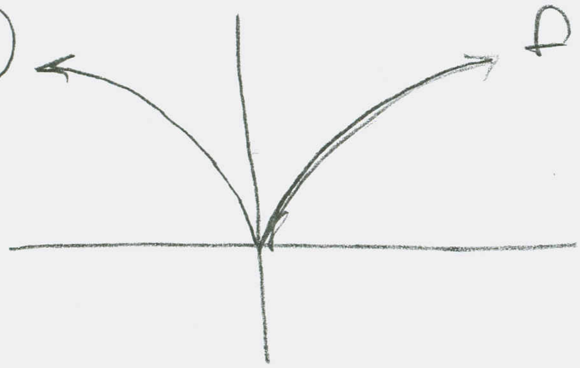
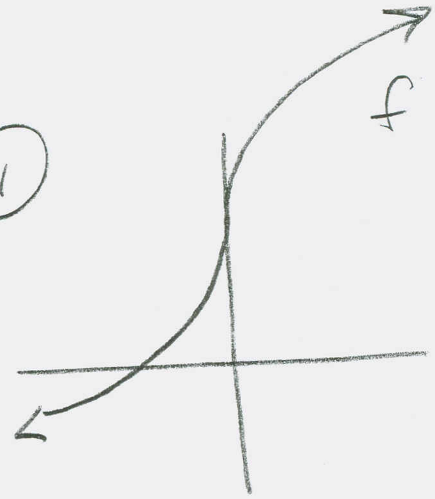
201 §3.2 #s 8, 11, 14, 21, 23, 24, 28, 32, 52

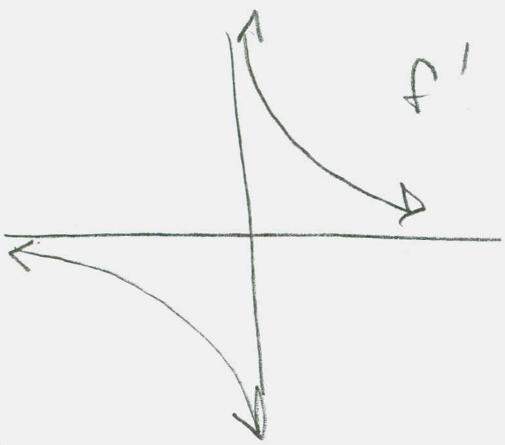
#s 4–11 Sketch the graph of f to sketch f'.

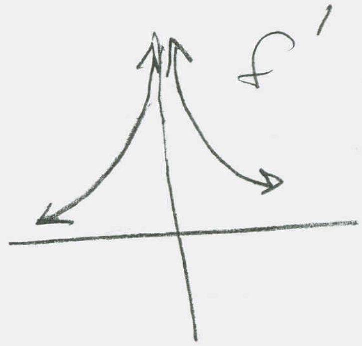
(8)



(11)

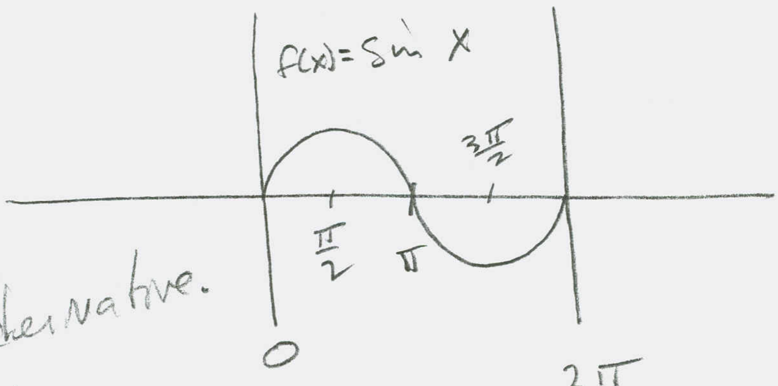




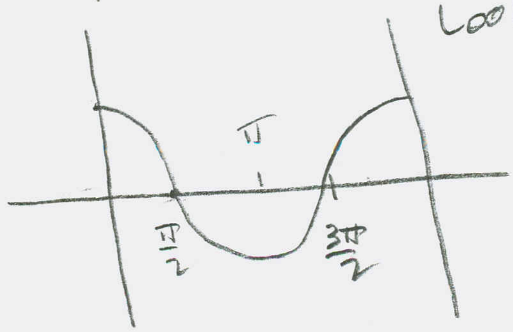


(14)

Sketch sin x & then its derivative. What's it look like?



