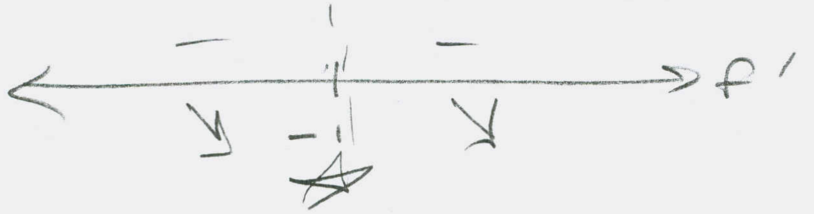
201 §4.4 II #s 43, 46, 48, 62

#s 43–46 Find H.A. of the curve and use them together with concavity and intervals of increase & decrease to sketch the curve.

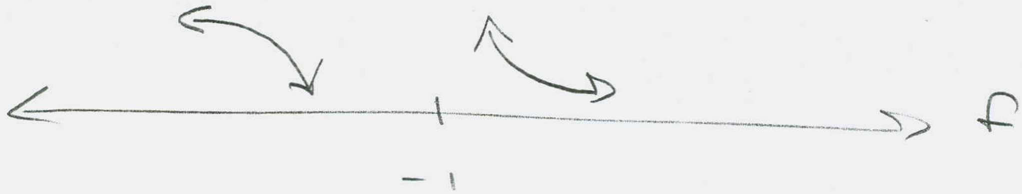
43) $f(x) = \dfrac{1-x}{1+x} \xrightarrow{|x| \to \infty} \boxed{-1 = y}$ is H.A.

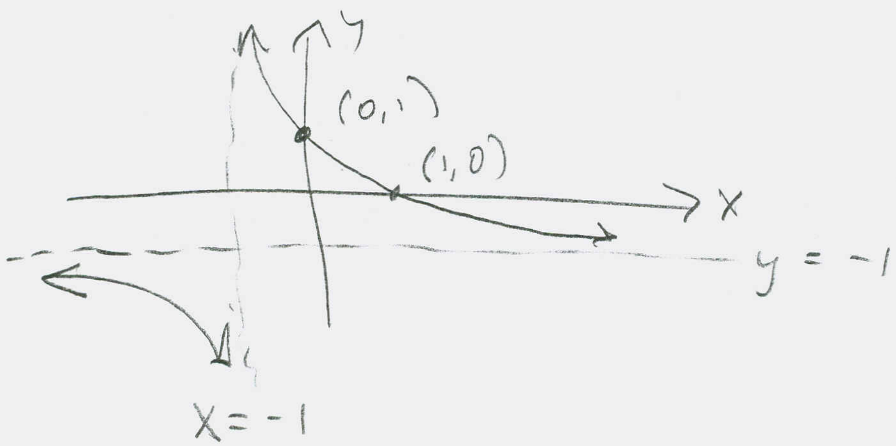
$$f'(x) = \frac{-1(x+1) - (1-x)(1)}{(x+1)^2} = \frac{-x-1-1+x}{(x+1)^2} = \frac{-2}{(x+1)^2}$$




Always decreasing on its domain

$$f''(x) = -2(-2(x+1)^{-3}) = \frac{4}{(x+1)^3}$$

201 §4.4 #46, 48, 62

(46) $f(x) = \dfrac{x}{\sqrt{x^2+1}} = x(x^2+1)^{-\frac{1}{2}} \longrightarrow$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{x}{|x|\sqrt{1+\frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{x}{x\sqrt{1+\frac{1}{x^2}}} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}} = \boxed{1 = y} \text{ HA!}$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x}{-x\sqrt{1+\frac{1}{x^2}}} = \lim_{x\to-\infty} \frac{-1}{\sqrt{1+\frac{1}{x^2}}} = \boxed{-1 = y}$$

HA!

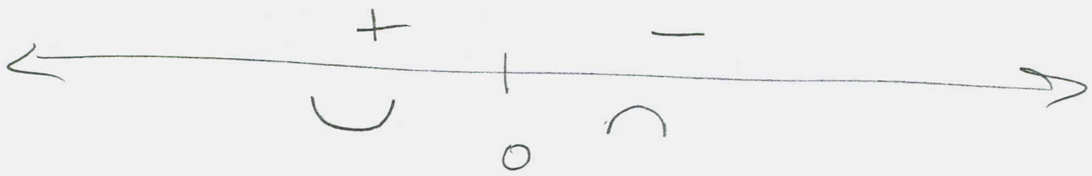
$$f'(x) = 1(x^2+1)^{-\frac{1}{2}} + x\left(-\tfrac{1}{2}(x^2+1)^{-\frac{3}{2}}\right)(2x)$$

$$= \frac{1}{\sqrt{x^2+1}} - \frac{x^2}{\left(\sqrt{x^2+1}\right)^3} = \frac{x^2+1-x^2}{\sqrt{x^2+1}^{\,3}} = \frac{1}{\sqrt{x^2+1}^{\,3}}$$

