201 §3.4 #s 2, 4, 9, 14, 18, 20, 23, 40, 46, 50

#s 1–16 Differentiate

(1) $f(x) = 3x^2 - 2\cos x$

$\Rightarrow \boxed{f'(x) = 6x + 2\sin x}$

(2) $f(x) = \sqrt{x}\,\sin x$

$= x^{\frac{1}{2}} \sin x$

$\Rightarrow f'(x) = \frac{1}{2} x^{-\frac{1}{2}} \sin x - x^{\frac{1}{2}} \cos x$

$= \boxed{\frac{1}{2\sqrt{x}} \sin x - \sqrt{x}\cos x}$

(4) $y = 2\csc x + 5\cos x$

$\Rightarrow \boxed{\frac{dy}{dx} = -2\csc x \cot x - 5\sin x}$

(9) $y = \frac{x}{2 - \tan x} \Rightarrow$

$\frac{dy}{dx} = \frac{1(2 - \tan x) - x(-\sec^2 x)}{(2 - \tan x)^2}$

$= \boxed{\frac{2 - \tan x + x\sec^2 x}{(2 - \tan x)^2}}$

ON TESTS, I'M NOT ASKING YOU to simplify. I'm highlighting TEST ANSWERS.

(14) $y = \csc\theta(\theta + \cot\theta) \longrightarrow$

$\boxed{\frac{dy}{dx} = -\csc\theta\cot\theta(\theta + \cot\theta) + \csc\theta(1 - \csc^2\theta)}$

$= -\theta\csc\theta\cot\theta - \csc\theta\cot^2\theta + \csc\theta - \csc^3\theta$

$= -\theta\csc\theta\cot\theta - \csc\theta(\csc^2\theta - 1) + \csc\theta - \csc^3\theta$

$= -\theta\csc\theta\cot\theta - \csc^3\theta + \csc\theta + \csc\theta - \csc^3\theta$

$= -\csc\theta\cot\theta - 2\csc^3\theta + 2\csc\theta$

201 §3.4 #s 14, 18, 20, 23, 40, 46, 50

The last is ONE WAY TO Attempt to simplify. I can keep going with this, but with no purpose other than to differentiate there is not much profit.

The book replaced $\csc^2\theta$ with a $1+\cot^2\theta$. In my work, I replaced $\cot^2\theta$ with a $\csc^2\theta - 1$. No real reason to go either route, unless you needed to take another derivative, or do something else with the result, and were hoping for something less messy.

(18) Prove that $\frac{d}{dx}\left[\sec x\right] = \sec x \tan x$

Pf $\frac{d}{dx}\left[\sec x\right] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{0\cdot\cos x - 1\cdot(-\sin x)}{(\cos x)^2}$

$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} = \sec x \tan x$ ■

(20) Prove, using defin of derivative, that

$\frac{d}{dx}\left[\cos x\right] = -\sin x$

201 §3.4 #s 20, 23, 40, 46, 50

Proof We use work from proof of $\frac{d}{dx}[\sin x] = \cos x$.

$$\frac{\cos(x+h) - \cos x}{h} = \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \frac{\cos x(\cos h - 1) - \sin x \sin h}{h}$$

$$= \cos x \left(\frac{\cos h - 1}{h}\right) - \sin x \left(\frac{\sin h}{h}\right)$$

$$\xrightarrow{h \to 0} (\cos x)(0) - (\sin x)(1) = -\sin x \quad \blacksquare$$

This proof used $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$ and

$$\lim_{h \to 0} \frac{\sin h}{h} = 1$$

#s 21-24 Find an equation of the tangent line to the curve @ the given point.

23 $f(x) = x + \cos x$ @ $(0, 1)$

$f'(x) = 1 - \sin x \Rightarrow m_{tan}\Big|_{x=0} = f'(0) = 1$

$y = m(x - x_1) + y_1$

$\boxed{y = 1(x-0) + 1}$ is fine $= x + 1 = y$ → Book answer

201 §3.4 #s 40, 46, 50

#s 39–48 Find the limit

40 $\lim\limits_{x\to 0} \dfrac{\sin(4x)}{\sin(6x)} = \lim\limits_{x\to 0}\left(\dfrac{\sin(4x)}{\sin(6x)} \cdot \dfrac{x}{x}\right)$

$= \lim\limits_{x\to 0}\left(\dfrac{\sin(4x)}{x} \cdot \dfrac{x}{\sin(6x)}\right) = \lim\limits_{h\to 0}\left(\dfrac{4}{4} \cdot \dfrac{\sin(4x)}{x}\right)\lim\limits_{h\to 0}\left(\dfrac{6}{6} \cdot \dfrac{x}{\sin(6x)}\right)$

$= \lim\limits_{h\to 0}\left(4 \cdot \dfrac{\sin(4x)}{4x}\right)\lim\limits_{h}\left(\dfrac{1}{6} \cdot \dfrac{6x}{\sin(6x)}\right) = (4 \cdot 1)\left(\dfrac{1}{6} \cdot 1\right)$

$= \boxed{\dfrac{2}{3}}$

I KNOW it's tempting to factor like this:

$\sin(4x) = 4\sin x$, but this is WRONG.

