

## 4.9 Solutions

5.  $f(x) = x^3 + 2x - 4 \Rightarrow f'(x) = 3x^2 + 2$ , so  $x_{n+1} = x_n - \frac{x_n^3 + 2x_n - 4}{3x_n^2 + 2}$ . Now  $x_1 = 1 \Rightarrow$

$$x_2 = 1 - \frac{1 + 2 - 4}{3 \cdot 1^2 + 2} = 1 - \frac{-1}{5} = 1.2 \Rightarrow x_3 = 1.2 - \frac{(1.2)^3 + 2(1.2) - 4}{3(1.2)^2 + 2} \approx 1.1797.$$

8.  $f(x) = x^5 + 2 \Rightarrow f'(x) = 5x^4$ , so  $x_{n+1} = x_n - \frac{x_n^5 + 2}{5x_n^4}$ . Now  $x_1 = -1 \Rightarrow$

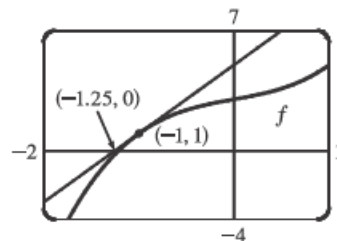
$$x_2 = -1 - \frac{(-1)^5 + 2}{5 \cdot (-1)^4} = -1 - \frac{1}{5} = -1.2 \Rightarrow x_3 = -1.2 - \frac{(-1.2)^5 + 2}{5(-1.2)^4} \approx -1.1529.$$

9.  $f(x) = x^3 + x + 3 \Rightarrow f'(x) = 3x^2 + 1$ , so  $x_{n+1} = x_n - \frac{x_n^3 + x_n + 3}{3x_n^2 + 1}$ .

Now  $x_1 = -1 \Rightarrow$

$$x_2 = -1 - \frac{(-1)^3 + (-1) + 3}{3(-1)^2 + 1} = -1 - \frac{-1 - 1 + 3}{3 + 1} = -1 - \frac{1}{4} = -1.25.$$

Newton's method follows the tangent line at  $(-1, 1)$  down to its intersection with the  $x$ -axis at  $(-1.25, 0)$ , giving the second approximation  $x_2 = -1.25$ .

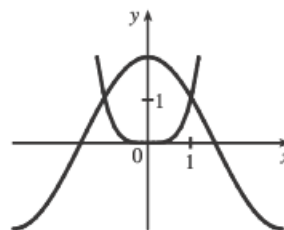


16.  $2 \cos x = x^4$ , so  $f(x) = 2 \cos x - x^4 \Rightarrow f'(x) = -2 \sin x - 4x^3 \Rightarrow$

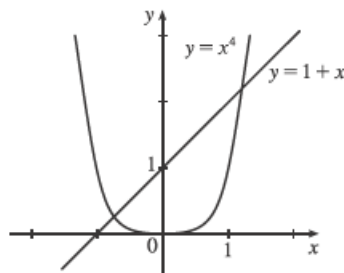
$$x_{n+1} = x_n - \frac{2 \cos x_n - x_n^4}{-2 \sin x_n - 4x_n^3}. \text{ From the figure, the positive root}$$

of  $2 \cos x = x^4$  is near 1.  $x_1 = 1 \Rightarrow x_2 \approx 1.014184, x_3 \approx 1.013958 \approx x_4$ .

So the positive root is 1.013958, to six decimal places.



17.



From the graph, we see that there appear to be points of intersection near

$x = -0.7$  and  $x = 1.2$ . Solving  $x^4 = 1 + x$  is the same as solving

$$f(x) = x^4 - x - 1 = 0. \quad f(x) = x^4 - x - 1 \Rightarrow f'(x) = 4x^3 - 1,$$

$$\text{so } x_{n+1} = x_n - \frac{x_n^4 - x_n - 1}{4x_n^3 - 1}.$$

$$x_1 = -0.7$$

$$x_2 \approx -0.725253$$

$$x_3 \approx -0.724493$$

$$x_4 \approx -0.724492 \approx x_5$$

$$x_1 = 1.2$$

$$x_2 \approx 1.221380$$

$$x_3 \approx 1.220745$$

$$x_4 \approx 1.220744 \approx x_5$$

To six decimal places, the roots of the equation are  $-0.724492$  and  $1.220744$ .

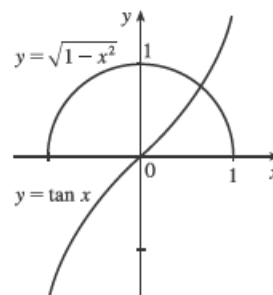
22. From the graph, there appears to be a point of intersection near  $x = 0.7$ .

Solving  $\tan x = \sqrt{1 - x^2}$  is the same as solving

$$f(x) = \tan x - \sqrt{1 - x^2} = 0. \quad f(x) = \tan x - \sqrt{1 - x^2} \Rightarrow$$

$$f'(x) = \sec^2 x + x/\sqrt{1 - x^2}, \text{ so } x_{n+1} = x_n - \frac{\tan x_n - \sqrt{1 - x_n^2}}{\sec^2 x_n + x_n/\sqrt{1 - x_n^2}}.$$

$x_1 = 0.7 \Rightarrow x_2 \approx 0.652356, x_3 \approx 0.649895, x_4 \approx 0.649889 \approx x_5$ . To six decimal places, the root of the equation is 0.649889.



29.  $f(x) = x^3 - 3x + 6 \Rightarrow f'(x) = 3x^2 - 3$ . If  $x_1 = 1$ , then  $f'(x_1) = 0$  and the tangent line used for approximating  $x_2$  is horizontal. Attempting to find  $x_2$  results in trying to divide by zero.

31. For  $f(x) = x^{1/3}$ ,  $f'(x) = \frac{1}{3}x^{-2/3}$  and

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^{1/3}}{\frac{1}{3}x_n^{-2/3}} = x_n - 3x_n = -2x_n.$$

Therefore, each successive approximation becomes twice as large as the previous one in absolute value, so the sequence of approximations fails to converge to the root, which is 0. In the figure, we have  $x_1 = 0.5$ ,

$x_2 = -2(0.5) = -1$ , and  $x_3 = -2(-1) = 2$ .

