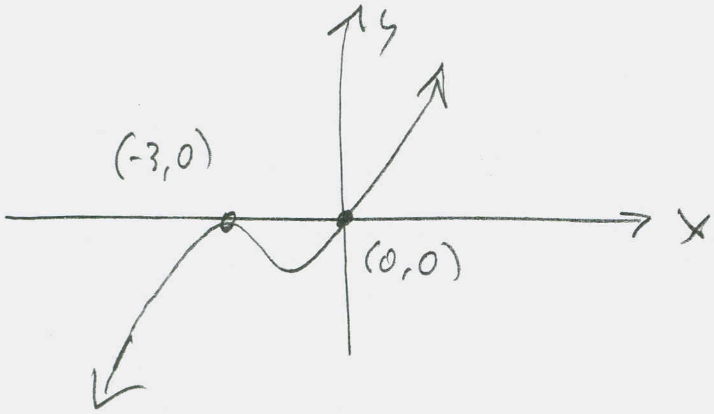
201 §4.5 #s 2, 7, 10, 13, 14, 21, 32, 39, 45, 52

#s 1-38 Use the guidelines to sketch the curve.

(2) $f(x) = x^3 + 6x^2 + 9x$

SET = 0 $\Longrightarrow$ $x(x^2+6x+9) = x(x+3)^2 = 0$



Now the issue is to find min & inflection point(s)

$f'(x) = 3x^2 + 12x + 9 = 3(x^2+4x+3) = 3(x+3)(x+1) \overset{SET}{=} 0$

$\Longrightarrow x = -3$ or $x = -1$

$f(-3) = 0 \rightsquigarrow \boxed{(-3,0) \text{ MAX}}$

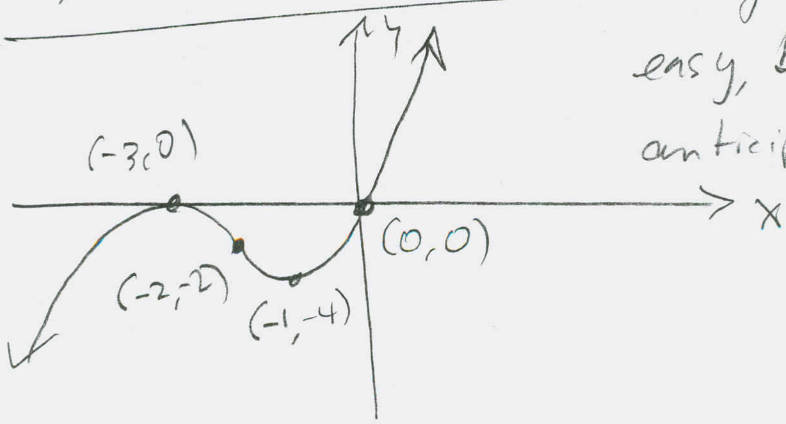
$f(-1) = -1 + 6 - 9 = -4 \rightsquigarrow \boxed{(-1,-4) \text{ MIN}}$

$f''(x) = 6x + 12 \overset{SET}{=} 0 \longrightarrow x = -2$

$f(-2) = (-2)^3 + 6(-2)^2 + 9(-2) = -8 + 24 - 18 = -2$

$\rightsquigarrow \boxed{(-2,-2) \text{ is I.P.}}$

Polynomials are easy, because we anticipate what they look like.



201 §4.5 #s 7, 10, 13, 14, 21, 32, 39, 45, 52

(7) $f(x) = 2x^5 - 5x^2 + 1$

$f'(x) = 10x^4 - 10x$

$f''(x) = 40x^3 - 10$

$f'(x) \overset{SET}{=} 0$

$10x(x^3 - 1) = 0$

$10x(x-1)(x^2+x+1) = 0$

$+ \quad - \quad +$ f'

$\nearrow \quad 0 \quad \searrow \quad 1 \quad \nearrow$

$x = 0, 1$

$f(0) = 1 \rightsquigarrow$ (0, 1) MAX

$f(1) = -2 \rightsquigarrow$ (1, −2) MIN

$f''(x) \overset{SET}{=} 0$

$10(4x^3 - 1) = 0$

$- \quad + \quad f''$

$\frown \quad .63 \quad \smile$

$4x^3 = 1$

$x^3 = \frac{1}{4}$

$x = \sqrt[3]{\frac{1}{4}} = \frac{\sqrt[3]{2}}{2}$

$f\left(\frac{\sqrt[3]{2}}{2}\right) = 2\left(\sqrt[3]{\frac{1}{4}}\right)^5 - 5\left(\sqrt[3]{\frac{1}{4}}\right)^2 + 1 \approx -.7858$

$= 2\left(\frac{1}{4^{5/3}}\right) - 5\left(\frac{1}{4^{2/3}}\right) + 1$ $\rightsquigarrow (.62996, -.7858)$

