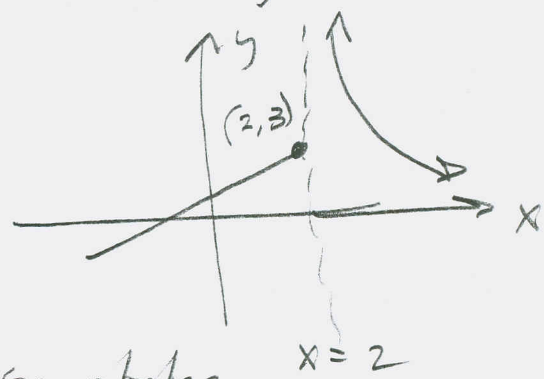
201 §4.4 I #s 2, 4, 7, 10, 15, 16, 20, 22, 31, 48, 51, 60

(2) (a) Can the graph of $y=f(x)$ intersect a vertical asymptote? Illustrate by a sketch

<u>YES</u>, but you have to be sneaky.

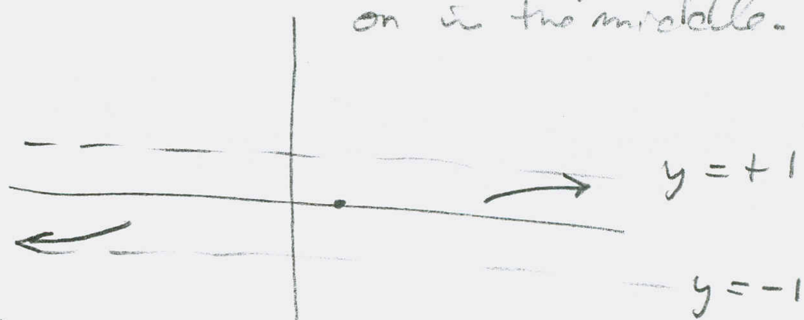
$$f(x)=\begin{cases} x+1 & \text{if } x \le 2 \\ \frac{1}{x-2} & \text{if } x > 2 \end{cases}$$



(b) How many horizontal asymptotes can a function have? Up to 2, but <u>usually just one</u>. Again, you'd have to be sneaky, with an absolute value, or something

$$\frac{|x^3-1|}{x^3+1}$$

Whatever else goes on in the middle.



(4) For the function g in the graph, state the following:

(a) $\lim_{x\to\infty} g(x) = 1$

(c) $\lim_{x\to 3} g(x) = g(3) \approx .3$

(b) $\lim_{x\to -\infty} g(x) = 2$

(d) $\lim_{x\to 0} g(x) = g(0) \approx 0$

201 §4.4 #s 7, 10, 15, 16, 20, 22, 31, 48, 51, 60

(7) Evaluate the limit and justify each step.

$$\lim_{x\to\infty} \frac{3x^2-x+4}{2x^2+5x-8} = \frac{3}{2}$$, since degree of denominator & numerator are the same, so we just look at the big stuff.

#s 9-30 Find the limit.

(10) $$\lim_{x\to\infty} \frac{3x+5}{x-4} = \boxed{3}$$

(15) $$\lim_{u\to\infty} \frac{4u^4+5}{(u^2-2)(2u^2-1)} = \lim_{u\to\infty} \frac{4u^4+5}{2u^4+\text{smaller}} = \frac{4}{2} = \boxed{2}$$

(16) $$\lim_{x\to-\infty} \frac{\sqrt{9x^6-x}}{x^3+1} = \lim_{x\to-\infty} \frac{\sqrt{9x^6\left(1-\frac{1}{9x^5}\right)}}{x^3\left(1+\frac{1}{x^3}\right)}$$

$$= \lim_{x\to-\infty} \frac{3|x|^3\sqrt{1-\frac{1}{9x^5}}}{x^3\left(1+\frac{1}{x^3}\right)} = \lim_{x\to-\infty} \frac{-3x^3\sqrt{1-\frac{1}{9x^5}}}{x^3\left(1+\frac{1}{x^3}\right)}$$

$$= \lim_{x\to-\infty} \frac{-3\sqrt{1-\frac{1}{9x^5}}}{1+\frac{1}{x^3}} = \boxed{-3}$$

