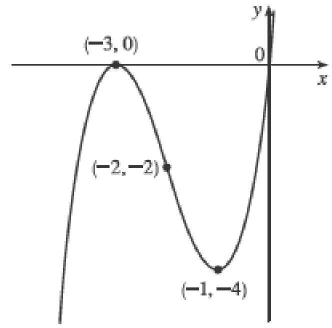
2. $y = f(x) = x^3 + 6x^2 + 9x = x(x+3)^2$ **A.** $D = \mathbb{R}$ **B.** x-intercepts are -3 and 0, y-intercept $= 0$ **C.** No symmetry **D.** No asymptote

E. $f'(x) = 3x^2 + 12x + 9 = 3(x+1)(x+3) < 0 \quad \Leftrightarrow \quad -3 < x < -1$, so f is decreasing on $(-3, -1)$ and increasing on $(-\infty, -3)$ and $(-1, \infty)$.

F. Local maximum value $f(-3) = 0$, local minimum value $f(-1) = -4$

G. $f''(x) = 6x + 12 = 6(x+2) > 0 \quad \Leftrightarrow \quad x > -2$, so f is CU on $(-2, \infty)$ and CD on $(-\infty, -2)$. IP at $(-2, -2)$

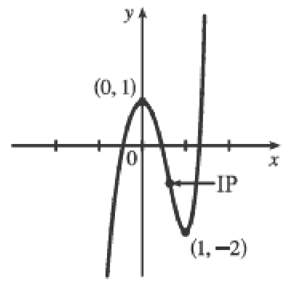
H.



7. $y = f(x) = 2x^5 - 5x^2 + 1$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 1$ **C.** No symmetry **D.** No asymptote

E. $f'(x) = 10x^4 - 10x = 10x(x^3 - 1) = 10x(x-1)(x^2 + x + 1)$, so $f'(x) < 0 \quad \Leftrightarrow \quad 0 < x < 1$ and $f'(x) > 0 \quad \Leftrightarrow \quad x < 0$ or $x > 1$. Thus, f is increasing on $(-\infty, 0)$ and $(1, \infty)$ and decreasing on $(0, 1)$. **F.** Local maximum value $f(0) = 1$, local minimum value $f(1) = -2$ **G.** $f''(x) = 40x^3 - 10 = 10(4x^3 - 1)$ so $f''(x) = 0 \quad \Leftrightarrow \quad x = 1/\sqrt[3]{4}$. $f''(x) > 0 \quad \Leftrightarrow \quad x > 1/\sqrt[3]{4}$ and $f''(x) < 0 \quad \Leftrightarrow \quad x < 1/\sqrt[3]{4}$, so f is CD on $\left(-\infty, 1/\sqrt[3]{4}\right)$ and CU on $\left(1/\sqrt[3]{4}, \infty\right)$. IP at $\left(\dfrac{1}{\sqrt[3]{4}}, 1 - \dfrac{9}{2\left(\sqrt[3]{4}\right)^2}\right) \approx (0.630, -0.786)$

H.



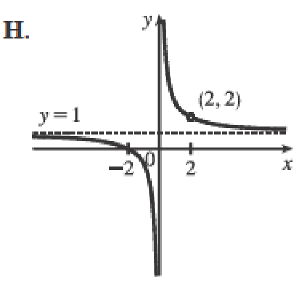
10. $y = f(x) = \dfrac{x^2 - 4}{x^2 - 2x} = \dfrac{(x+2)(x-2)}{x(x-2)} = \dfrac{x+2}{x} = 1 + \dfrac{2}{x}$ for $x \neq 2$. There is a hole in the graph at $(2, 2)$.

A. $D = \{x \mid x \neq 0, 2\} = (-\infty, 0) \cup (0, 2) \cup (2, \infty)$ **B.** y-intercept: none; x-intercept: -2 **C.** No symmetry

D. $\displaystyle\lim_{x \to \pm\infty} \frac{x+2}{x} = 1$, so $y = 1$ is a HA. $\displaystyle\lim_{x \to 0^-} \frac{x+2}{x} = -\infty$, $\displaystyle\lim_{x \to 0^+} \frac{x+2}{x} = \infty$, so $x = 0$ is a VA. **E.** $f'(x) = -2/x^2 < 0 \;\; [x \neq 0, 2]$ so f is decreasing on $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$. **F.** No extrema

G. $f''(x) = 4/x^3 > 0 \quad \Leftrightarrow \quad x > 0$, so f is CU on $(0, 2)$ and $(2, \infty)$ and CD on $(-\infty, 0)$. No IP.