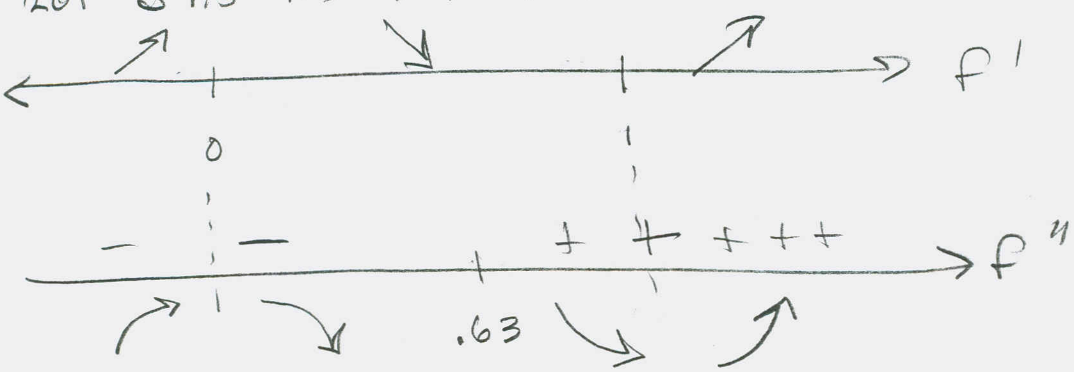
$\approx \boxed{(.63, -.79) \text{ I.P.}}$

$= \frac{2}{2^{\frac{10}{3}}} - \frac{5}{2^{\frac{4}{3}}} + 1$

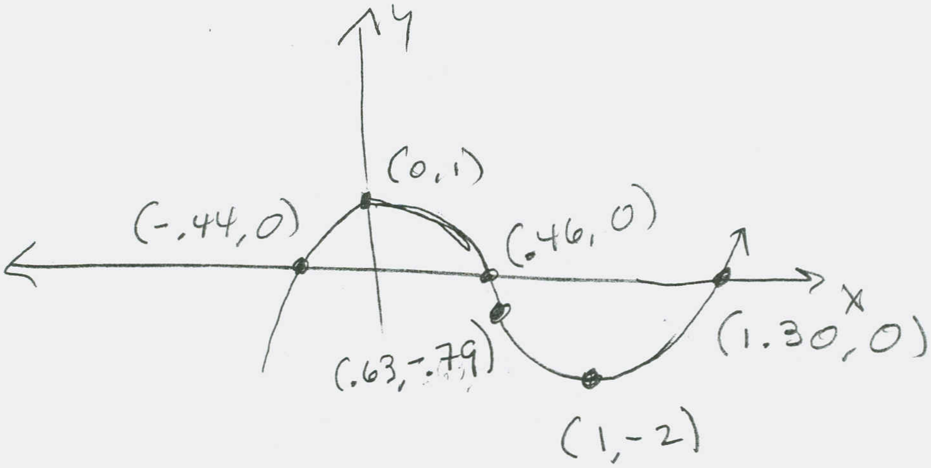
$= 2^{1-\frac{10}{3}} - \frac{5}{2^{1+\frac{1}{3}}} + 1 = 2^{-\frac{7}{3}} - \frac{5}{2\sqrt[3]{2}} + 1$

$= \frac{1}{2^2\sqrt[3]{2}} - \frac{5}{2\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} + 1 \approx$

201 §4.5 #s 7, 10, 13, 14, 21, 32, 39, 45, 52



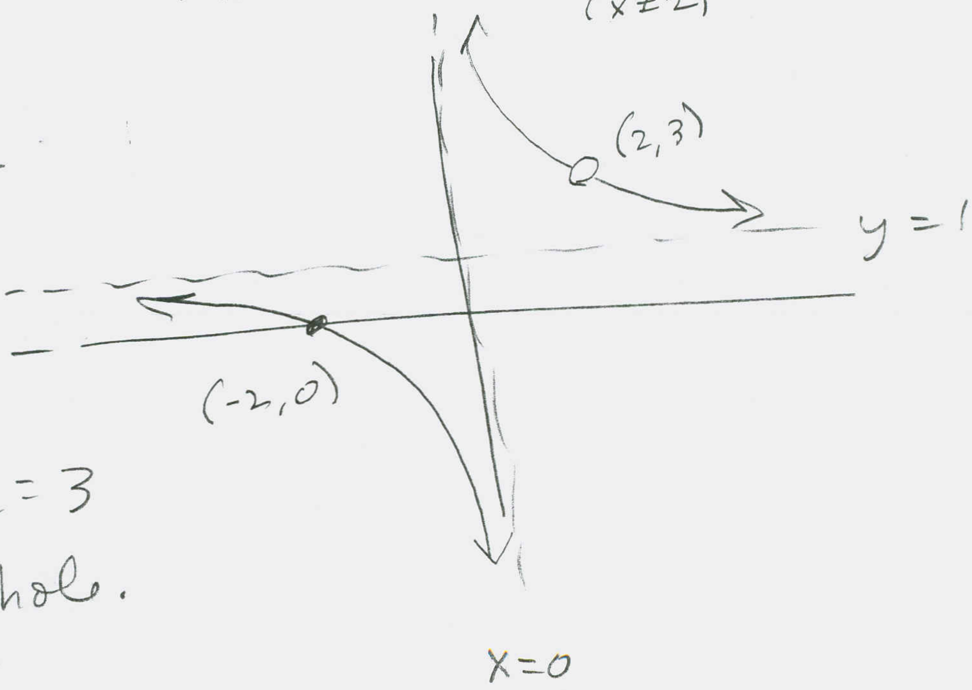
$f(0) = 1 \rightsquigarrow (0,1)$



(10) $f(x) = \dfrac{x^2-4}{x^2-2x} = \dfrac{(x-2)(x+2)}{x(x-2)} = \dfrac{x+2}{x} = 1 + \dfrac{2}{x} = R^*(x)$

$(x \neq 2)$

$$\begin{array}{r|rr} 0 & 1 & 2 \\ & & 0 \\ \hline & 1 & 2 \end{array}$$



$R^*(2) = 1 + \frac{2}{2} = 3$

(2,3) is hole.

