201 §3.5 #s 4, 11, 18, 27, 35, 48, 54, 67, 70, 88

(4) Write the composite function in the form $f(g(x))$. Then find its derivative.

$y = \tan(\sin x)$

$f(x) = \tan x \Longrightarrow f'(x) = \sec^2 x$

$g(x) = \sin x \Longrightarrow g'(x) = \cos x$

$\Longrightarrow y' = f'(g(x))\, g'(x) = \left(\sec^2(\sin x)\right)\left(\cos x\right)$

#s 7-46 Find the derivative of the function.

(11) $g(t) = \dfrac{1}{(t^4+1)^3} = (t^4+1)^{-3} \Longrightarrow$

$\boxed{g'(t) = -3(t^4+1)^{-4}(4t^3)}$ STOP!

$= \dfrac{-12t^3}{(t^4+1)^4}$

(18) $h(t) = (t^4-1)^3(t^3+1)^4 \Longrightarrow$

$\boxed{h'(t) = 3(t^4-1)^2(4t^3)\left(t^3+1\right)^4 + (t^4-1)^3(4(t^3+1)^3(3t^2))}$

STOP

$= 12t^3(t^4-1)^2(t^3+1)^4 + 12t^2(t^4-1)^3(t^3+1)^3$

$= 12t^2(t^4-1)^2(t^3+1)^3\left[t(t^3+1) + t^4-1\right]$

$= 12t^2(t^4-1)^2(t^3+1)^3\left[t^4+t+t^4-1\right]$

$= 12t^2(t^4-1)^2(t^3+1)^3\left[2t^4+t-1\right]$

201 §3.5 #s 27, 35, 48, 54, 67, 70, 88

27 $y = \dfrac{r}{\sqrt{r^2+1}} = \dfrac{r}{(r^2+1)^{\frac{1}{2}}} \Longrightarrow$

$$\boxed{y' = \frac{1(r^2+1)^{\frac{1}{2}} - r\left(\frac{1}{2}(r^2+1)^{-\frac{1}{2}}(2r)\right)}{\left((r^2+1)^{1/2}\right)^2}}$$

$$= \frac{(r^2+1)^{\frac{1}{2}} - r^2(r^2+1)^{-1/2}}{r^2+1}$$

$$= \frac{(r^2+1)^{-\frac{1}{2}}\left[r^2+1-r^2\right]}{r^2+1}$$

FACTOR OUT $(r^2+1)^{-\frac{1}{2}}$

$$= \frac{1}{(r^2+1)^{3/2}} = \frac{1}{\sqrt{(r^2+1)^3}} = y'$$

Scratch

$(r^2+1)^{\frac{1}{2}}(r^2+1) = (r^2+1)^{\frac{3}{2}}$

35 $y = \left(\dfrac{1-\cos x}{1+\cos x}\right)^4 \Longrightarrow$

$$\boxed{y' = 4\left(\frac{1-\cos x}{1+\cos x}\right)^3\left[\frac{\sin x(1+\cos x) - (1-\cos x)(-\sin x)}{(1+\cos x)^2}\right]}$$

$$= 4\left(\frac{1-\cos x}{1+\cos x}\right)^3\left[\frac{\sin x + \sin x\cos x + \sin x - \sin x\cos x}{(1+\cos x)^2}\right]$$

$$= 4\left(\frac{1-\cos x}{1+\cos x}\right)^3\left[\frac{2\sin x}{(1+\cos x)^2}\right] = \frac{8\sin x(1-\cos x)^3}{(1+\cos x)^5}$$

201 §3.5 #s 48, 54, 67, 70, 88

48 Find the 2^{nd} derivative of the function.

$y = \sin^2(\pi t) \Longrightarrow y' = (2\sin(\pi t))(\cos \pi t)(\pi)$

$= 2\pi \sin(\pi t)\cos(\pi t)$

$= \pi \sin(2\pi t)$ (via $\sin(2x) = 2\sin x \cos x$)

$\Longrightarrow \boxed{y'' = (\pi \cos(2\pi t))(2\pi)}$

$= 2\pi^2 \cos(2\pi t)$

The hard way:

$\frac{d}{dt}\left[2\pi \sin(\pi t)\cos(\pi t)\right] =$

$2\pi(\cos(\pi t))(\pi)(\cos(\pi t)) + 2\pi(\sin(\pi t))(-\sin(\pi t))(\pi)$

$= 2\pi^2\left[\cos^2(\pi t) - \sin^2(\pi t)\right]$

$= 2\pi^2\left[\cos(2\pi t)\right]$ (via $\cos^2 x - \sin^2 x = \cos(2x)$)

#s 51-54 Find an eq'n of the tangent line to the curve at the given point.