No critical values Always positive. $\mathcal{D} = \mathbb{R}$.

$$f''(x) = \frac{d}{dx}\left[(x^2+1)^{-\frac{3}{2}}\right] = -\tfrac{3}{2}(x^2+1)^{-\frac{5}{2}}(2x)$$

$$= \frac{-3x}{(x^2+1)^{5/2}}$$



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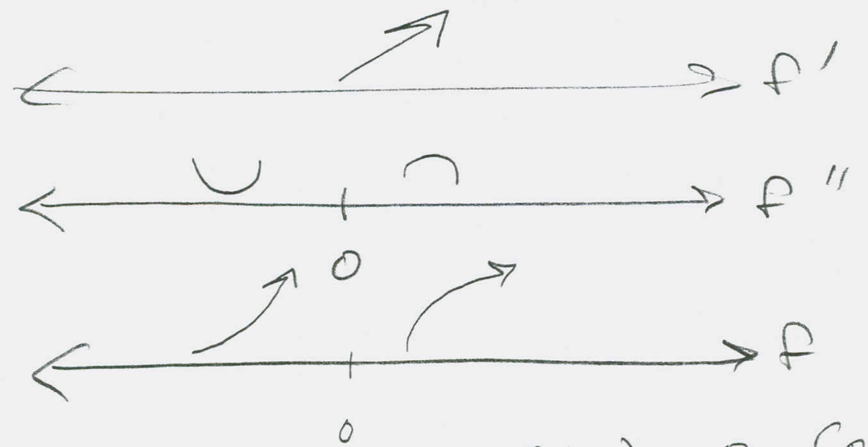
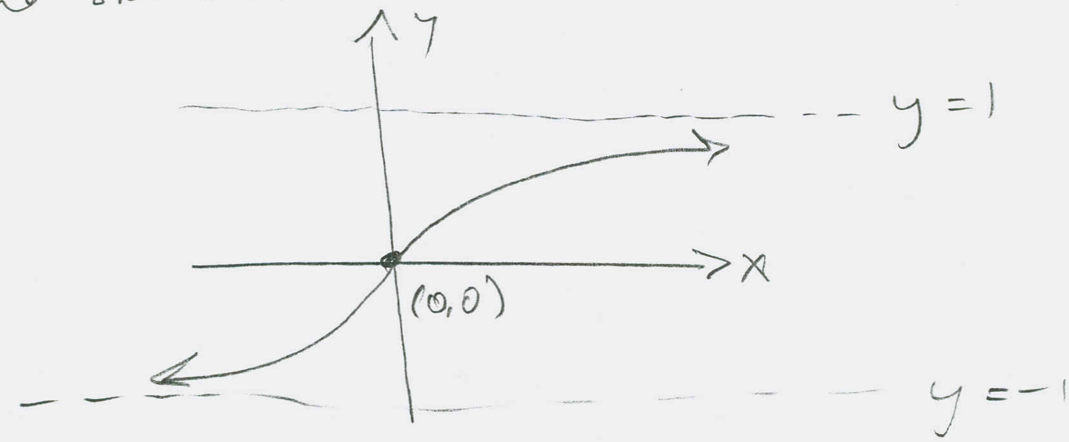
(46)




$f(0) = 0 \quad (0,0)$

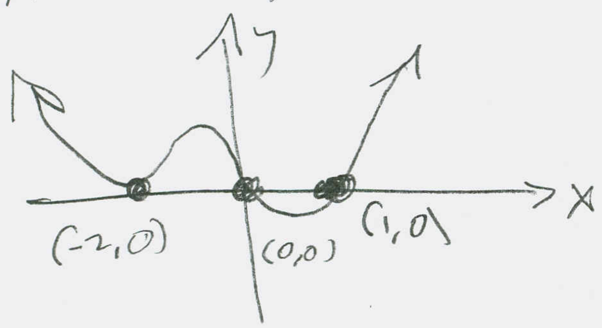
Final sketch




(48) Find limits as $x \to \infty$ & $x \to -\infty$. Use this info & intercepts to give a rough sketch