201 §4.5 #s 13, 14, 21, 32, 39, 45, 52

13 $f(x) = \dfrac{x}{x^2+9}$ $\quad f(0)=0 \rightsquigarrow (0,0)$

$$f'(x) = \frac{x^2+9-(x(2x))}{(x^2+9)^2} = \frac{-x^2+9}{(x^2+9)^2} = \frac{-(x-3)(x+3)}{(x^2+9)^2}$$

$\boxed{\begin{array}{l} y=0 \text{ is H.A.} \\ \text{No V.A.} \end{array}}$

$$f''(x) = \frac{-2x(x^2+9)^2 - (-x^2+9)(2(x^2+9)(2x))}{(x^2+9)^4}$$

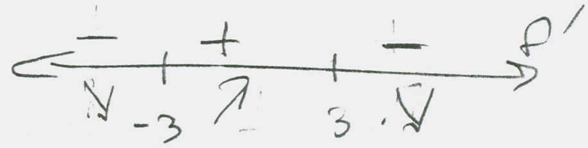
$$= \frac{-2x(x^2+9) - 4x(-x^2+9)}{(x^2+9)^3}$$

$$= \frac{-2x^3-18x+4x^3-36x}{(x^2+9)^3} = \frac{2x^3-54x}{(x^2+9)^3} = \frac{2x(x^2-27)}{(x^2+9)^3}$$

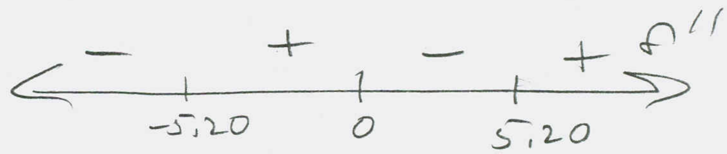
$f'(x)=0 \Longrightarrow x = \pm 3$

$f(3) = \frac{3}{18} = \frac{1}{6} \rightsquigarrow \boxed{\text{MAX } \left(3, \tfrac{1}{6}\right)}$

$f(-3) = -\frac{1}{6} \rightsquigarrow \boxed{\left(-3, -\tfrac{1}{6}\right) \text{ MIN}}$



