201 §4.8 #s 5, 8, 9, 16, 17, 22, 29, 31

#s 5-8 Use Newton, and x_1 to find x_3, To do this like the test, round x_2 to 4 places and x_3 to 4 places.

(5) $f(x) = x^3 + 2x - 4$, $x_1 = 1$. Find the root.

$f'(x) = 3x^2 + 2 \Rightarrow$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x^3+2x-4}{3x^2+2}$$

$$x_2 = 1 - \frac{1^3 + 2(1) - 4}{3(1)^2 + 2} = 1.2$$

$$x_3 = 1.2 - \frac{(1.2)^3 + 2(1.2) - 4}{3(1.2)^2 + 2} \approx 1.79746835$$

$$\approx \boxed{1.7975 \approx x_3}$$

(8) $f(x) = x^5 + 2$, $x_1 = -1$

$f'(x) = 5x^4$

$$x_K = x_K - \frac{f(x_K)}{f'(x_K)} = -1 - \frac{(-1)^5+2}{5(-1)^4} = -1 - \frac{-1+2}{5}$$

$$= -1 - \frac{1}{5} = -\frac{6}{5} = -1.2 = x_2$$

$$x_3 \approx -1.152901235 \approx \boxed{-1.1529 \approx \cancel{x_3}}$$

201 §4.8 #s 9, 16, 17, 22, 29, 31

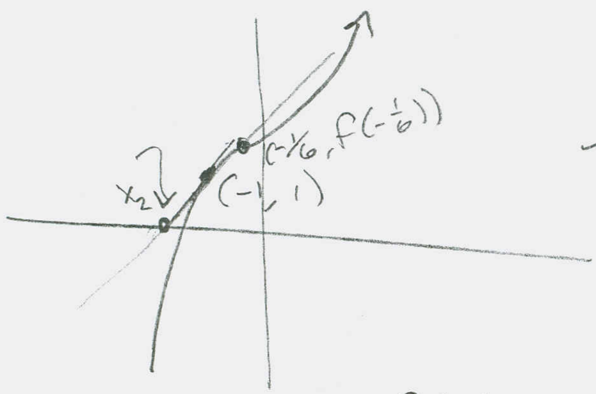
(9) Look @ $f(x) = x^3 + x + 3$ and its tangent line @ $x = x_1 = -1$ & explain how x_2 is obtained

$f'(x) = 3x^2 + 1 \Longrightarrow f'(-1) = 4 \rightsquigarrow m_{tan} = 4$

$f(-1) = -1 - 1 + 3 = 1 \rightsquigarrow (-1, 1) = (x_1, y_1)$

$y = 4(x - (-1)) + 1 = 4(x+1) + 1 = 4x + 5 = L(x)$

($f''(x) = 6x + 1 \Longrightarrow$ IP @ $x = -\frac{1}{6}$



x_2 is where the tangent line to $f(x)$ has its zero.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = x_1 - \frac{x_1^3 + x_1 + 3}{3x_1^2 + 1}$$

$$x_1 = -1 \Longrightarrow x_2 = -1 - \frac{(-1)^3 + (-1) + 3}{3(-1)^2 + 1} = -1 - \frac{-1-1+3}{3+1} = -1 - \frac{1}{4}$$

$= \frac{-5}{4} = -1.2$, again!

This is close to $x = -1.213412$, which graphers gives as the root.

201 §4.8 #s 16, 17, 22, 29, 31

(16) Find the positive root of $2\cos(x) - x^4 = f(x)$ to 6 places.

$f(x) = 2\cos(x) - x^4$

$f'(x) = -2\sin(x) - 4x^3$

$$x_{k+1} = x_k - \frac{2\cos(x) - x^4}{-2\sin(x) - 4x^3}$$

$x_1 = 1$

$x_2 \approx 1.014183606$

$x_3 \approx 1.013957673$

$x_4 \approx 1.03957614$

$x_5 \approx 1.03957614$

$\therefore \boxed{x \approx 1.039576}$

#s 17-22 Use Newton to find ALL roots. to 6 places.