(48) $f(x) = x^3(x+2)^2(x-1)$

As $x \to \infty$, $f(x) \longrightarrow (x^3)(x^2)(x) = x^6$



201 § 4.4 # 62

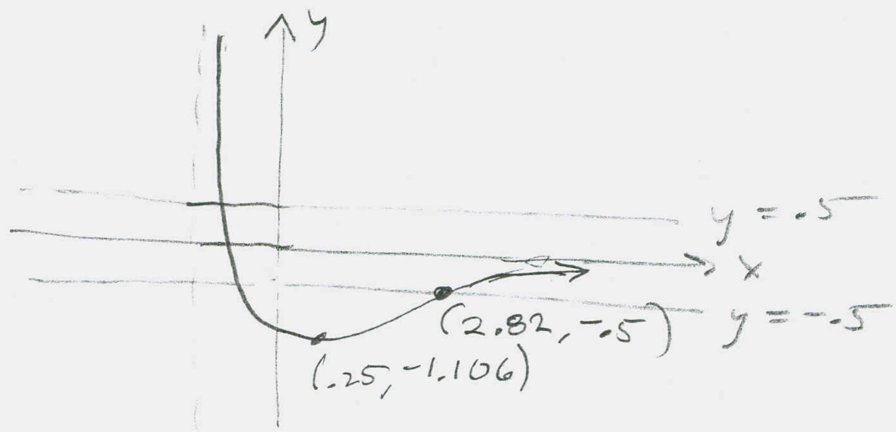
62 Find values of N corresponding to $\varepsilon = .5$ & $\varepsilon = 0.1$. For those who are interested, I have done a FORMAL argument on another sheet. For OUR purposes, we find $N \ni \left| \frac{\sqrt{4x^2+1}}{x+1} - 2 \right| < \varepsilon$ for the desired values of $\varepsilon$ by graphing.

$$f(x) = \frac{\sqrt{4x^2+1}}{x+1} - 2,$$

$$g(x) = -.5$$

$$h(x) = +.5$$

Then find the intersection:




$x = -1$

The intersection is @ $(2.82, -.5)$

$$\boxed{\text{Use } N = 3 \text{ and } \forall\, x > 3,\ |f(x)| < .5}$$

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Now, can we be SURE that $f(x)$ stays inside the $y=-.5$ to $y=+.5$ "tube?" Not from what we've done, but we can do some calculus to make sure

$$f'(x) = \frac{d}{dx}\left[(4x^2+1)^{\frac{1}{2}}(x+1)^{-1} - 2\right]$$

$$= \tfrac{1}{2}(4x^2+1)^{-\frac{1}{2}}(8x)(x+1)^{-1} + (4x^2+1)^{\frac{1}{2}}(-1(x+1)^{-2})$$

$$= \frac{4x}{(x+1)\sqrt{4x^2+1}} - \frac{\sqrt{4x^2+1}}{(x+1)^2} = \frac{4x(x+1)-(4x^2+1)}{(x+1)^2\sqrt{4x^2+1}}$$

$$= \frac{4x^2+4x-4x^2-1}{(x+1)^2\sqrt{4x^2+1}} = \frac{4x-1}{(x+1)^2\sqrt{4x^2+1}}$$

Note this has a critical value @ $x=\frac{1}{4}$, which corresponds to the minimum in the sketch, and this is the ONLY critical value.

So $f'(x)>0 \;\forall\; x>\frac{1}{4}$. This means it can't "wiggle out" of the tube. But what will keep it from growing bigger than $y=+.5$?

This is a good question, but I'm leaving it open.

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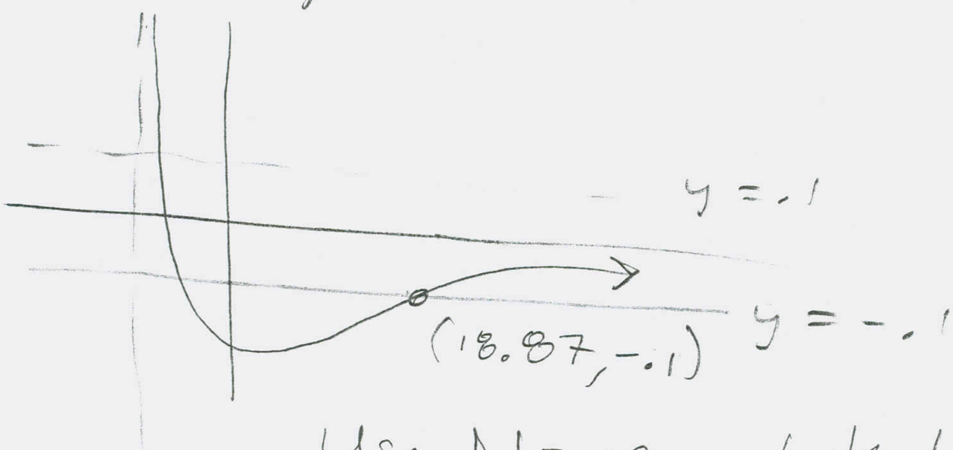
As for $\varepsilon = .1$, we compare $f(x)$ to $g(x) = -.1$ and $h(x) = +.1$

Very similar graph, only it takes longer for $f(x)$ to get in the tube.




Use $N = 19$ and that ought to do it.

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Now let's CLOSE the question of "wiggle"

$$\frac{\sqrt{4x^2+1} - 2(x+1)}{x+1} < 0 \quad \text{when?}$$

$$\frac{4x^2+1 - 2^2(x+1)^2}{(x+1)\left(\sqrt{4x^2+1} + 2(x+1)\right)} = \frac{4x^2+1 - 4x^2 - 8x - 4}{(x+1)\left(\sqrt{4x^2+1} + 2(x+1)\right)}$$

$$= \frac{-8x-3}{(x+1)\left(\sqrt{4x^2+1} + 2(x+1)\right)}$$

As long as $x > 0$, the denominator will never be zero and the numerator will always be negative.

$\therefore$ We'll always be below the x-axis!