$f''(x)=0 \Longrightarrow$



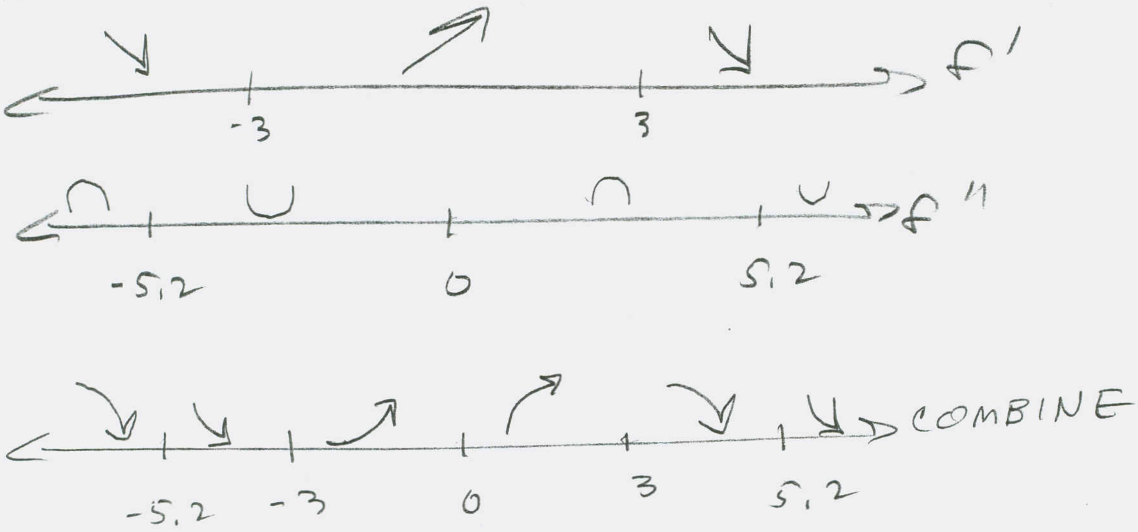
$x=0$ OR $x^2-27=0$

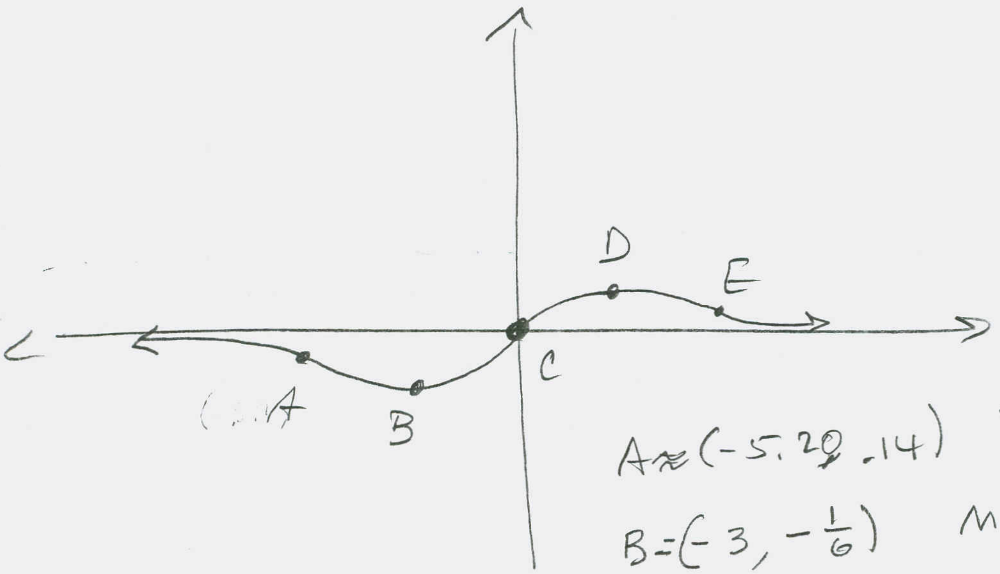
$x = \pm\sqrt{27}$

$x = \pm 3\sqrt{3} \approx \pm 5.19615 2423$

201 §4.5 #s 13, 14, 21, 32, 39, 45, 52

13 cont'd





$A \approx (-5.20, .14)$ IP

$B = (-3, -\frac{1}{6})$ MIN

$C = (0, 0)$ IP & x-int & y-int.

$D = (3, \frac{1}{6})$ MAX

$E \approx (5.20, -.14)$ IP

201 §4.5 #s 14, 21, 32, 39, 45, 52

(14) $f(x) = \dfrac{x^2}{x^2+9}$ is even. $\mathcal{D} = \mathbb{R}$, $\boxed{y = 1 \text{ is H.A.}}$

x-int = y-int = $(0,0)$

$$f'(x) = \frac{2x(x^2+9) - x^2(2x)}{(x^2+9)^2} = \frac{2x^3+18x-2x^2}{(x^2+9)^2}$$

$$= \frac{18x}{(x^2+9)^2}$$

$$f''(x) = \frac{18(x^2+9)^2 - 18x(2(x^2+9)(2x))}{(x^2+9)^4}$$

$$= \frac{18(x^2+9)[x^2+9-4x^2]}{(x^2+9)^4} = \frac{18[9-3x^2]}{(x^2+9)^3}$$

$f'(x) \overset{SET}{=} 0 \implies x = 0$

$\boxed{(0,0) \text{ is MIN}}$

$-$ | $+$ f'

↘ 0 ↗

$f''(x) \overset{SET}{=} 0 \implies -3(x^2-3) = 0 \implies x = \pm\sqrt{3}$

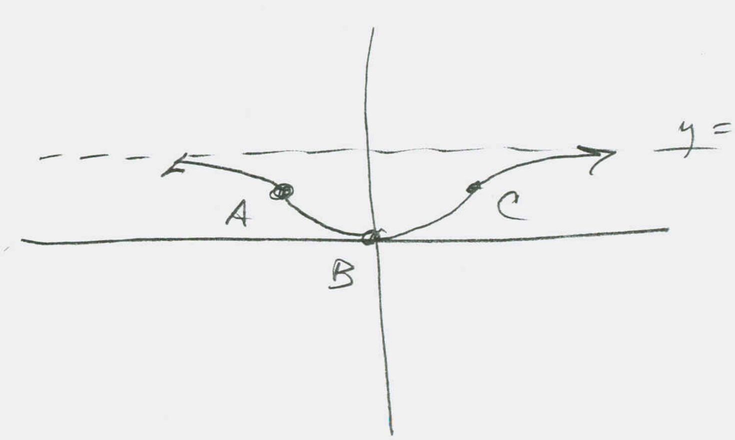
$-$ | $+$ | $-$ f''

$\cap$ $-\sqrt{3}$ $\cup$ $\sqrt{3}$ $\cap$

$f(\sqrt{3}) = \frac{3}{12} = \frac{1}{4}$

$\boxed{(\pm\sqrt{3}, \tfrac{1}{4}) \text{ is IP}}$

$A = (-\sqrt{3}, \frac{1}{4})$ IP
$B = (0,0)$ MIN
$C = (\sqrt{3}, \frac{1}{4})$ IP



201 §4.5 #s 21, 32, 39, 45, 52

(21) $f(x)=\sqrt{x^2+x-2} = (x^2+x-2)^{\frac{1}{2}}$

$f(0) \nexists$

Need $x^2+x-2 \geq 0 \implies (x+2)(x-1) > 0 \implies$

$\mathcal{D} = (-\infty,-2] \cup [1,\infty)$

$$f'(x) = \tfrac{1}{2}(x^2+x-2)^{-\frac{1}{2}}(2x+1) = \frac{2x+1}{2\sqrt{x^2+x-2}} = (2x+1)\left(\tfrac{1}{2}\right)(x^2+x-2)^{-\frac{1}{2}}$$

