201 §3.6 I #s 4, 7, 10, 19, 25, 28, 32, 33

#s 1-4 (a) Find y' implicitly

(b) Solve eq'n explicitly for y & find y' explicitly

(c) Check by plugging in y from (b) into answer for part (a)

4 $\cos x + \sqrt{y} = 5$

(a) $-\sin x + \frac{1}{2} y^{-\frac{1}{2}} y' = 0$

$$\frac{1}{2} \frac{y'}{\sqrt{y}} = + \sin x$$

$$\boxed{y' = +2\sqrt{y} \sin x}$$

(b) $\sqrt{y} = 5 - \cos x$

$$y = (5 - \cos x)^2 = (\cos x - 5)^2$$

$$\Rightarrow y' = 2(\cos x - 5)(-\sin x)$$

(c) $y' = +2\sqrt{y} \sin x = +2\sqrt{(\cos x - 5)^2} \sin x$

$= +2 \, |\cos x - 5| \sin x = +2(-5 + \cos x) \sin x$ ✓

(Here, $|\cos x - 5| = -(\cos x - 5)$, since $-5 + \cos x < 0$ always.)

✓ $= -2(\cos x - 5) \sin x$

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#s 5-20 Find $\frac{dy}{dx} = y'$ by implicit differentiation

(7) $x^2 + xy - y^2 = 4 \implies$

$2x + 1y + xy' - 2yy' = 0$

$y'(x-2y) = -2x - y$

$\boxed{y' = \frac{-2x-y}{x-2y}}$ OR $\frac{2x+y}{2y-x}$

(10) $y^5 + x^2y^3 = 1 + x^4y \implies$

$5y^4y' + 2xy^3 + x^2(3y^2y') = 4x^3y + x^4y'$

$5y^4y' + 3x^2y^2y' - x^4y' = 4x^3y - 2xy^3$

$y'(5y^4 + 3x^2y^2 - x^4) = 4x^3y - 2xy^3$

$\boxed{y' = \frac{4x^3y - 2xy^3}{5y^4 + 3x^2y^2 - x^4}}$

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19

$y\cos x = 1 + \sin(xy) \Longrightarrow$

$y'\cos x - y\sin x = (\cos(xy))(1y + xy')$

$y'\cos x - y\sin x = y\cos(xy) + xy'\cos(xy)$

GET ALL y's on one side

$y'\cos x - xy'\cos(xy) = y\cos(xy) + y\sin x$

$$\boxed{y' = \frac{y\cos(xy) + y\sin x}{\cos x - \cos(xy)}}$$

25 Use implicit differentiation to find an eq'n of the tangent line to the curve at the given point.

$x^2 + xy + y^2 = 3$; $(1,1)$ (Ellipse)

$2x + 1y + xy' + 2yy' = 0$

$y'(x+2y) = -2x - y$

$y' = \dfrac{-2x-y}{x+2y} \Longrightarrow$ @ $(1,1)$,

$y' = \dfrac{-2(1)-1}{1+2(1)} = \dfrac{-3}{3} = -1 = m$

$\boxed{y = -1(x-1) + 1} = -x + 2$

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28 $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ @ $(-3\sqrt{3}, 1)$

$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}y' = 0 \Longrightarrow$

$\frac{2}{3}y^{-\frac{1}{3}}y' = -\frac{2}{3}x^{-\frac{1}{3}}$

$y' = \dfrac{-\frac{2}{3}x^{-\frac{1}{3}}}{\frac{2}{3}y^{-\frac{1}{3}}} = -\dfrac{x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}} = -\dfrac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}} = -\dfrac{\sqrt[3]{y}}{\sqrt[3]{x}}$

$\Longrightarrow y'\Big|_{\substack{x=-3\sqrt{3} \\ y=1}} = -\dfrac{\sqrt[3]{1}}{\sqrt[3]{-3\sqrt{3}}} = -\dfrac{1}{-\left(3(3)^{\frac{1}{2}}\right)^{\frac{1}{3}}}$

$= \dfrac{1}{3^{\frac{1}{3}}3^{\frac{1}{6}}} = \dfrac{1}{3^{\frac{1}{3}+\frac{1}{6}}} = \dfrac{1}{3^{\frac{2+1}{6}}} = \dfrac{1}{3^{\frac{3}{6}}} = \dfrac{1}{3^{\frac{1}{2}}}$

