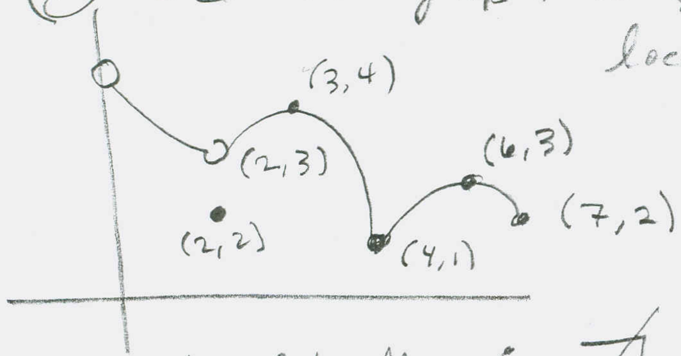
201 §4.1 #s 6, 7, 10, 13, 16, 22, 27, 33, 42, 52, 53

(6) Use the graph to state the absolute & local max & min values



Absolute Max: $\nexists$

Absolute Min: $f(4) = 1$

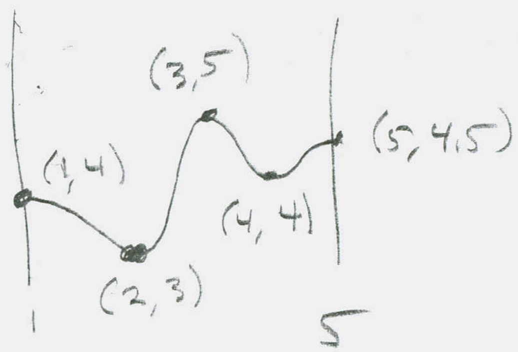
Local Max: $f(3) = 4$, $f(6) = 3$

Local Min: $f(2) = 2$, $f(4) = 1$

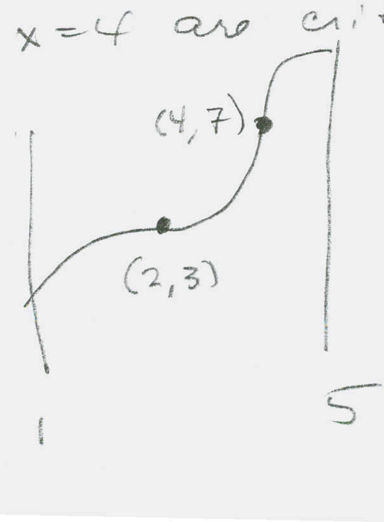
$f(7) = 2$ is not a local min, b/c there's nothing going on to its right.

#s 7–10 Sketch the graph of a function f that is cnt^{s} on [1, 5] and has the given properties.

(7) Absolute min @ 2,
" max @ 3
Local min @ 4



(10) f has no local extremes, by x = 2 & x = 4 are critical numbers

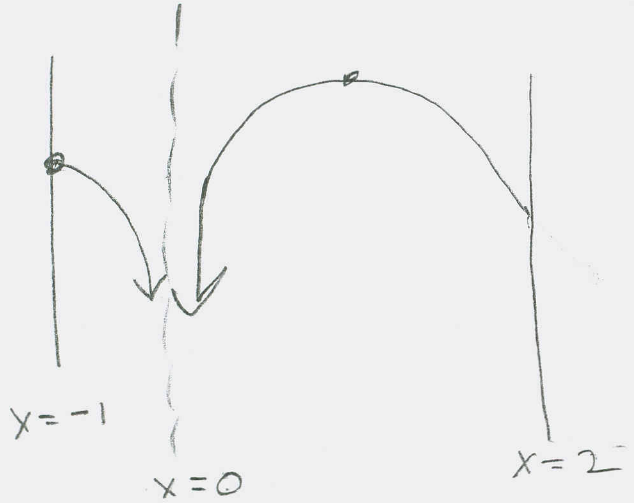


$f' \nexists$ @ $x = 4$ (vertical)