$f'(x) =$



Looks like cos x!

201 §3.2 #s 21, 23, 24, 28, 32, 52

#s 17-27 Find the derivative of the function using the definition of derivative. State the domain of the function and its derivative.

(21) $f(x) = \frac{1}{2}x - \frac{1}{3}$ $\quad \mathcal{D} = (-\infty, \infty)$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{2}(x+h) - \frac{1}{3} - \left(\frac{1}{2}x - \frac{1}{3}\right)}{h}$$

$$= \frac{\frac{1}{2}x + \frac{1}{2}h - \frac{1}{3} - \frac{1}{2}x + \frac{1}{3}}{h} = \frac{\frac{1}{2}h}{h} = \frac{1}{2} \xrightarrow{h \to 0} \boxed{\frac{1}{2} = f'(x)}$$

$(h \neq 0)$

$$\boxed{\mathcal{D}(f') = (-\infty, \infty)}$$

$$\boxed{\begin{array}{l} \mathcal{D}(g) = [-\frac{1}{2}, \infty) \\ \mathcal{D}(g') = (-\frac{1}{2}, \infty) \end{array}}$$

(23) $g(x) = \sqrt{1+2x}$

$$\frac{g(x+h) - g(x)}{h} = \frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h}$$

$$= \left(\frac{\sqrt{1+2x+2h} - \sqrt{1+2x}}{h}\right)\left(\frac{\sqrt{1+2x+2h} + \sqrt{1+2x}}{\sqrt{1+2x+2h} + \sqrt{1+2x}}\right)$$

$$= \frac{1+2x+2h - (1+2x)}{h\left(\sqrt{1+2x+2h} + \sqrt{1+2x}\right)} = \frac{2h}{h\left(\sqrt{1+2x+2h} + \sqrt{1+2x}\right)}$$

$$= \frac{2}{\sqrt{1+2x+2h} + \sqrt{1+2x}} \xrightarrow{h \to 0} \frac{2}{2\sqrt{1+2x}} = \boxed{\frac{1}{\sqrt{1+2x}} = g'(x)}$$

$(h \neq 0)$

201 §3.2 #s 24, 28, 32, 52

(24) $f(x) = \dfrac{x+3}{1-3x}$

$$\boxed{\mathcal{D} = \mathbb{R} \setminus \{\tfrac{1}{3}\} = (-\infty, \tfrac{1}{3}) \cup (\tfrac{1}{3}, \infty)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[f(x+h)-f(x)\right]$$

$$= \frac{1}{h}\left[\frac{x+h+3}{1-3(x+h)} - \frac{x+3}{1-3x}\right] = \frac{1}{h}\left[\frac{(x+h+3)(1-3x)-(x+3)(1-3x-3h)}{(1-3x-3h)(1-3x)}\right]$$

$$= \frac{1}{h}\left[\frac{x-3x^2+h-3hx+3-9x-x+3x^2+3xh-3+9x+9h}{(1-3x-3h)(1-3x)}\right]$$

$$= \frac{1}{h}\left[\frac{h-3hx+3xh+9h}{(1-3x-3h)(1-3x)}\right] = \frac{1}{h}\left[\frac{10h}{(1-3x-3h)(1-3x)}\right]$$

$$= \frac{10}{(1-3x-3h)(1-3x)} \xrightarrow{h \to 0} \boxed{\frac{10}{(1-3x)^2} = f'(x)}$$