(17) $x^4 - x - 1 = 0$

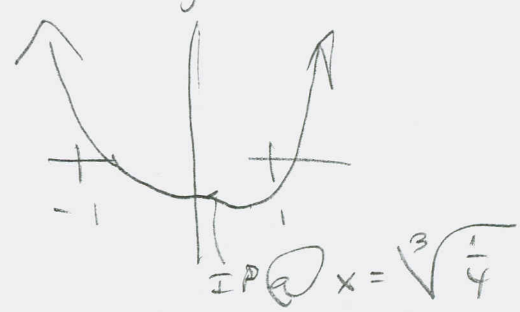
$f(x) = x^4 - x - 1$

$f'(x) = 4x^3 - 1$

$x_1 = -1, 1$

$$x_{k+1} = x_k - \frac{x^4 - x - 1}{4x^3 - 1}$$

Rough sketch:



$x_1 = -1$:

$x_2 \approx -.8$

$x_3 \approx -.731233595 8$

$x_4 \approx -.72454848$

$x_5 \approx -.724491963$

$x_6 \approx -.724491959$

$\boxed{x \approx -.724492}$

$x_1 = 1$

$x_2 = .\overline{3}$

$x_3 \approx 1.23580786$

$x_4 \approx 1.221058994$

$x_5 \approx 1.220744226$

$x_6 \approx 1.220744085$

$\boxed{x \approx 1.220744}$

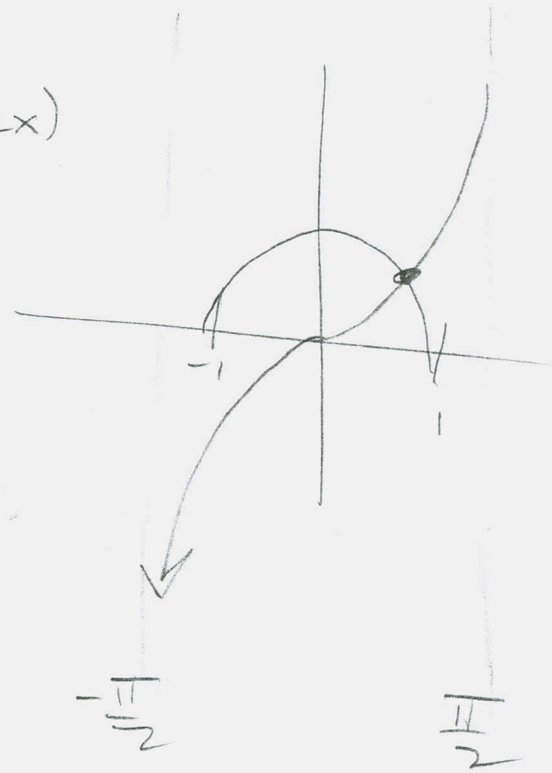
201 §4.8 #s 22, 29, 31

22 $\tan x = \sqrt{1-x^2}$

$f(x) = \tan x - \sqrt{1-x^2}$

$f'(x) = \sec^2 x + \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$

$= \sec^2 x + \frac{x}{\sqrt{1-x^2}}$



Use $x_1 = .5$ as guess ($x_1 = .5$)

$x_2 \approx .6704464655$

$x_3 \approx .6502985916$

$x_4 \approx .649889108$

$x_5 \approx .6498889467$

$\boxed{x \approx .649889}$ is result to 6 places

29 Explain why Newton's doesn't work for

$x^3 - 3x + 6 = 0$ if $x_1 = 1$

$f'(x) = 3x^2 - 3 = 0$ when $x = 1$, so x_2 would be undefined if $x_1 = 1$.

31 What goes wrong with Newton for $f(x) = \sqrt[3]{x}$ for <u>any</u> x_1?

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} \longrightarrow$

$x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k^{\frac{1}{3}}}{\frac{1}{3}x_k^{-\frac{2}{3}}} = x_k - 3x_k = -2x_k = x_{k+1}$

So x_{k+1} is $2x_k$ & we go away from the zero!