{1}{6}t^3 - \frac{4}{15}t^{5/2} + Ct + D \implies f(t) = \frac{1}{24}t^4 - \frac{8}{105}t^{7/2} + \frac{1}{2}Ct^2 + Dt + E$$

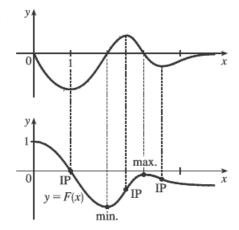
31.
$$f'(t) = 2\cos t + \sec^2 t \implies f(t) = 2\sin t + \tan t + C$$
 because $-\pi/2 < t < \pi/2$. $f(\frac{\pi}{3}) = 2(\sqrt{3}/2) + \sqrt{3} + C = 2\sqrt{3} + C$ and $f(\frac{\pi}{3}) = 4 \implies C = 4 - 2\sqrt{3}$, so $f(t) = 2\sin t + \tan t + 4 - 2\sqrt{3}$.

40.
$$f'''(x) = \cos x \implies f''(x) = \sin x + C$$
. $f''(0) = C$ and $f''(0) = 3 \implies C = 3$. $f''(x) = \sin x + 3 \implies f'(x) = -\cos x + 3x + D$. $f'(0) = -1 + D$ and $f'(0) = 2 \implies D = 3$. $f'(x) = -\cos x + 3x + 3 \implies f(x) = -\sin x + \frac{3}{2}x^2 + 3x + E$. $f(0) = E$ and $f(0) = 1 \implies E = 1$. Thus, $f(x) = -\sin x + \frac{3}{2}x^2 + 3x + 1$.

44. We know right away that c cannot be f's antiderivative, since the slope of c is not zero at the x-value where f = 0. Now f is positive when a is increasing and negative when a is decreasing, so a is the antiderivative of f.

4.9 I Solutions

45.



The graph of F must start at (0,1). Where the given graph, y = f(x), has a

local minimum or maximum, the graph of F will have an inflection point.

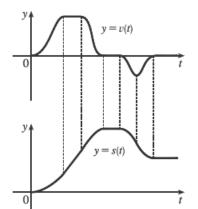
Where f is negative (positive), F is decreasing (increasing).

Where f changes from negative to positive, F will have a minimum.

Where f changes from positive to negative, F will have a maximum.

Where f is decreasing (increasing), F is concave downward (upward).

46.



Where v is positive (negative), s is increasing (decreasing).

Where v is increasing (decreasing), s is concave upward (downward).

Where v is horizontal (a steady velocity), s is linear.