$(h \neq 0)$

$\boxed{\mathcal{D}(f') = \mathcal{D}(f) \text{ for this one.}}$

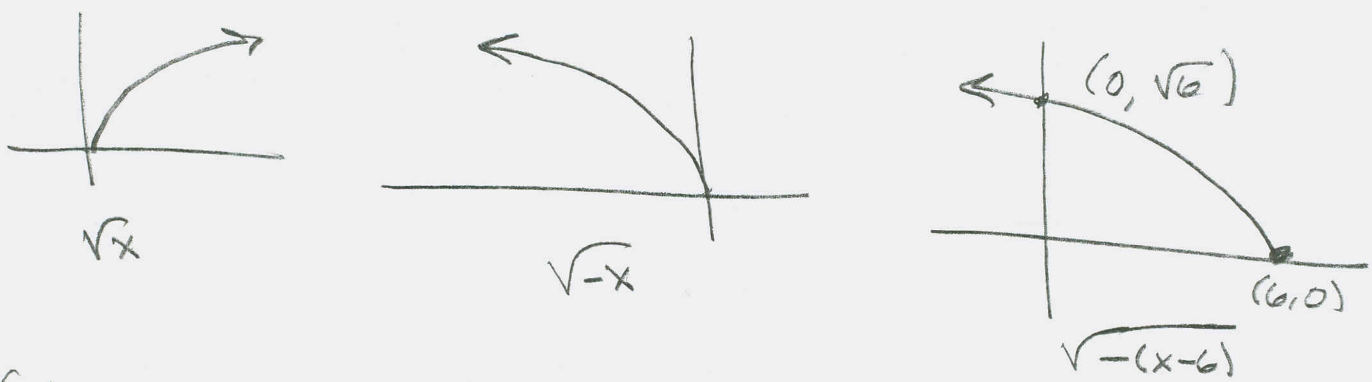
Check: $\dfrac{d}{dx}\left[\dfrac{x+3}{1-3x}\right] = \dfrac{(1-3x)-(x+3)(-3)}{(1-3x)^2}$

$$= \frac{1-3x+3x+9}{(1-3x)^2} = \frac{10}{(1-3x)^2} \checkmark$$

201 §3.2 #s 28, 32, 52

(28) $f(x)=\sqrt{6-x} = \sqrt{-(x-6)}$

(a) $\sqrt{x} \longrightarrow \sqrt{-x} \longrightarrow \longrightarrow \sqrt{-(x-6)}$



(b) Use (a) to graph $f'(x)$

(c) Find $f'(x)$:

$$\frac{\sqrt{6-(x+h)} - \sqrt{6-x}}{h}$$

$$=\left(\frac{\sqrt{6-x-h} - \sqrt{6-x}}{h}\right)\left(\frac{\sqrt{6-x-h}+\sqrt{6-x}}{\sqrt{6-x-h}+\sqrt{6-x}}\right)$$

$$= \frac{6-x-h-(6-x)}{h\left(\sqrt{6-x-h}+\sqrt{6-x}\right)} = -\frac{1}{\sqrt{6-x-h}+\sqrt{6-x}}$$

$(h \neq 0)$

$$\xrightarrow{h \to 0} \boxed{-\frac{1}{2\sqrt{6-x}} = f'(x)}$$

$$\boxed{\begin{array}{l}\mathcal{D}(f) = (-\infty, 6] \\ \mathcal{D}(f') = (-\infty, 6)\end{array}}$$

201 §3.2 #s 32, 52

32 We're given a table of values for P(t) = percentage of Americans under 18 at time t = years after 1950. P'(t) column added

Yr	t	P(t)	P'(t)
1950	0	31.1	.46
1960	10	35.7	.145
1970	20	34.0	−.385
1980	30	28.0	−.415
1990	40	25.7	−.115
2000	50	25.7	0

(a) What's P'(t) mean? It's the rate of change in % of under-18 Americans, per year.

(b) See Table for P'(t) values.

For P'(t), I guess we try taking average slopes? 1950 ⇝ t=0 is special, as is the last one. For all the others, use h = ± 10 & take the average.

$$\frac{P(10)-P(0)}{10-0} = \frac{35.7-31.1}{10} = \frac{4.6}{10} = .46$$

$$\frac{P(20)-P(10)}{10} = \frac{34.0-35.7}{10} = -.17 \quad \text{AVG: } \frac{.46-.17}{2} = .145$$

$$\frac{P(30)-P(20)}{10} = \frac{28.0-34.0}{10} = -.6 \quad \text{AVG: } \frac{-.6-.17}{2} = -.385$$