$= \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3} = m$

$$\boxed{y = \frac{\sqrt{3}}{3}\left(x + 3\sqrt{3}\right) + 1}$$

32 (a) $y^2 = x^3 + 3x^2$ is the Tschirnhausen cubic.
Find eq'n of tangent line @ $(1, -2)$

$2yy' = 3x^2 + 6x \Longrightarrow$

$y'\Big|_{(1,-2)} = \dfrac{3x^2+6x}{2y}\Big|_{(1,-2)} = \dfrac{3(1)^2+6(1)}{2(-2)} = \dfrac{9}{-4} = -\dfrac{9}{4} = y'$

$$\boxed{y = -\frac{9}{4}(x-1) - 2}$$

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32 cont'd

(b) Where does the curve have horizontal tangents?

$$y' = \frac{3x^2+6x}{2y} \overset{\text{SET}}{=} 0 \Longrightarrow$$

$$3x^2+6x=0$$

$$3x(x+2)=0 \longrightarrow$$

$x \in \{-2, 0\}$. Now

$x=0 \Longrightarrow y^2 = 0^3+3(0)^2=0 \Longrightarrow y=0$

and, there, $y' \nexists$.

$x=-2 \Longrightarrow$

$$y^2 = (-2)^3+3(-2)^2 = -8+12=4 \longrightarrow$$

$$y = \pm\sqrt{4} = \pm 2$$

FINAL ANSWER: Horizontal tangents at the points $\boxed{(-2,2),\ (-2,-2)}$

(c) We illustrate with a graph from a CAS.

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(33) Find y'' by implicit differentiation.

$9x^2+y^2=9$

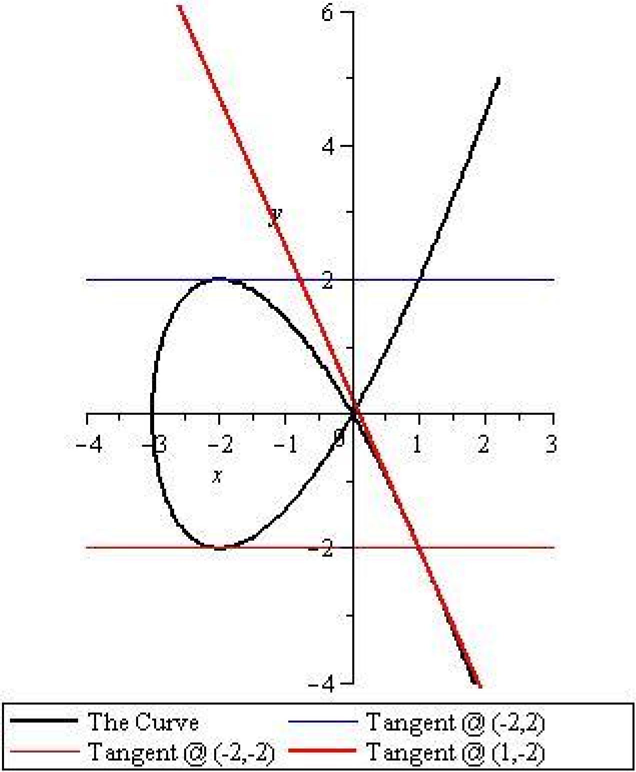
$18x+2yy'=0 \Longrightarrow y'=-\frac{9x}{y}$

$18+2(y')^2+2yy''=0$

$y''=\frac{-2(y')^2-18}{2y}=\boxed{\frac{-2\left(-\frac{9x}{y}\right)^2-18}{2y}}$

$=\frac{-2\left(\frac{81x^2}{y^2}\right)-18\cdot\frac{y^2}{y^2}}{2y}=\frac{\frac{162x^2-18y^2}{y^2}}{2y}$

$=\frac{2(81x^2-9y^2)}{2y^3}=\frac{81x^2-9y^2}{y^3}$



6
4
2
-2
-4
y
-4
-3
-2
-1
0
1
2
3
x
The Curve
Tangent @ (-2,2)
Tangent @ (-2,-2)
Tangent @ (1,-2)