54 $y = \sqrt{x^2+5}$ @ $(2, 3)$

$y = (x^2+5)^{\frac{1}{2}} \Longrightarrow$

$y' = \frac{1}{2}(x^2+5)^{-\frac{1}{2}}(2x) \Longrightarrow$

$= \dfrac{x}{\sqrt{x^2+5}}$

201 §3.5 #s 54, 67, 70, 88

54 cnt'd

$$y'\Big|_{x=2} = y'(2) = \frac{2}{\sqrt{2^2+5}} = \frac{2}{\sqrt{9}} = \frac{2}{3} = m$$

Acceptable notation

$$\therefore \boxed{y = \frac{2}{3}(x-2)+3}$$ is tangent line

67 $\S$ f is difb$^{\underline{l}}$ $\forall x \in \mathbb{R}$. Let $F(x) = f(\cos x)$ and $G(x) = \cos(f(x))$.
Find (a) $F'(x)$ and (b) $G'(x)$

(a) $F'(x) = \left(f'(\cos x)\right)(-\sin x)$

(b) $G'(x) = \left(-\sin(f(x))\right)\left(f'(x)\right)$

70 If g is twice-difb$^{\underline{l}}$ and $f(x) = x\,g(x^2)$ find f'' in terms of g, g' & g''.

$$f'(x) = g(x^2) + x\,g'(x^2)(2x)$$
$$= g(x^2) + 2x^2 g'(x^2) \quad \Longrightarrow$$

$$\boxed{f''(x) = \left(g'(x^2)\right)(2x) + 4x\,g'(x^2) + (2x^2)\left(g''(x^2)\right)(2x)}$$
$$= 2x\,g'(x^2) + 4x\,g'(x^2) + 4x^2 g''(x^2)$$

§ 3.5 #88

(c) If $g(x) = \sin|x|$, find $g'(x)$ and sketch the graphs of $g(x)$ and $g'(x)$. Where is g NOT difbl.

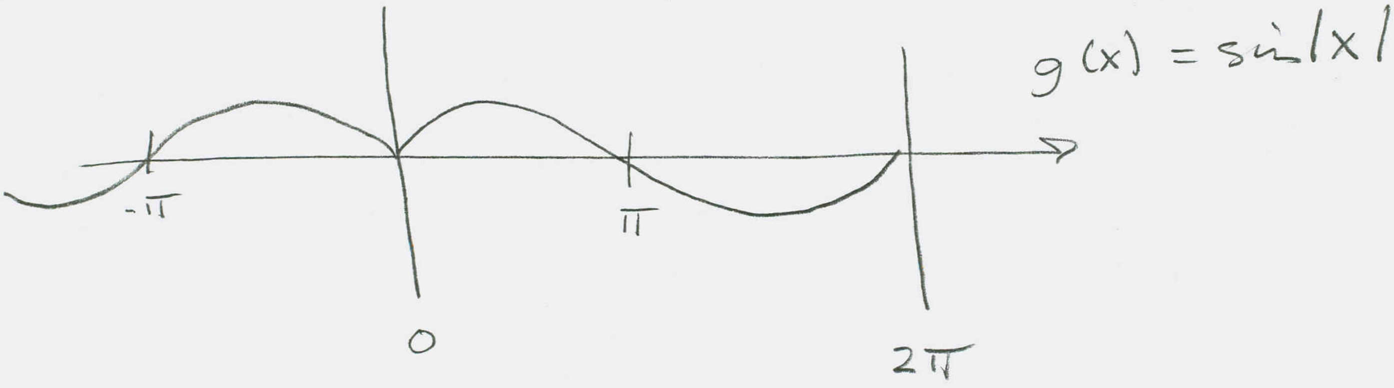
I THINK the author wanted to see $\frac{d}{dx}\left[\sin\left((x^2)^{\frac{1}{2}}\right)\right]$, which I didn't demonstrate to the class, yet. For a piecewise-defined version of this, see lecture notes

$$\frac{d}{dx}\left[\sin\left((x^2)^{\frac{1}{2}}\right)\right] = \left(\cos\left((x^2)^{\frac{1}{2}}\right)\right)\left(\frac{1}{2}(x^2)^{-\frac{1}{2}}\right)(2x)$$

$$= \frac{x\cos\left((x^2)^{\frac{1}{2}}\right)}{\sqrt{x^2}} = \frac{x\cos|x|}{|x|}$$

It's not differentiable whenever $|x|=0$, i.e., whenever $x=0$.

$$\frac{x\cos|x|}{|x|} = \begin{cases} \dfrac{x\cos x}{x} = \cos x & \text{if } x \geq 0 \\[2ex] \dfrac{x\cos(-x)}{-x} = -\cos x & \text{if } x < 0 \end{cases}$$



201 §3.5 #88

88 Discussed extensively in class.

We use the idea that $|x| = \sqrt{x^2}$ to evaluate absolute value functions in (a) & (b)

(a) $\frac{d}{dx}\left[|x|\right] = \frac{d}{dx}\left[\sqrt{x^2}\right] = \frac{d}{dx}\left[(x^2)^{\frac{1}{2}}\right]$

