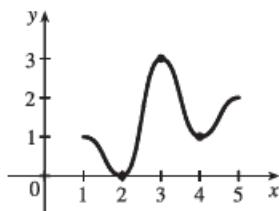


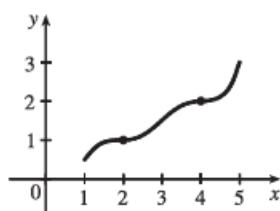
## 4.1 Solutions

6. There is no absolute maximum value; absolute minimum value is  $g(4) = 1$ ; local maximum values are  $g(3) = 4$  and  $g(6) = 3$ ; local minimum values are  $g(2) = 2$  and  $g(4) = 1$ .

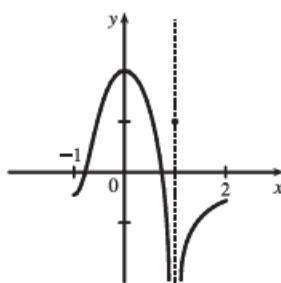
7. Absolute minimum at 2, absolute maximum at 3,  
local minimum at 4



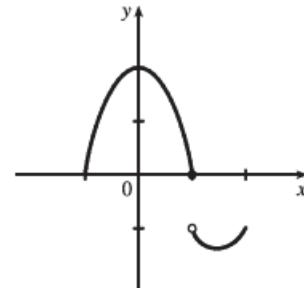
10.  $f$  has no local maximum or minimum, but 2 and 4 are critical numbers



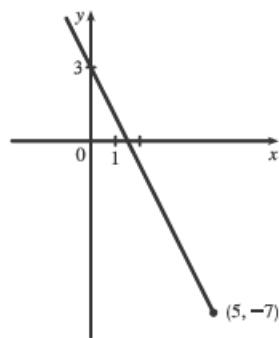
13. (a) Note: By the Extreme Value Theorem,  $f$  must *not* be continuous; because if it were, it would attain an absolute minimum.



(b)



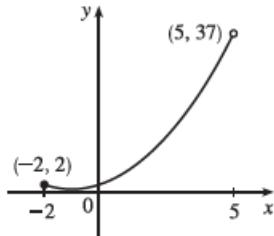
16.  $f(x) = 3 - 2x$ ,  $x \leq 5$ . Absolute minimum  $f(5) = -7$ ;  
no local minimum. No absolute or local maximum.



## 4.1 Solutions

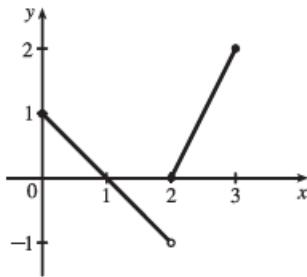
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22.  $f(x) = 1 + (x + 1)^2$ ,  $-2 \leq x < 5$ . No absolute or local maximum. Absolute and local minimum  $f(-1) = 1$ .



$$27. f(x) = \begin{cases} 1-x & \text{if } 0 \leq x < 2 \\ 2x-4 & \text{if } 2 \leq x \leq 3 \end{cases}$$

Absolute maximum  $f(3) = 2$ ; no local maximum. No absolute or local minimum.



$$33. s(t) = 3t^4 + 4t^3 - 6t^2 \Rightarrow s'(t) = 12t^3 + 12t^2 - 12t. \quad s'(t) = 0 \Rightarrow 12t(t^2 + t - 1) \Rightarrow$$

$t = 0$  or  $t^2 + t - 1 = 0$ . Using the quadratic formula to solve the latter equation gives us

$$t = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2} \approx 0.618, -1.618. \quad \text{The three critical numbers are } 0, \frac{-1 \pm \sqrt{5}}{2}.$$

$$42. g(\theta) = 4\theta - \tan \theta \Rightarrow g'(\theta) = 4 - \sec^2 \theta. \quad g'(\theta) = 0 \Rightarrow \sec^2 \theta = 4 \Rightarrow \sec \theta = \pm 2 \Rightarrow \cos \theta = \pm \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, \text{ and } \frac{4\pi}{3} + 2n\pi \text{ are critical numbers.}$$

Note: The values of  $\theta$  that make  $g'(\theta)$  undefined are not in the domain of  $g$ .

$$52. f(x) = \frac{x^2 - 4}{x^2 + 4}, [-4, 4]. \quad f'(x) = \frac{(x^2 + 4)(2x) - (x^2 - 4)(2x)}{(x^2 + 4)^2} = \frac{16x}{(x^2 + 4)^2} = 0 \Leftrightarrow x = 0. \quad f(\pm 4) = \frac{12}{20} = \frac{3}{5} \text{ and } f(0) = -1. \quad \text{So } f(\pm 4) = \frac{3}{5} \text{ is the absolute maximum value and } f(0) = -1 \text{ is the absolute minimum value.}$$

53.  $f(t) = t\sqrt{4-t^2}$ ,  $[-1, 2]$ .

$$f'(t) = t \cdot \frac{1}{2}(4-t^2)^{-1/2}(-2t) + (4-t^2)^{1/2} \cdot 1 = \frac{-t^2}{\sqrt{4-t^2}} + \sqrt{4-t^2} = \frac{-t^2 + (4-t^2)}{\sqrt{4-t^2}} = \frac{4-2t^2}{\sqrt{4-t^2}}.$$

$f'(t) = 0 \Rightarrow 4-2t^2 = 0 \Rightarrow t^2 = 2 \Rightarrow t = \pm\sqrt{2}$ , but  $t = -\sqrt{2}$  is not in the given interval,  $[-1, 2]$ .

$f'(t)$  does not exist if  $4-t^2 = 0 \Rightarrow t = \pm 2$ , but  $-2$  is not in the given interval.  $f(-1) = -\sqrt{3}$ ,  $f(\sqrt{2}) = 2$ , and

$f(2) = 0$ . So  $f(\sqrt{2}) = 2$  is the absolute maximum value and  $f(-1) = -\sqrt{3}$  is the absolute minimum value.