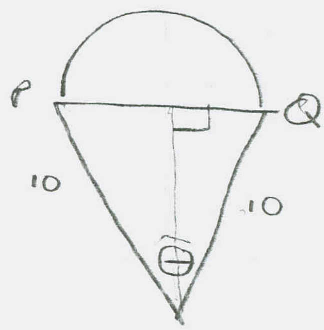
Here's a ~~slightly~~ more compact version:

$$\frac{\sin(4x)}{\sin(6x)} = \underbrace{\frac{\sin(4x)}{4x} \cdot \frac{6x}{\sin(6x)}}_{\text{What you want}} \cdot \underbrace{\frac{4}{6}}_{\text{To keep it equal}} \xrightarrow{x\to 0} 1 \cdot 1 \cdot \frac{4}{6} = \frac{2}{3}$$

201 § 3.4 #s 46, 50

(46) $\displaystyle\lim_{x\to 0} \frac{\sin(x^2)}{x} = \lim_{x\to 0} x \cdot \frac{\sin(x^2)}{x^2} = 0 \cdot 1 = 0$

(50) A semicircle with diameter $|PQ|$ sits on an isosceles triangle to make an ice-cream cone. If $A(\theta)$ = Area of the semicircle & $B(\theta)$ = area of the triangle, find $\displaystyle\lim_{\theta\to 0^+} \frac{A(\theta)}{B(\theta)}$



semicircle: $\dfrac{\pi r^2}{2} = \dfrac{\pi\left(\frac{|PQ|}{2}\right)^2}{2}$

$= \dfrac{\pi |PQ|^2}{8}$ = Area of $\frown$

triangle: $\frac{1}{2} bh = \frac{1}{2} |PQ| h$

$\dfrac{h}{10} = \cos\left(\frac{\theta}{2}\right) \Rightarrow h = 10\cos\left(\frac{\theta}{2}\right) = 10\sqrt{\dfrac{1+\cos x}{2}}$

$\Rightarrow$ Area of ∇ $= \frac{1}{2} |PQ| \cdot 10 \sqrt{\dfrac{1+\cos x}{2}}$

$= 5 |PQ| \sqrt{\dfrac{1+\cos x}{2}}$ Did I need this? Let's see...

$\dfrac{A(\theta)}{B(\theta)} = \dfrac{\frac{\pi |PQ|^2}{8}}{\frac{1}{2}|PQ| \cdot 10\cos\left(\frac{\theta}{2}\right)}$ NOPE. Won't need it.

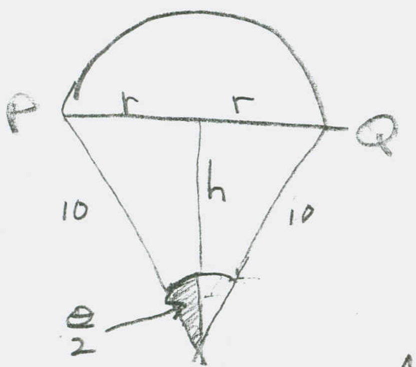
* sigh *

201 S' 3.4 #50

$$\frac{A(\theta)}{B(\theta)} = \frac{\pi\,|PQ|}{8\cdot 5\cos\left(\frac{\theta}{2}\right)} \xrightarrow{\theta\to 0} 0\text{, since}$$

$$\pi\,|PQ| \xrightarrow{\theta\to 0} 0 \quad \& \quad 40\cos\left(\frac{\theta}{2}\right) \xrightarrow{\theta\to 0} 40$$

The text is more clever in its solution.



Area of $\cap$ $= \frac{1}{2}\pi r^2$

Area of ∇ $= rh = r\cdot 10\cos\frac{\theta}{2}$

This gives $\displaystyle\frac{A(\theta)}{B(\theta)} = \frac{\frac{1}{2}\pi r^2}{10r\cos\left(\frac{\theta}{2}\right)} = \frac{\frac{1}{2}\pi\, r}{10\cos\frac{\theta}{2}}$

$$= \frac{\frac{1}{2}\pi\cdot 10\sin\left(\frac{\theta}{2}\right)^{*}}{10\cos\left(\frac{\theta}{2}\right)} = \frac{1}{2}\pi\tan\left(\frac{\theta}{2}\right) \xrightarrow{\theta\to 0} 0.$$

$*$ $r = 10\sin\left(\frac{\theta}{2}\right)$, since $\frac{r}{10} = \sin\left(\frac{\theta}{2}\right)$.

I always have to go here before I go here. Lots of people instantly see $r = 10\sin\left(\frac{\theta}{2}\right)$ without thinking. That's OK.