201 §3.8 #s 2, 5, 10, 21, 24, 27, 28

(2a) A = area of circle with radius r, and r is changing, (a) find $\frac{dA}{dt}$

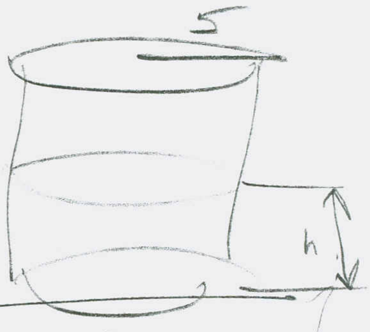
$$A = \pi r^2$$

$$\frac{dA}{dt} = \frac{d}{dt}\left[\pi r^2\right] = 2\pi r \frac{dr}{dt}$$

(b) If the radius is expanding at a constant rate of $1 \frac{m}{s}$, how fast is area increasing when radius is 30 m?

$$\left.\frac{dA}{dt}\right|_{r=30} = 2\pi(30)(1) = \boxed{60\pi \ \frac{m^2}{s}}$$

(5) A cylindrical tank w/ radius 5 m is being filled with water at a rate of $3 \frac{m^3}{min}$. How fast is the height of the water increasing?



$$V = \pi r^2 h = 25\pi h \implies$$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt} \implies$$

$$3 = 25\pi \frac{dh}{dt} \implies \boxed{\frac{dh}{dt} = \frac{3}{25\pi} \ \frac{m}{min}}$$

201 §3.8 #5 ~~2,5~~, 10, 21, 24, 27, 28

(10) A particle moves along $y = \sqrt{1+x^2}$. As it reaches (2,3), the y-coordinate is increasing at a rate of $4\,\frac{cm}{s}$. How fast is the x-coordinate changing at that instant?

WANT $\left.\frac{dx}{dt}\right|_{\substack{x=2\\ y=3\\ y'=4}}$

This one has a $\frac{dx}{dt} = x'$ in it, because we're differentiating w.r.t. "t".

$y = (x^2+1)^{\frac{1}{2}} \Longrightarrow$

$\frac{dy}{dt} = \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)\frac{dx}{dt}$. Let $x=2$, $y=3$, $\frac{dy}{dt}=4$.

Then $4 = \frac{1}{2}(2^2+1)^{-\frac{1}{2}}(2(2))\left(\frac{dx}{dt}\right)$

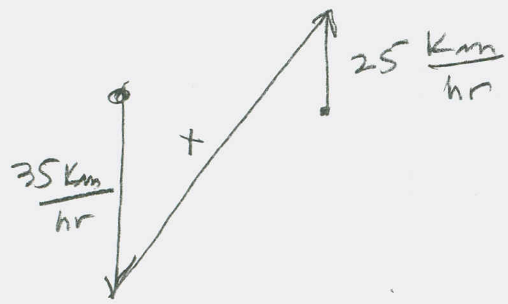
$4 = \left(\frac{1}{2\sqrt{5}}\right)(4)\frac{dx}{dt}$

$\frac{2\sqrt{5}}{4}\cdot 4 = \boxed{\frac{dx}{dt} = 2\sqrt{5}\ \frac{cm}{s}}$

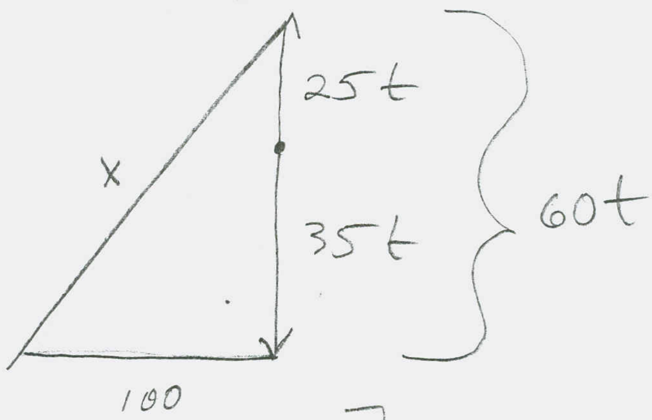
(21) At noon, ship A is 100 Km west of ship B. Ship A is sailing south at $35\,\frac{Km}{h}$ and Ship B is sailing north at 25 Km/hr. How fast is the distance between them changing @ 4:00 pm?

201 §3.8 #s 21, 24, 27, 28

@ moon



want $\left.\dfrac{dx}{dt}\right|_{t=4}$



$$\frac{d}{dt}\left[x^2 = 100^2 + (60t)^2\right]$$

$$2x\,x' = 2(60t)\cdot(60)(1) \longrightarrow \frac{dt}{dt} = t' = 1$$

$$\Longrightarrow x' = \frac{7200t}{2x} = \frac{3600t}{x}$$

Now, $t = 4 \Longrightarrow 60t = 240$, so

$$x^2 = 100^2 + 240^2 = 10000 + 57600$$

$$x^2 = 67600 \Longrightarrow$$

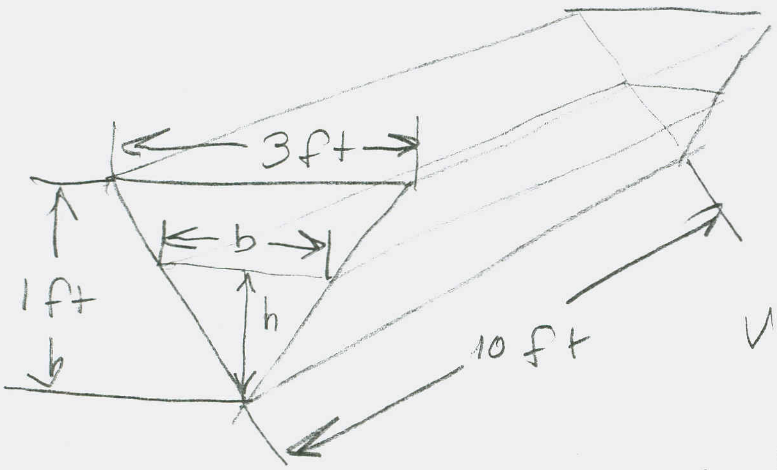
$$x = 260 \text{ Km} \qquad (\text{Take } x \geq 0)$$