H.



13. $y = f(x) = x/(x^2+9)$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercept: $f(x) = 0 \Leftrightarrow x = 0$

C. $f(-x) = -f(x)$, so f is odd and the curve is symmetric about the origin. **D.** $\lim_{x\to\pm\infty} [x/(x^2+9)] = 0$, so $y = 0$ is a HA; no VA **E.** $f'(x) = \dfrac{(x^2+9)(1) - x(2x)}{(x^2+9)^2} = \dfrac{9-x^2}{(x^2+9)^2} = \dfrac{(3+x)(3-x)}{(x^2+9)^2} > 0 \Leftrightarrow -3 < x < 3$, so f is increasing on $(-3, 3)$ and decreasing on $(-\infty, -3)$ and $(3, \infty)$. **F.** Local minimum value $f(-3) = -\frac{1}{6}$, local maximum value $f(3) = \frac{1}{6}$

$$f''(x) = \frac{(x^2+9)^2(-2x) - (9-x^2)\cdot 2(x^2+9)(2x)}{[(x^2+9)^2]^2} = \frac{(2x)(x^2+9)[-(x^2+9) - 2(9-x^2)]}{(x^2+9)^4} = \frac{2x(x^2-27)}{(x^2+9)^3}$$

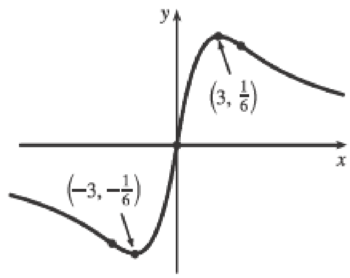
$$= 0 \quad \Leftrightarrow \quad x = 0, \pm\sqrt{27} = \pm 3\sqrt{3}$$

G. $$f''(x) = \frac{(x^2+9)^2(-2x) - (9-x^2)\cdot 2(x^2+9)(2x)}{[(x^2+9)^2]^2} = \frac{(2x)(x^2+9)\left[-(x^2+9) - 2(9-x^2)\right]}{(x^2+9)^4}$$

$$= \frac{2x(x^2-27)}{(x^2+9)^3} = 0 \quad \Leftrightarrow \quad x = 0, \pm\sqrt{27} = \pm 3\sqrt{3}$$

$f''(x) > 0 \Leftrightarrow -3\sqrt{3} < x < 0$ or $x > 3\sqrt{3}$, so f is CU on $(-3\sqrt{3}, 0)$ and $(3\sqrt{3}, \infty)$, and CD on $(-\infty, -3\sqrt{3})$ and $(0, 3\sqrt{3})$. There are three inflection points: $(0, 0)$ and $\left(\pm 3\sqrt{3}, \pm\frac{1}{12}\sqrt{3}\right)$.

H.



14. $y = f(x) = x^2/(x^2+9)$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercept: $f(x) = 0 \Leftrightarrow x = 0$

C. $f(-x) = f(x)$, so f is even and symmetric about the y-axis. **D.** $\lim_{x\to\pm\infty} \left[x^2/(x^2+9)\right] = 1$, so $y = 1$ is a HA; no VA

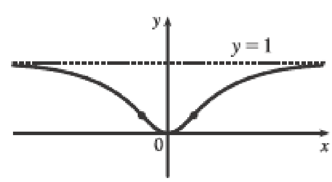
E. $f'(x) = \dfrac{(x^2+9)(2x) - x^2(2x)}{(x^2+9)^2} = \dfrac{18x}{(x^2+9)^2} > 0 \Leftrightarrow x > 0$, so f is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$. **F.** Local minimum value $f(0) = 0$; no local maximum

G. $$f''(x) = \frac{(x^2+9)^2(18) - 18x\cdot 2(x^2+9)\cdot 2x}{[(x^2+9)^2]^2} = \frac{18(x^2+9)[(x^2+9) - 4x^2]}{(x^2+9)^4} = \frac{18(9-3x^2)}{(x^2+9)^3}$$

$$= \frac{-54(x+\sqrt{3})(x-\sqrt{3})}{(x^2+9)^3} > 0 \quad \Leftrightarrow \quad -\sqrt{3} < x < \sqrt{3}$$

so f is CU on $(-\sqrt{3}, \sqrt{3})$ and CD on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$. There are two inflection points: $\left(\pm\sqrt{3}, \frac{1}{4}\right)$.