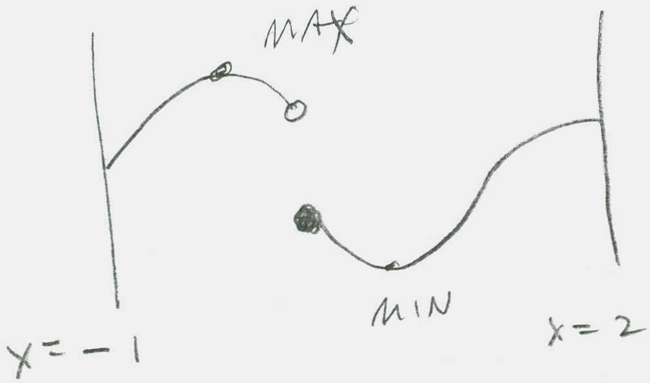
$f'(2) = 0$ (terrace)

201 § 4.1 #s 13, 16, 22, 27, 33, 42, 52, 53

13 (a) Sketch the graph of a function on [-1, 2] that has an absolute max but no absolute min.



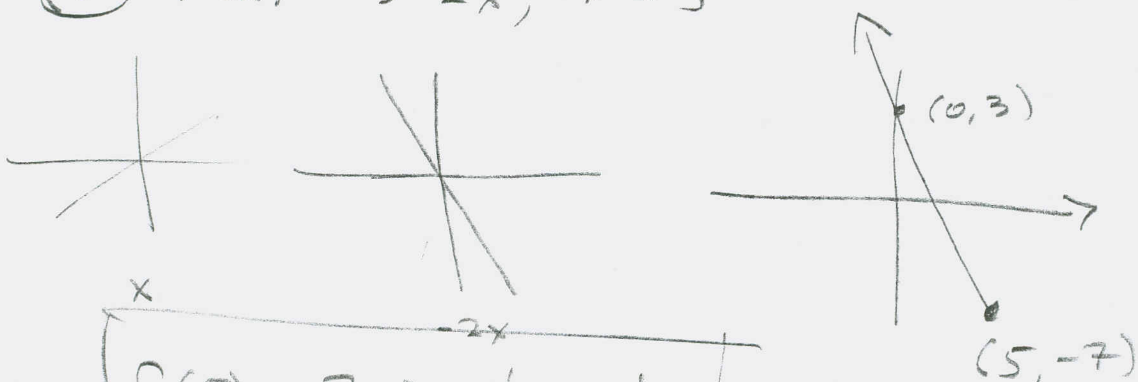
(b) Sketch the graph of a function that is discontinuous, but has both an absolute max and min on [-1, 2]



201 §4.1 #s 16, 22, 27, 33, 42, 52, 53

#s 15-28 Sketch the graph of f and use your sketch to find absolute extreme values of f.

(16) $f(x) = 3-2x, \ x \le 5$



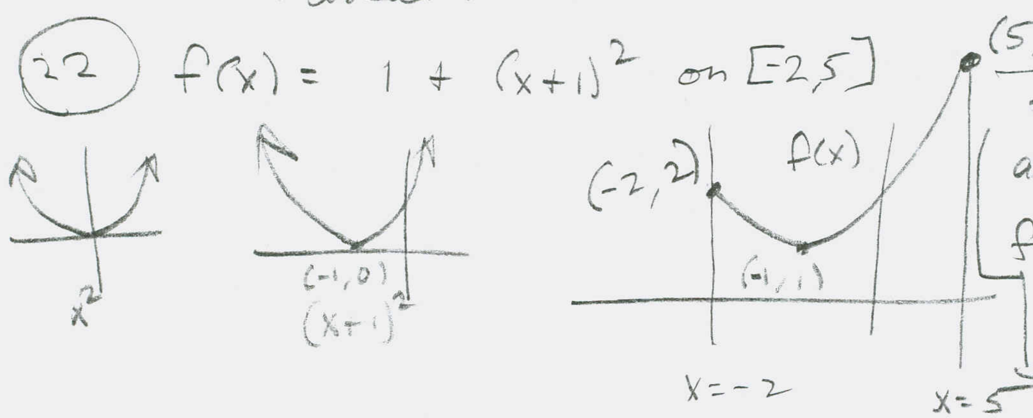
$\boxed{f(5) = -7 \text{ is abs. min.}}$

$3-2(5) = 3-10 = -7$

No abs. max.