$$\Longrightarrow x' = \frac{dx}{dt} = \frac{3600(4)}{260} = \frac{14400}{260} = 55.\overline{384615}$$

$$\approx \boxed{55.4 \ \frac{\text{Km}}{\text{hr}}}$$

201 §3.8 #s 24, 27, 28

(24) Done in class. A trough 10 ft long, 3 feet across and with ends that are isosceles triangles is being filled at a (constant) rate of $12 \frac{ft^3}{min}$, how fast is the water level rising when the water is 6 inches deep? Oh. And it's 1 foot tall



want $\left.\frac{dh}{dt}\right|_{h=\frac{1}{2}ft=6''}$

Volume = area of ends times length

$$V = \frac{1}{2} bh \cdot 10$$

By similar triangles, $\frac{3}{1} = \frac{b}{h} \Longrightarrow 3h = b \Longrightarrow$

$$V = 5(3h)(h)$$

$$V = 15h^2 \Longrightarrow$$

$$\frac{dV}{dt} = 15(2h)\frac{dh}{dt} = 30h\frac{dh}{dt}$$

$$12 = 30\left(\tfrac{1}{2}\right)\frac{dh}{dt} \Longrightarrow \boxed{\frac{dh}{dt} = \frac{12}{15} = \frac{4}{5} \text{ ft per minute}}$$

TOTAL VOL: $5(3) = 15 ft^3$

201 §3.8 #s 27, 28

(27) Gravel's being dumped at a rate of 30 ft^3/min, and its coarseness is such that its base diameter & height are always the same. How fast is the height of the pile increasing when the pile is 10 feet high?

Want $\left.\frac{dh}{dt}\right|_{h=10}$

$$\text{Volume} = V = \frac{1}{3}\pi r^2 h$$

$$\text{Diameter} = 2r = h \Longrightarrow r = \frac{h}{2} \Longrightarrow$$

$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$$

$$= \frac{1}{12}\pi h^3 \Longrightarrow$$

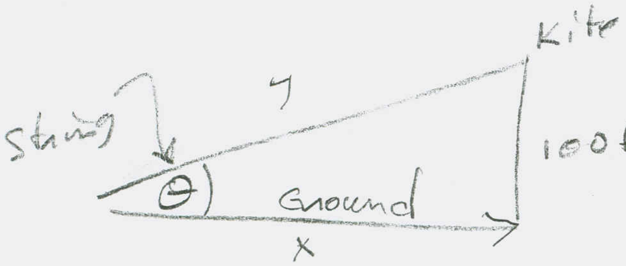
$$\frac{dV}{dt} = \frac{3}{12}\pi h^2 \frac{dh}{dt} \Longrightarrow$$

$$30 = \frac{1}{4}\pi (10)^2 \left.\frac{dh}{dt}\right|_{h=10} = \frac{100}{4}\pi \left.\frac{dh}{dt}\right|_{h=10} = 25\pi \left.\frac{dh}{dt}\right|_{h=10}$$

$$\Longrightarrow \left.\frac{dh}{dt}\right|_{h=10} = \frac{30}{25\pi} \frac{ft}{min} \approx \boxed{.3819718634 \ \frac{ft}{min}}$$

201 §3.8 #28

28 Done in class a couple ways



$\frac{dx}{dt} = 8 \frac{ft}{s}$ given

Find $\left.\frac{d\theta}{dt}\right|_{y=200}$

Preliminaries:

$y = 200 \Longrightarrow x = \sqrt{200^2 - 100^2}$

$= \sqrt{(200-100)(200+100)}$

$= \sqrt{(100)(300)}$

$= 100\sqrt{3} = \left. x \right|_{y=200}$

$\frac{d\theta}{dt}$ should be small and negative

Method 1

$\tan\theta = \frac{100}{x}$

$\sec^2\theta \frac{d\theta}{dt} = -\frac{100}{x^2}\frac{dx}{dt}$

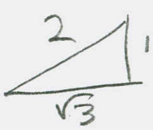
$\boxed{\text{Let } y = 200} \Longrightarrow$

$\sec^2\theta \frac{d\theta}{dt} = -\frac{100}{(100\sqrt{3})^2} \cdot 8$

Need θ:

$\theta = \text{Arctan}\left(\frac{100}{100\sqrt{3}}\right) =$

$= \text{Arctan}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$



$\Longrightarrow$

$\sec^2\left(\frac{\pi}{6}\right)\frac{d\theta}{dt} = -\frac{8}{300} \Longrightarrow$

$\frac{d\theta}{dt} = -\frac{2}{75}\cdot\left(\frac{3}{4}\right) = \boxed{-\frac{1}{50} = \left.\frac{d\theta}{dt}\right|_{y=200}} \frac{\text{rads}}{\text{sec}}$

Method 2:

$\cot\theta = \frac{x}{100}$

$-\csc^2\theta \frac{d\theta}{dt} = \frac{1}{100}\frac{dx}{dt}$

$-2^2 \left.\frac{d\theta}{dt}\right|_{y=200} = \frac{1}{100} \cdot 8$

$\boxed{\left.\frac{d\theta}{dt}\right|_{y=200} = -\frac{2}{100} = -\frac{1}{50} \frac{\text{rads}}{\text{sec.}}}$