H.



21. $y = f(x) = \sqrt{x^2 + x - 2} = \sqrt{(x+2)(x-1)}$ **A.** $D = \{x \mid (x+2)(x-1) \geq 0\} = (-\infty, -2] \cup [1, \infty)$

B. y-intercept: none; x-intercepts: -2 and 1 **C.** No symmetry **D.** No asymptote

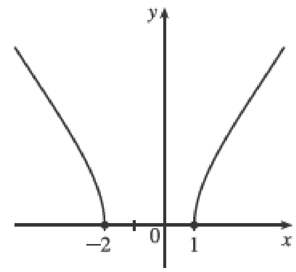
E. $f'(x) = \frac{1}{2}(x^2 + x - 2)^{-1/2}(2x + 1) = \dfrac{2x+1}{2\sqrt{x^2 + x - 2}}$, $f'(x) = 0$ if $x = -\frac{1}{2}$, but $-\frac{1}{2}$ is not in the domain.

$f'(x) > 0 \quad \Rightarrow \quad x > -\frac{1}{2}$ and $f'(x) < 0 \quad \Rightarrow \quad x < -\frac{1}{2}$, so (considering the domain) f is increasing on $(1, \infty)$ and f is decreasing on $(-\infty, -2)$. **F.** No local extrema

G.
$$f''(x) = \frac{2(x^2 + x - 2)^{1/2}(2) - (2x+1) \cdot 2 \cdot \frac{1}{2}(x^2 + x - 2)^{-1/2}(2x+1)}{\left(2\sqrt{x^2 + x - 2}\right)^2}$$
$$= \frac{(x^2 + x - 2)^{-1/2}\left[4(x^2 + x - 2) - (4x^2 + 4x + 1)\right]}{4(x^2 + x - 2)}$$
$$= \frac{-9}{4(x^2 + x - 2)^{3/2}} < 0$$

so f is CD on $(-\infty, -2)$ and $(1, \infty)$. No IP

H.



32. $y = f(x) = x + \cos x$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 1$; the x-intercept is about -0.74 and can be found using Newton's method **C.** No symmetry **D.** No asymptote **E.** $f'(x) = 1 - \sin x > 0$ except for $x = \frac{\pi}{2} + 2n\pi$, so f is increasing on $\mathbb{R}$. **F.** No local extrema

G. $f''(x) = -\cos x$. $f''(x) > 0 \quad \Rightarrow \quad -\cos x > 0 \quad \Rightarrow \quad \cos x < 0 \quad \Rightarrow$

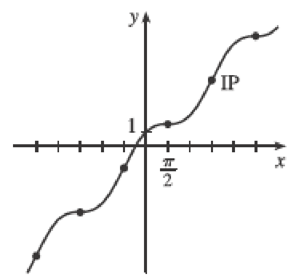
x is in $\left(\frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi\right)$ and $f''(x) < 0 \quad \Rightarrow$

x is in $\left(-\frac{\pi}{2} + 2n\pi, \frac{\pi}{2} + 2n\pi\right)$, so f is CU on $\left(\frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi\right)$ and CD on

$\left(-\frac{\pi}{2} + 2n\pi, \frac{\pi}{2} + 2n\pi\right)$. IP at $\left(\frac{\pi}{2} + n\pi, f\left(\frac{\pi}{2} + n\pi\right)\right) = \left(\frac{\pi}{2} + n\pi, \frac{\pi}{2} + n\pi\right)$

[on the line $y = x$]

H.

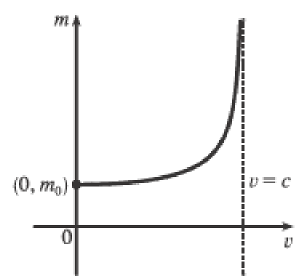


39. $m = f(v) = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$. The m-intercept is $f(0) = m_0$. There are no v-intercepts. $\lim_{v \to c^-} f(v) = \infty$, so $v = c$ is a VA.

$$f'(v) = -\tfrac{1}{2}m_0(1 - v^2/c^2)^{-3/2}(-2v/c^2) = \frac{m_0 v}{c^2(1 - v^2/c^2)^{3/2}} = \frac{m_0 v}{\dfrac{c^2(c^2 - v^2)^{3/2}}{c^3}} = \frac{m_0 c v}{(c^2 - v^2)^{3/2}} > 0,$$ so f is increasing on $(0, c)$. There are no local extreme values.