201 § 4.4 #s 20, 22, 31, 48, 57, 60

(20) $\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right) = -\infty+\infty = ?$

$$= \lim_{x\to-\infty}\left(\left(\frac{x+\sqrt{x^2+2x}}{1}\right)\left(\frac{x-\sqrt{x^2+2x}}{x-\sqrt{x^2+2x}}\right)\right)$$

$$= \lim_{x\to-\infty}\frac{x^2-x^2-2x}{x-\sqrt{x^2+2x}} = \lim_{x\to-\infty}\frac{-2x}{x-\sqrt{x^2+2x}}$$

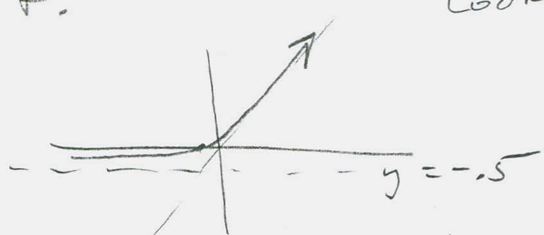
$$= \lim_{x\to-\infty}\frac{-2x}{x-|x|\sqrt{1+\frac{2}{x}}} = \lim_{x\to-\infty}\frac{-2x}{x+x\sqrt{1+\frac{2}{x}}}$$

$$= \lim_{x\to-\infty}\left(\frac{-2x}{x\left(1+\sqrt{1+\frac{2}{x}}\right)}\right) = \lim_{x\to\infty}\frac{-2}{1+\sqrt{1+\frac{2}{x}}} = \frac{-2}{2} = \boxed{-1}$$

(22) $\lim_{x\to\infty}\cos x \;\boxed{\nexists\,.}$

(31) (a) Estimate $\lim_{x\to-\infty}\left(\sqrt{x^2+x+1}+x\right)$ by graphing it.

Looks like about $y=-.5$ is H.A.



(b) With a table, I get darn close to $y=-.5$. To 4 places: $-.5000$! ☺

(c) Find the value

201 §4.4 #s 31, 48, 51, 60

31 (c) cont'd

$$\lim_{x\to-\infty}\left(\sqrt{x^2+x+1}+x\right)=\lim_{x\to-\infty}\frac{x^2+x+1-x^2}{\sqrt{x^2+x+1}-x}$$

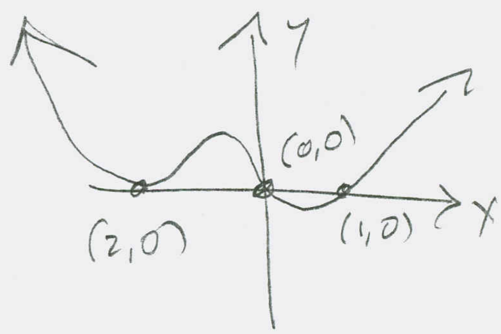
$$=\lim_{x\to-\infty}\frac{x+1}{|x|\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-x}$$

$$=\lim\frac{x+1}{-x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-x}=\lim_{x\to-\infty}\frac{x\left(1+\frac{1}{x}\right)}{-x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right)}$$

$$=\lim_{x\to-\infty}\frac{1+\frac{1}{x}}{-\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right)}=\frac{1}{-(1+1)}=\boxed{-\frac{1}{2}}$$

48 Find $\lim_{x\to\infty}$ and $\lim_{x\to-\infty}$ of the following.

Use this info, with intercepts, to give a rough sketch.



