201 §3.3 I #s 2, 4, 9, 14, 22, 23, 33, 49, 53, 66

#s 1-20 Differentiate the function

2. $f(x) = \sqrt{30} \implies f'(x) = 0$

4. $F(x) = \frac{3}{4}x^8 \implies f'(x) = 8\left(\frac{3}{4}\right)x^7 = 6x^7$

9. $V(r) = \frac{4}{3}\pi r^3 \implies V'(r) = 4\pi r^2$

14. $f(t) = \sqrt{t} - \frac{1}{\sqrt{t}} = t^{\frac{1}{2}} - t^{-\frac{1}{2}} \implies$

$f'(t) = \frac{1}{2}t^{-\frac{1}{2}} - \left(-\frac{1}{2}\right)t^{-\frac{3}{2}} = \frac{1}{2}t^{-\frac{1}{2}} + \frac{1}{2}t^{-\frac{3}{2}}$

$= \frac{1}{2\sqrt{t}} + \frac{1}{2\sqrt{t^3}}$

22. Find the derivative of $F(x) = \frac{x - 3x\sqrt{x}}{\sqrt{x}}$ in two ways:

(i) Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$F(x) = \frac{x - 3x\left(x^{\frac{1}{2}}\right)}{x^{1/2}} = \frac{x - 3x^{\frac{3}{2}}}{x^{\frac{1}{2}}} \longrightarrow$

$F'(x) = \frac{\left(1 - \frac{9}{2}x^{\frac{1}{2}}\right)\left(x^{\frac{1}{2}}\right) - \left(x - 3x^{\frac{3}{2}}\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{\left(x^{\frac{1}{2}}\right)^2}$

$= \frac{x^{\frac{1}{2}} - \frac{9}{2}x - \left(\frac{1}{2}x^{\frac{1}{2}} - \frac{3}{2}x\right)}{x} = \frac{x^{\frac{1}{2}} - \frac{9}{2}x - \frac{1}{2}x^{\frac{1}{2}} + \frac{3}{2}x}{x}$

$= \frac{\frac{1}{2}x^{\frac{1}{2}} - 3x}{x} = \boxed{\frac{1}{2}x^{-\frac{1}{2}} - 3}$

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(ii) Simplifying 1st:

$$F(x) = \frac{x - 3x^{\frac{3}{2}}}{x^{\frac{1}{2}}} = x^{\frac{1}{2}} - 3x \longrightarrow$$

$$\boxed{F'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 3}$$

#s 23-42 Differentiate:

23 $V(x) = (2x^3+3)(x^4-2x) \longrightarrow$

$$V'(x) = (6x^2)(x^4-2x) + (2x^3+3)(4x^3-2)$$

$$= 6x^6 - 12x^3 + 8x^6 - 4x^3 + 12x^3 - 6$$

$$= \boxed{14x^6 - 4x^3 - 6 = V'(x)}$$

33 $f(t) = \dfrac{t^2+2}{t^4-3t^2+1} \longrightarrow$

$$f'(t) = \frac{2t(t^4-3t^2+1) - (t^2+2)(4t^3-6t)}{(t^4-3t^2+1)^2}$$

$$= \frac{2t^5 - 6t^3 + 2t - (4t^5 - 6t^3 + 8t^3 - 12t)}{(\quad)^2}$$

$$= \frac{2t^5 - 6t^3 + 2t - 4t^5 - 2t^3 + 12t}{(\quad)^2} = \boxed{\frac{-2t^5 - 8t^3 + 14t}{(t^4-3t^2+1)^2}}$$

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(49) Find an eq'n of the tangent line to $y = \frac{2x}{x+1}$ @ $(1,1)$:

$$y' = \frac{2(x+1) - 2x(1)}{(x+1)^2} = f'(x) \longrightarrow$$

$$m_{tan} @ (1,1) = f'(1) = \frac{2(2)-2}{3^2} = \frac{2}{9}$$

$$\underline{y = m(x - x_1) + y_1}$$

$$\boxed{y = \frac{2}{9}(x-1) + 1} = \frac{2}{9}x - \frac{2}{9} + \frac{9}{9} =$$

$$\boxed{y = \frac{2}{9}x + \frac{7}{9}}$$

(53) Find eq'n of tangent line & normal line to the curve $y = x + \sqrt{x}$ @ $(1,2)$

$$y' = 1 + \frac{1}{2}x^{-\frac{1}{2}} = f'(x) \longrightarrow$$

$$m_{tan} = f'(1) = 1 + \frac{1}{2}(1)^{-\frac{1}{2}} = \frac{3}{2} = m \longrightarrow$$

$$m_{\perp} = -\frac{2}{3}$$

Tan Line: $y = \frac{3}{2}(x-1) + 2 = \frac{3}{2}x - \frac{3}{2} + \frac{4}{2}$

$$\boxed{y = \frac{3}{2}x + \frac{1}{2}}$$

Normal Line: $y = -\frac{2}{3}(x-1) + 2 = -\frac{2}{3}x + \frac{2}{3} + \frac{6}{3}$

$$\boxed{y = -\frac{2}{3}x + \frac{8}{3}}$$

201 §3.3 #66

66 If $h(2)=4$ & $h'(2)=-3$, Find

$$\frac{d}{dx}\left[\frac{h(x)}{x}\right]\Bigg|_{x=2}$$

$$\frac{d}{dx}\left[\frac{h(x)}{x}\right] = \frac{(h'(x))(x)-(h(x))(1)}{x^2} =$$

$$= \frac{x\,h'(x)-h(x)}{x^2} \Longrightarrow$$

$$\frac{d}{dx}\left[\frac{h(x)}{x}\right]\Bigg|_{x=2} = \frac{2\,h'(2)-h(2)}{2^2} = \frac{(2)(-3)-4}{4}$$

$$= \frac{-6-4}{4} = \frac{-10}{4} = \boxed{-\frac{5}{2} = \frac{d}{dx}\left[\frac{h(x)}{x}\right]\Bigg|_{x=2}}$$