$$f''(v) = \frac{(c^2 - v^2)^{3/2}(m_0 c) - m_0 c v \cdot \frac{3}{2}(c^2 - v^2)^{1/2}(-2v)}{[(c^2 - v^2)^{3/2}]^2}$$
$$= \frac{m_0 c(c^2 - v^2)^{1/2}[(c^2 - v^2) + 3v^2]}{(c^2 - v^2)^3} = \frac{m_0 c(c^2 + 2v^2)}{(c^2 - v^2)^{5/2}} > 0,$$

so f is CU on $(0, c)$. There are no inflection points.



45. $y = \dfrac{4x^3 - 2x^2 + 5}{2x^2 + x - 3}$. Long division gives us:

$$\begin{array}{r|l} & 2x - 2 \\ \hline 2x^2 + x - 3 & 4x^3 - 2x^2 \qquad + 5 \\ & 4x^3 + 2x^2 - 6x \\ \hline & -4x^2 + 6x + 5 \\ & -4x^2 - 2x + 6 \\ \hline & 8x - 1 \end{array}$$

Thus, $y = f(x) = \dfrac{4x^3 - 2x^2 + 5}{2x^2 + x - 3} = 2x - 2 + \dfrac{8x - 1}{2x^2 + x - 3}$ and $f(x) - (2x - 2) = \dfrac{8x - 1}{2x^2 + x - 3} = \dfrac{\frac{8}{x} - \frac{1}{x^2}}{2 + \frac{1}{x} - \frac{3}{x^2}}$

[for $x \neq 0$] $\to 0$ as $x \to \pm\infty$. So the line $y = 2x - 2$ is a SA.

52. $y = f(x) = \dfrac{(x+1)^3}{(x-1)^2} = \dfrac{x^3 + 3x^2 + 3x + 1}{x^2 - 2x + 1} = x + 5 + \dfrac{12x - 4}{(x-1)^2}$

A. $D = \{x \in \mathbb{R} \mid x \neq 1\} = (-\infty, 1) \cup (1, \infty)$ **B.** y-intercept: $f(0) = 1$; x-intercept: $f(x) = 0 \Rightarrow x = -1$ **C.** No symmetry **D.** $\lim\limits_{x \to 1} f(x) = \infty$, so $x = 1$ is a VA.

$\lim\limits_{x \to \pm\infty} [f(x) - (x + 5)] = \lim\limits_{x \to \pm\infty} \dfrac{12x - 4}{x^2 - 2x + 1} = \lim\limits_{x \to \pm\infty} \dfrac{\frac{12}{x} - \frac{4}{x^2}}{1 - \frac{2}{x} + \frac{1}{x^2}} = 0$, so the line $y = x + 5$ is a SA.

E. $f'(x) = \dfrac{(x-1)^2 \cdot 3(x+1)^2 - (x+1)^3 \cdot 2(x-1)}{[(x-1)^2]^2}$

$$= \frac{(x-1)(x+1)^2[3(x-1) - 2(x+1)]}{(x-1)^4} = \frac{(x+1)^2(x-5)}{(x-1)^3}$$

so $f'(x) > 0$ when $x < -1$, $-1 < x < 1$, or $x > 5$, and $f'(x) < 0$ when $1 < x < 5$. f is increasing on $(-\infty, 1)$ and $(5, \infty)$ and decreasing on $(1, 5)$.

F. Local minimum value $f(5) = \frac{216}{16} = \frac{27}{2}$, no local maximum

G. $f''(x) = \dfrac{(x-1)^3[(x-1)^2 + (x-5) \cdot 2(x+1)] - (x+1)^2(x-5) \cdot 3(x-1)^2}{[(x-1)^3]^2}$

$$= \frac{(x-1)^2(x+1)\{(x-1)[(x+1) + 2(x-5)] - 3(x+1)(x-5)\}}{(x-1)^6}$$

$$= \frac{(x+1)\{(x-1)[3x-9] - 3(x^2 - 4x - 5)\}}{(x-1)^4} = \frac{(x+1)(24)}{(x-1)^4}$$

so $f''(x) > 0$ if $-1 < x < 1$ or $x > 1$, and $f''(x) < 0$ if $x < -1$. Thus, f is CU on $(-1, 1)$ and $(1, \infty)$ and CD on $(-\infty, -1)$. IP at $(-1, 0)$

H.