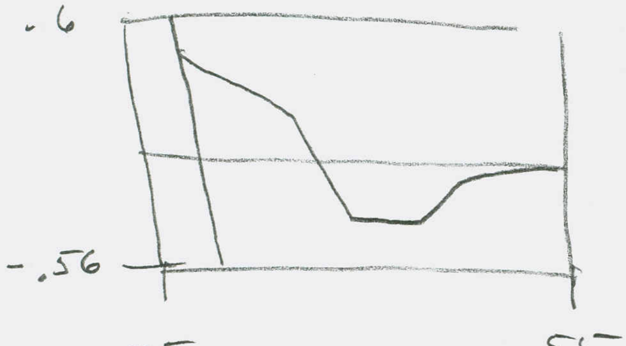
$$\frac{P(40)-P(30)}{10} = \frac{25.7-28.0}{10} = -.23 \quad \text{AVG: } \frac{-.23-.6}{2} = -.415$$

$$\frac{P(50)-P(40)}{10} = \frac{25.7-25.7}{10} = 0 \quad \text{AVG: } -\frac{.23}{2} = -.115$$

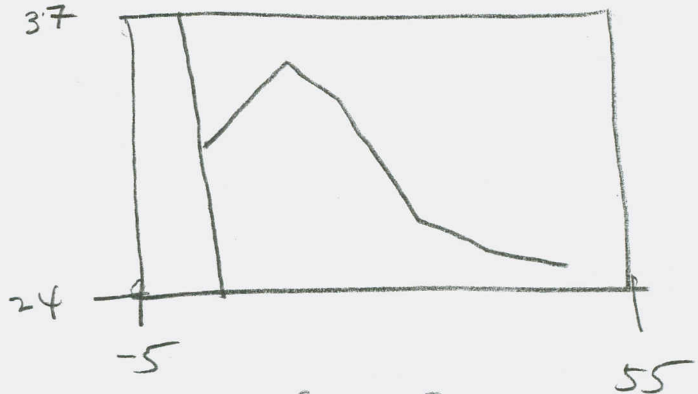
201 §3.2 # 32, 52

32 (c) Graphs of $P'(t)$ & $P(t)$

$P'(t)$:



$P(t)$:

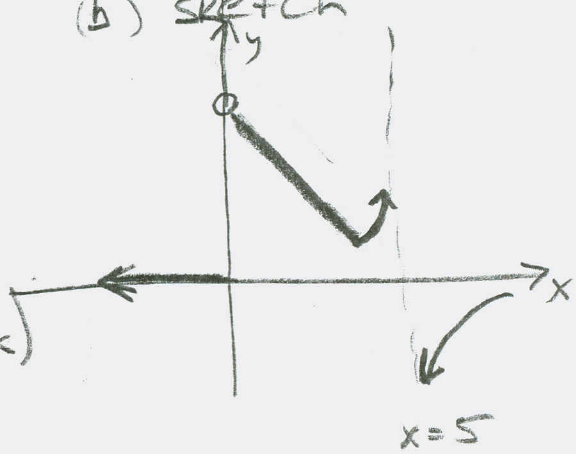


(d) You might get more accurate values for
$P'(t)$ by sketching P' just by looking
at P. Another way would be to fit
an actual function to the data & just
take the derivative of that function.

52 Left- & Right-hand derivatives. We use
them for the piecewise function

(b) sketch

$$f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 5-x & \text{if } 0 < x < 4 \\ \dfrac{1}{5-x} & \text{if } x \ge 4 \end{cases}$$



(a) $f'_-(4) = -1$ (Slope of $y = 5-x$)

$$f'_+(x) = \lim_{h \to 0^+} \frac{1}{h}\left[\frac{1}{5-(x+h)} - \frac{1}{5-x}\right]$$

201 §3.2 #52

$$= \lim_{h\to 0^+} \frac{1}{h}\left[\frac{5-x-(5-x-h)}{(5-x-h)(5-x)}\right]$$

$$= \lim_{h\to 0^+} \frac{1}{h}\left[\frac{h}{(5-x-h)(5-x)}\right] = \lim_{h\to 0^+} \frac{1}{(5-x-h)(5-x)} = \frac{1}{(5-x)^2}$$

$$\Longrightarrow f'_+(4) = \frac{1}{(5-4)^2} = \frac{1}{1} = \boxed{1 = f'_+(4)}$$

$$\boxed{-1 = f'_-(4)}$$

(c) f is discontinuous @ $\boxed{x=0}$, → Not cnts

$\lim_{x\to 0^-} f(x) = 0 \neq 5 = \lim_{h\to 0^+} f(x)$, and $\boxed{x=5}$ (∞)

(d) f is not difbl @ $x=0$ & $x=4$ &
jump. $f_- \neq f_+$

$x=5$.
(∞)

Not smooth!