No local max

(22) $f(x) = 1 + (x+1)^2$ on $[-2,5]$



f(-1) = 1 is abs. min.

f(5) = 37 is abs max.

local min f(-1) = 1

~~#s 29-42 Find the critical #s~~

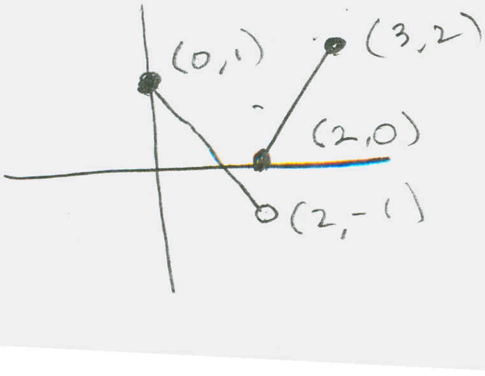
(27) $f(x) = \begin{cases} 1-x & \text{if } 0 \le x < 2 \\ 2x-4 & \text{if } 2 \le x \le 3 \end{cases}$

$f_-(2) = -1 \rightsquigarrow (2,-1)$ ○

$f_+(2) = 0 \rightsquigarrow (2,0)$ ●

$f(0) = 1$

$f(3) = 2$



No local extremes

f(3) = 2 is Abs. Max.

No Abs. Min!

201 §4.1 #s 33, 42, 52, 53

#s 29–42 Find the critical #s of the function.

(33) $s(t) = 3t^4 + 4t^3 - 6t^2$

$\Longrightarrow s'(t) = 12t^3 + 12t^2 - 12t$

$= 12t(t^2 + t - 1) \overset{\text{SET}}{=} 0 \Longrightarrow$

$t = 0$ OR $t^2 + t - 1 = 0$

$t^2 + t + \left(\frac{1}{2}\right)^2 = 1 + \frac{1}{4}$

$\left(t + \frac{1}{2}\right)^2 = \frac{5}{4}$

$t = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$

critical #s:

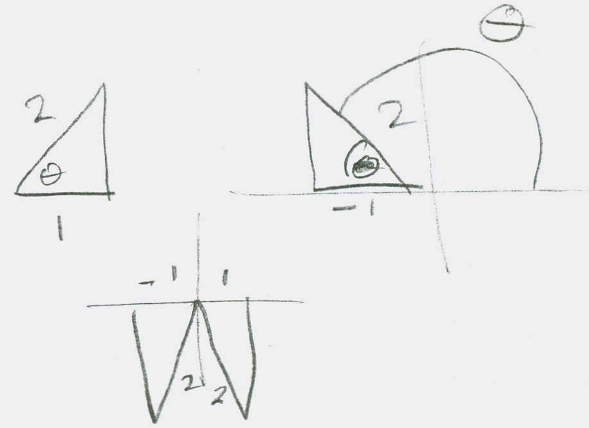
$\left\{0, \frac{-1 \pm \sqrt{5}}{2}\right.$

(42) $g(\theta) = 4\theta - \tan\theta$

$g'(\theta) = 4 - \sec^2\theta \overset{\text{SET}}{=} 0$

$\Longrightarrow \sec^2\theta = 4$

$\Longrightarrow \sec\theta = \pm 2$



Scratch: $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$$\theta = \left.\begin{matrix} \frac{\pi}{3} \pm 2n\pi \\ \frac{2\pi}{3} \pm 2n\pi \\ \frac{4\pi}{3} \pm 2n\pi \\ \frac{5\pi}{3} \pm 2n\pi \end{matrix}\right\} \text{where } g' = 0$$

What about $g' \nexists$?