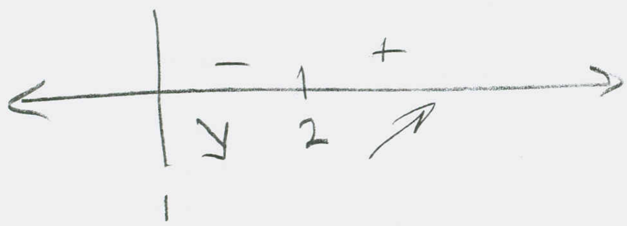
$$f''(x) = (x^2+x-2)^{-\frac{1}{2}} + x\left(-\tfrac{1}{2}(x^2+x-2)^{-\frac{3}{2}}\right)(2x)$$

$$= \frac{1}{\sqrt{x^2+x-2}} - \frac{x^2}{\sqrt{(x^2+x-2)^3}} = \frac{x^2+x-2-x^2}{\sqrt{(x^2+x-2)^3}}$$

$$= \frac{x-2}{\left(\sqrt{x^2+x-2}\right)^3}$$

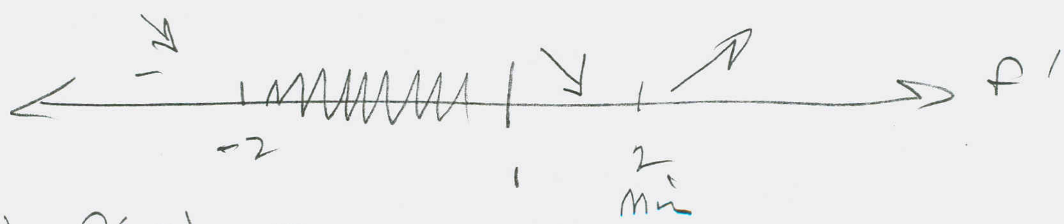
$f'(x) \overset{SET}{=} 0 \implies x=2$

$f'(x) \nexists$ when $x=-2, 1$ (Endpoints of $\mathcal{D}$)
Special



To the left of $x=-2$:

$f'(-3) = \dfrac{-3}{pos} = $ "<0" ↘



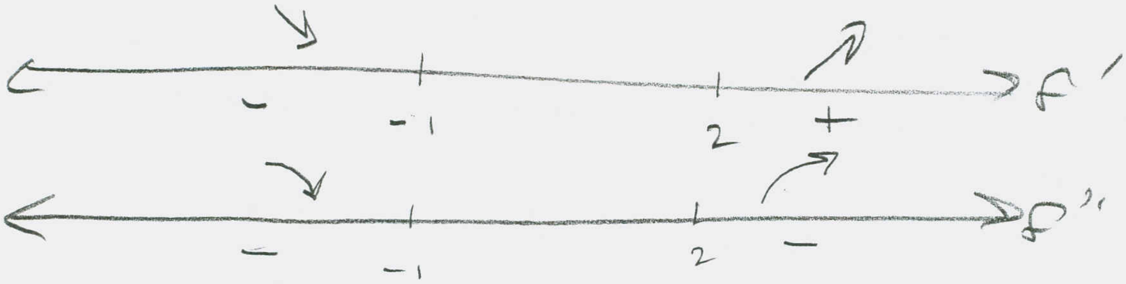
$f(1) = f(-2) = 0$, $f(2) = 2$

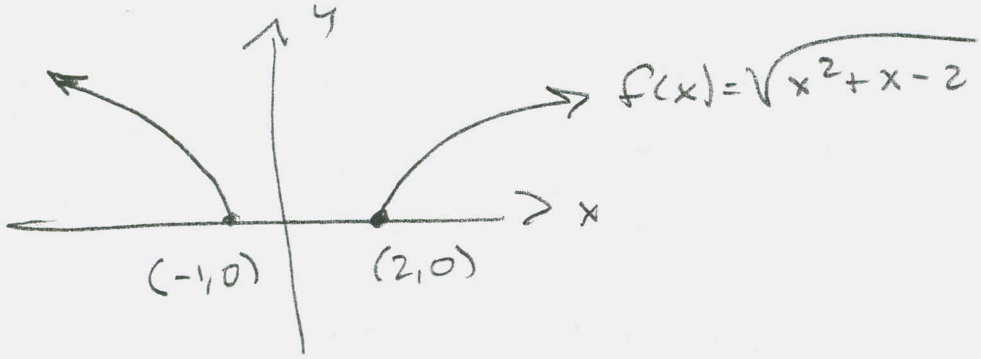
201 §4.5 #s 21, 32, 39, 45, 52

21 cnt'd

so $f''(x) < 0$ on its domain, and

$\mathcal{D}(f'') = \mathcal{D}(f') = (-\infty, -1) \cup (2, \infty)$





32 $y = x \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$y' = \tan x + x \sec^2 x \overset{\text{SET}}{=} 0$ & discover we haven't analytical techniques to solve: Use graphing calculator.

$y'' = \sec^2 x + \sec^2 x + x(2 \sec x)(\sec x \tan x)$

$= \sec^2 x [\, 2 + x \tan x \,] \overset{\text{SET}}{=} 0$ & same deal.

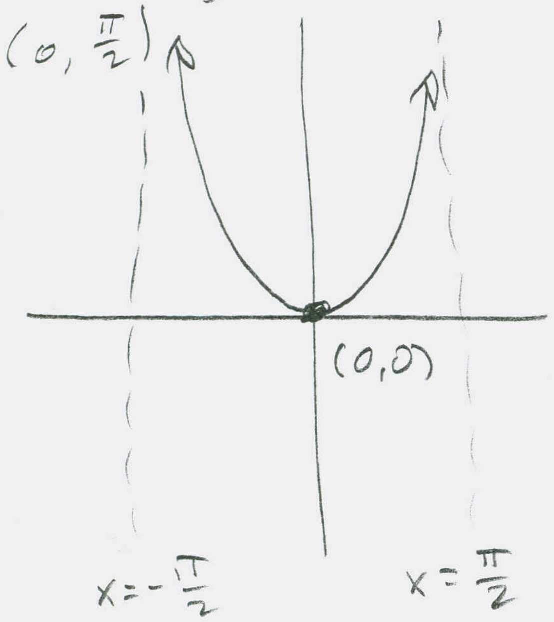
$y' = 0$ when $x = 0$ is apparent

$y'' = 0$ is NOT so apparent. $\sec^2 x > 0 \ \forall x$, and, $2 + x \tan x > 0 \ \forall x$, also, since $x \tan x$ is even function ≥ 0 on $(-\frac{\pi}{2}, \frac{\pi}{2})$. I guess this wasn't so hard.