$y=x^3(x+2)^2(x-1)$

$=x^3(x^2)(x)$ + smaller

$=x^6$ + smaller

as $x\to\infty$ ↗

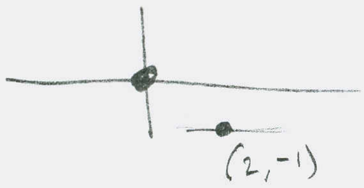
as $x\to-\infty$ ↖

End behavior is like $y=x^6$ ↖ ... ↗

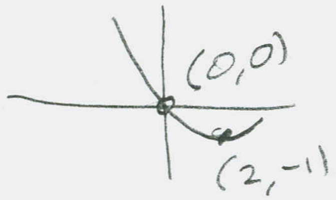
201 §4.4 #s 51, 60

(51) Sketch the graph of a function that satisfies all of the given conditions

$f'(2) = 0,\ f(2) = -1,\ f(0) = 0$

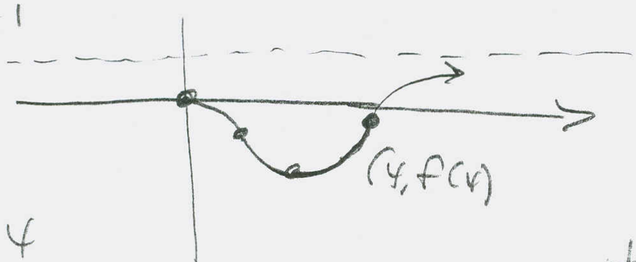


$f'(x) < 0$ if $0 < x < 2$, $f'(x) > 0$ if $x > 2$



$f''(x) < 0$ If $0 \le x < 1$ or if $x > 4$

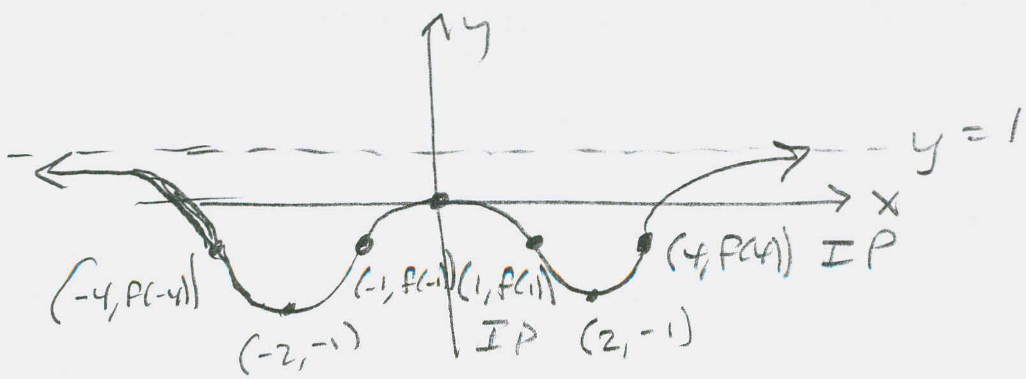
$\lim_{x \to \infty} f(x) = 1$



$f''(x) > 0$ for $1 < x < 4$

No telling where the x-int. is to the right (or left) of y-axis

$f(-x) = f(x) \Rightarrow$ EVEN



201 §4.4 #60

60 A tank contains 5000 L of pure water. Brine containing 30 $\frac{g\ salt}{L}$ is pumped into the tank at $\frac{25\ L}{min}$ rate. Show that the concentration of salt t minutes after the start is

$$C(t) = \frac{30t}{200+t}$$

Tank never fills, so no overflow / mixing issues.

After t min, we have 500 + 25t L of mixture and its concentration is

$$C(t) = \frac{\left(30\ \frac{g\ salt}{L}\right)(25t\ L)}{(5000+25t)\ L} = \frac{25(30)t}{25(200+t)} =$$

$$= \frac{30t}{200+t}.$$

(b) $\lim_{t \to \infty} C(t) = \frac{30}{1} = \boxed{30\ g/L}$