$= \frac{1}{2}(x^2)^{-\frac{1}{2}}(2x) = \frac{x}{(x^2)^{\frac{1}{2}}} = \frac{x}{|x|}$ ■

(b) The same idea for $|\sin x| = \sqrt{\sin^2 x}$

$= (\sin^2 x)^{\frac{1}{2}} \Longrightarrow$

$\frac{d}{dx}\left[|\sin x|\right] = \frac{d}{dx}\left[(\sin^2 x)^{\frac{1}{2}}\right] = \frac{1}{2}(\sin^2 x)^{-\frac{1}{2}}(2\sin x) \cdot$
$\cdot \cos x$

$= \frac{\sin x \cos x}{(\sin^2 x)^{\frac{1}{2}}} = \frac{\sin x \cos x}{|\sin x|}$

It's NOT differentiable when $\sin x = 0$:

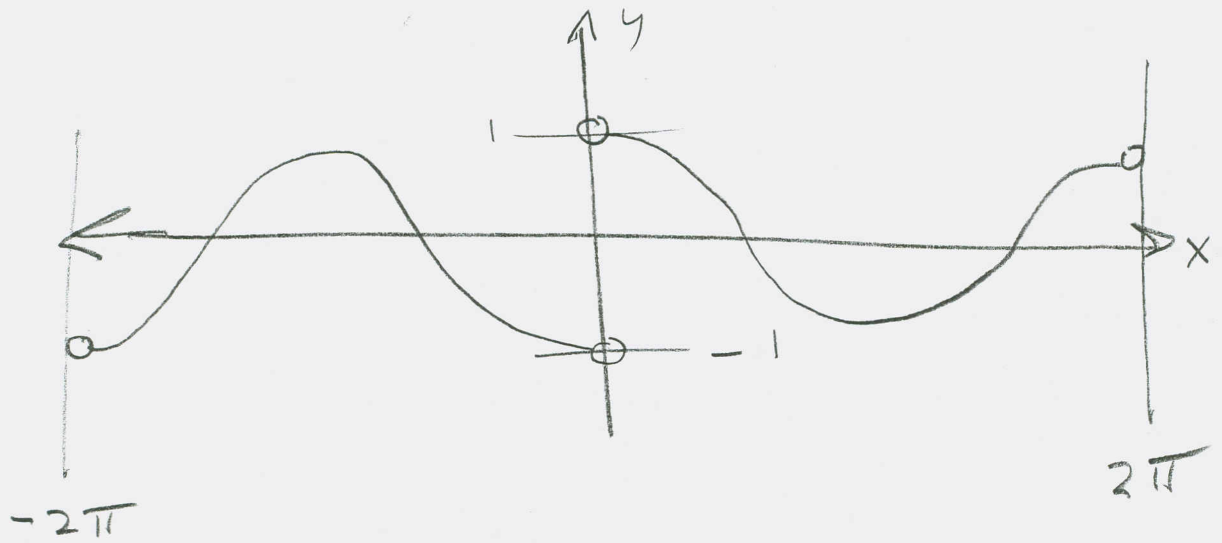
$x = 0, \pm\pi, \pm 2\pi, \ldots$, i.e.,

$\left\{ x \mid \boxed{x = n\pi, \ n \in \mathbb{Z}} \right\}$ is set where its NOT diff'bl

I think I originally said $\pm n\pi$, $n \in \mathbb{Z}$, but $\mathbb{Z}$ contains positive and negative whole numbers, so while I wasn't wrong, I wasn't ~~precise~~ ~~elegant~~ efficient :)

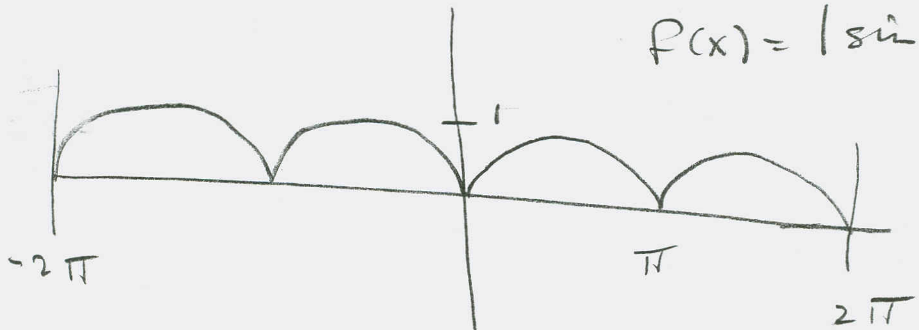
201 § 3.5 #88

Sketch of $g'(x) = \dfrac{x \cos|x|}{|x|}$



Left out sketch for (b)

$f(x) = |\sin x|$



For $f'(x) = \dfrac{\sin x \cos x}{|\sin x|} = \begin{cases} \cos x & \text{if } \sin x > 0 \\ -\cos x & \text{"} \ \sin x < 0 \end{cases}$