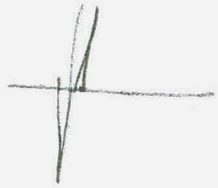
Need to find those values of θ, also.

201 §' 4.1 #s 42, 52, 53

$$g'(\theta) = 4 - \sec^2\theta = 4 - \frac{1}{\cos^2\theta} = \frac{4\cos^2\theta - 1}{\cos^2\theta}$$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$



$\theta =$ odd multiples of $\frac{\pi}{2}$ i.e. $\pm\frac{2n+1}{2}\pi$

Critical numbers:

$$\boxed{\frac{\pi}{3} \pm 2n\pi, \quad \frac{2\pi}{3} \pm 2n\pi, \quad \frac{4\pi}{3} \pm 2n\pi, \quad \frac{5\pi}{3} \pm 2n\pi \quad \pm(2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}}$$

#s 45-56 Find Abs max & min of f on $[a,b]$

52 $\quad f(x) = \frac{x^2-4}{x^2+4}$ on $[-4, 4]$

$$f(-4) = \frac{16-4}{16+4} = \frac{12}{20} = \frac{3}{5} = f(4)$$

$$f'(x) = \frac{2x(x^2+4) - (x^2-4)(2x)}{(x^2+4)^2} \overset{SET}{=} 0$$

$$\Longrightarrow 2x^3 + 8x - (2x^3 - 8x) = 0 \Longrightarrow 2x^3 + 8x - 2x^3 + 8x = 0$$

$$\Longrightarrow 16x = 0 \Longrightarrow x = 0$$

$$f(0) = \frac{-4}{4} = \boxed{-1 = f(0) = \text{Abs. MIN}}$$

$$\boxed{\begin{aligned} \tfrac{3}{5} &= f(4) \\ \tfrac{3}{5} &= f(-4) \end{aligned}} \longrightarrow \text{Tied for abs. max.}$$

201 §4.1 #53

53) $f(t) = t\sqrt{4-t^2}$ on $[-1, 2]$

$f(-1) = -1\sqrt{4-(-1)^2} = -\sqrt{3}$

$f(2) = 2\sqrt{4-2^2} = 0$

$f'(t) = \frac{d}{dt}\left[t(4-t^2)^{\frac{1}{2}}\right]$

$= 1(4-t^2)^{\frac{1}{2}} + t\left(\frac{1}{2}(4-t^2)^{-\frac{1}{2}}(-2t)\right)$

$= (4-t^2)^{\frac{1}{2}} - \frac{\boxed{t}}{(4-t^2)^{\frac{1}{2}}}$

Here's where I screwed up. The mistake is fixed in the notes: 110301-4-1.pdf

Click above to link to it.

$= \frac{(4-t^2)^{\frac{1}{2}}(4-t^2)^{\frac{1}{2}} - t}{(4-t^2)^{\frac{1}{2}}} = \frac{4-t^2-t}{(4-t^2)^{\frac{1}{2}}}$ SET 0

$\Rightarrow -t^2 - t + 4 = 0$

$t^2 + t - 4 = 0$

$t^2 + t + \left(\frac{1}{2}\right)^2 = 4 + \frac{1}{4}$

$\left(t + \frac{1}{2}\right)^2 = \frac{17}{4}$

$t + \frac{1}{2} = \pm\frac{\sqrt{17}}{2}$

$\boxed{t = \frac{-1 \pm \sqrt{17}}{2}}$

$f'(t)$ is undefined when

$(4-t^2)^{\frac{1}{2}} = 0 \Rightarrow$

$4 = t^2 \Rightarrow$

$\boxed{\pm 2 = t}$

Critical #s:

$\boxed{t = \frac{-1 \pm \sqrt{17}}{2}, \pm 2}$