201 §4.5 #s 32, 39, 45, 52

(32) Cntd. What else can be gleaned from $f'' > 0$? Hmmmm. f' must be increasing on its domain.

$\therefore$, since $f'(0) = 0$, we have f is decreasing on $(-\frac{\pi}{2}, 0)$ and increasing on



Concave up, everywhere. undefined (vertical asymptotes) @ $x = \pm\frac{\pi}{2}$

Even function.

$f(0) = 0$

(39) Given $m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ = mass of a particle with mass @ rest, m_0, moving at velocity v, and c = speed of light.

We sketch m as a function of v.

$m = m_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \longrightarrow$

$$\frac{dm}{dv} = -\frac{1}{2} m_0 \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}}\left(-\frac{2v}{c^2}\right) = \frac{m_0 v}{c^2}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}}$$

201 §4.5 #s 39, 45, 52

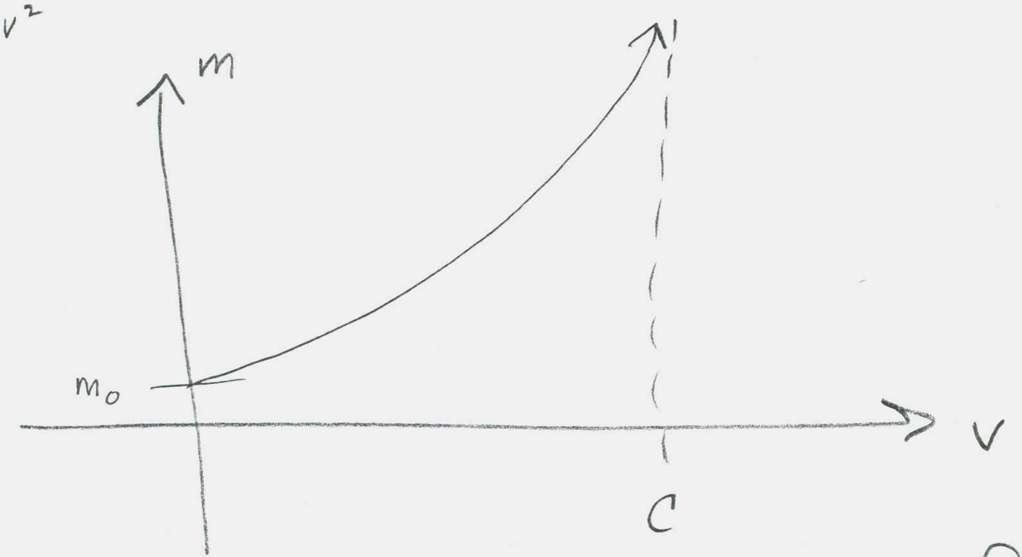
39 cnt'd

$$\frac{d^2m}{dv^2} = \frac{m_0}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}} + \frac{m_0 v}{c^2}\left(-\frac{3}{2}\left(1-\frac{v^2}{c^2}\right)^{-\frac{5}{2}}\left(-\frac{2v}{c^2}\right)\right)$$

$$= \frac{m_0}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}} + \frac{3m_0v^2}{c^4}\left(1-\frac{v^2}{c^2}\right)^{-\frac{5}{2}}$$

$\frac{dm}{dv} > 0$ on its domain: Mass grows as velocity increases.

$\frac{d^2m}{dv^2} > 0$ on its domain: Concave up.



Mass $\longrightarrow \infty$ as $v \longrightarrow$ speed of light.

201 §4.5 #s 45, 52

45 Find an eq'n of the slant asymptote. Don't sketch the curve.

$$y = \frac{4x^3-2x^2+5}{2x^2+x-3}$$

$$\begin{array}{r|l} & 2x - 2 \\ 2x^2+x-3 & 4x^3-2x^2+0x+5 \\ & -(4x^3+2x^2-6x) \\ \hline & \quad -4x^2+6x+5 \end{array}$$

You could stop right here, for the purposes of this exercise. $\boxed{y = 2x-2 \text{ is S.A.}}$

I'll take it on out and show you what we're looking at

$$\begin{array}{r|l} & 2x - 2 \ \ r\ \ 8x-1 \\ 2x^2+x-3 & 4x^3-2x^2+0x+5 \\ & -(4x^3+2x^2-6x) \\ \hline & \quad -4x^2+6x+5 \\ & \quad -(-4x^2-2x+6) \\ \hline & \qquad 8x-1 \end{array}$$

This means that

$$y = \frac{4x^3-2x^2+5}{2x^2+x-3} = 2x-2+\frac{8x-1}{2x^2+x-3}$$

This part vanishes for large $|x|$.

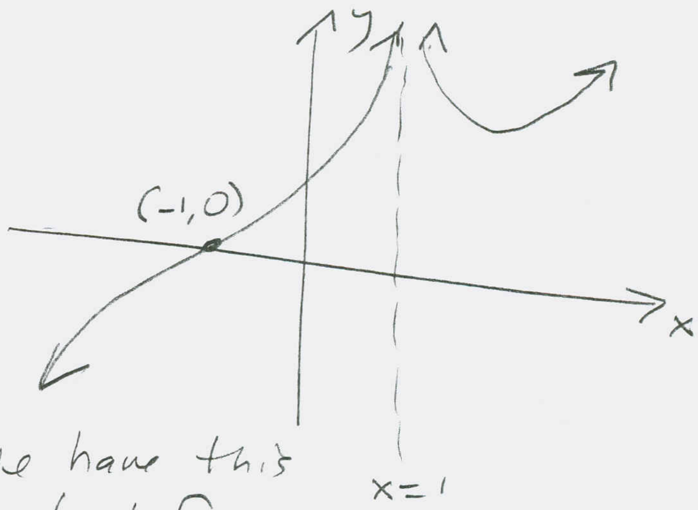
201 §4.5 #52

52 Sketch, using all your skills!

$$y = f(x) = \frac{(x+1)^3}{(x-1)^2}$$

$\mathcal{D} = \mathbb{R} \setminus \{1\}$

$$\boxed{\begin{array}{l} \text{V.A.: } x = 1 \\ \text{x-int: } (-1,0) \\ \text{y-int: } (0,1) \end{array}}$$



we have this much, before figuring derivatives & slant asymptote.

Slant: $(x+1)^3 = x^3 + 3x^2 + 3x + 1$

$(x-1)^2 = x^2 - 2x + 1$

Divide:

$$\begin{array}{r} x \quad +5 \\ x^2 - 2x + 1 \overline{\big)\, x^3 + 3x^2 + 3x + 1} \\ -(x^3 - 2x^2 + x) \\ \hline 5x^2 + 2x + 1 \end{array}$$

$\boxed{y = x + 5 \text{ is S.A.}}$

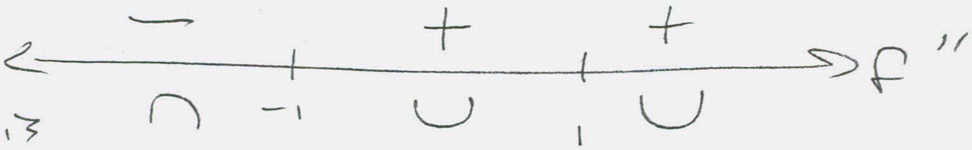
$$f'(x) = \frac{3(x+1)^2(x-1)^2 - 2(x+1)^3(x-1)}{(x-1)^4} = \frac{3(x+1)^2(x-1) - 2(x+1)^3}{(x-1)^3}$$

$$= \frac{(x+1)^2(3(x-1) - 2(x+1))}{(x-1)^3} = \frac{(x+1)^2(3x - 3 - 2x - 2)}{(x-1)^3}$$

201 §4.5 #52

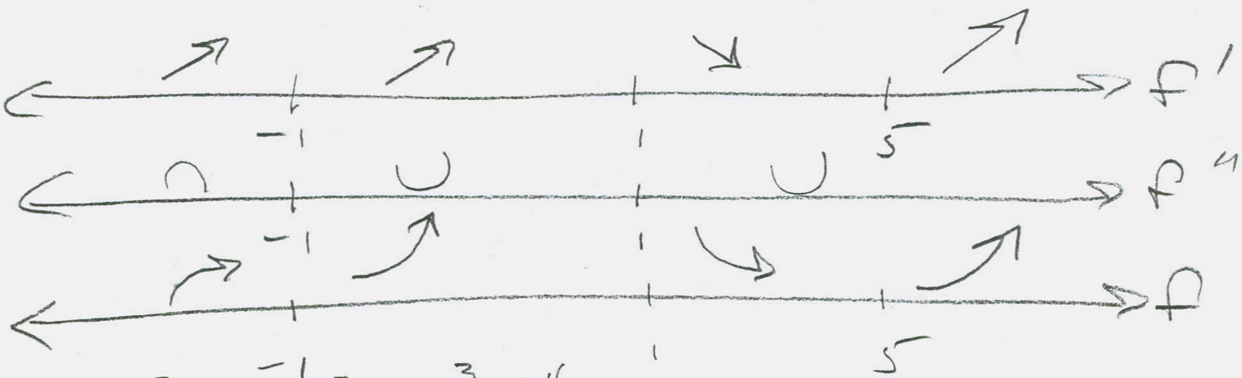
52 cnt'd

So f is



is the sign pattern, since x=1 has multiplicity 4 and x=−1 has multiplicity 4.

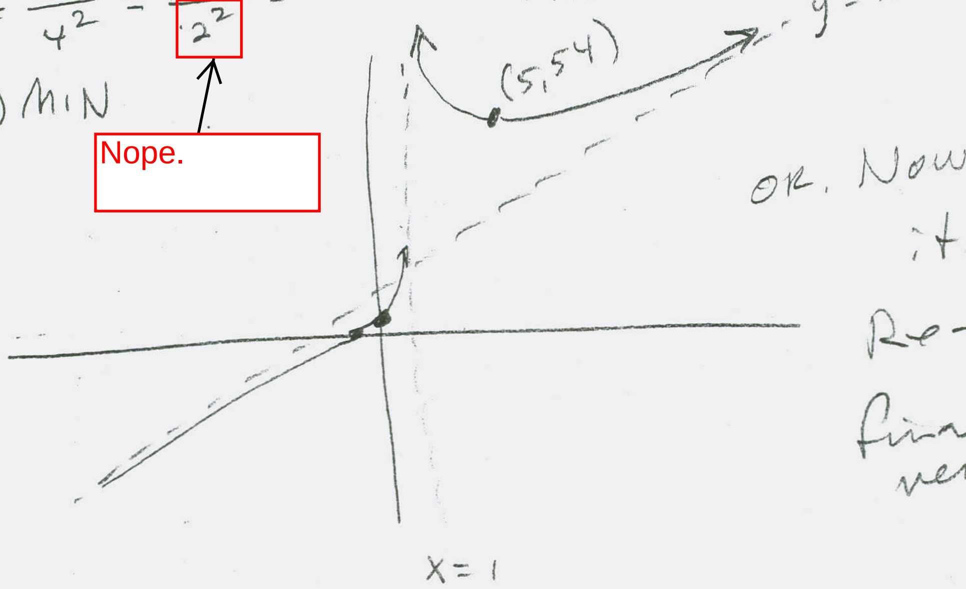
So f' & f'' combined:



$f(5) = \frac{6^3}{4^2} = \frac{2^3 \cdot 3^3}{\boxed{2^2}} = 2 \cdot 3^3 = 54$

(5,54) MIN

Nope.

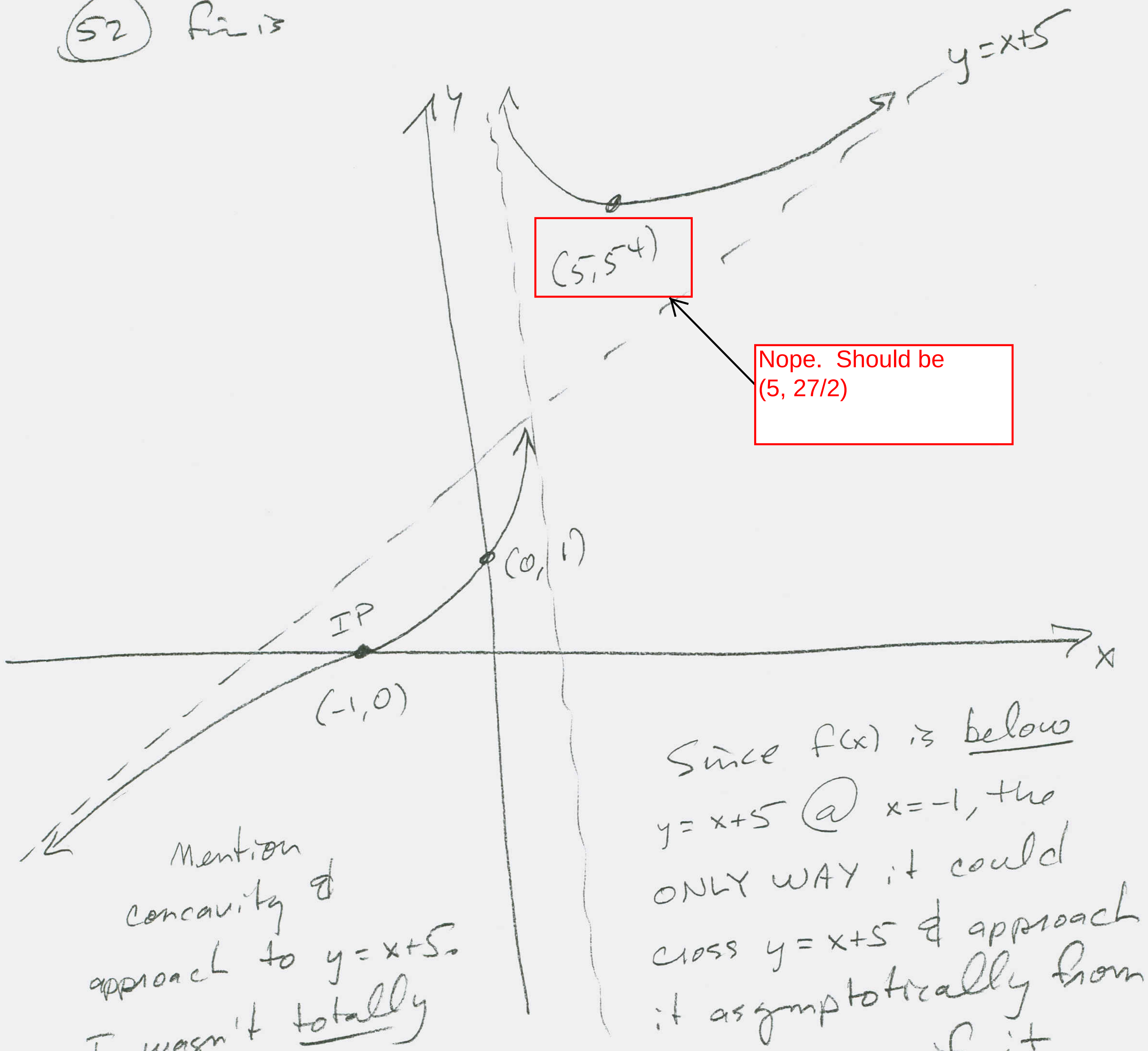


OK. Now I see it.

Re-sketch final version

201 §4.5 #52

52 fin 13



Since f(x) is <u>below</u> y = x+5 @ x = −1, the ONLY WAY it could cross y = x+5 & approach it asymptotically from <u>above</u> is if it had more IPs than we found.

That's all...

Mention concavity & approach to y = x+5.

I wasn't <u>totally</u> full of it when answering Damiel's question about how f(x) must approach y = x+5 as x → −∞.