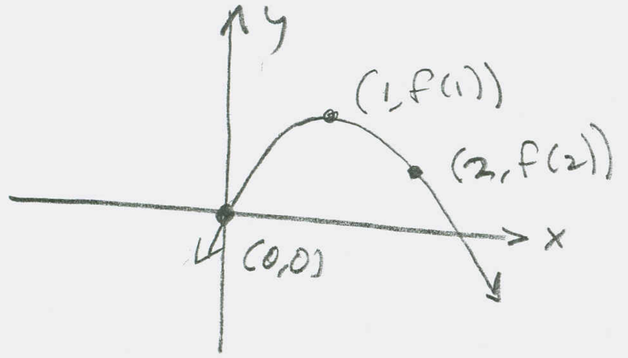
201 §3.1 II #s 19, 21, 25, 31, 32, 44

(19) Sketch the graph of a function f, which satisfies:

$f(0)=0$, $f'(0)=3$, $f'(1)=0$, & $f'(2)=-1$



(21) $f(x)=3x^2-5x$. Find $f'(2)$ & use it to find an eq'n of the tangent line to f @ (2,2)

$$\frac{f(x)-f(2)}{x-2} = \frac{3x^2-5x-2}{x-2} = \frac{(3x+1)(x-2)}{x-2}$$

$$= 3x+1 \xrightarrow{x \to 2} 3(2)+1 = 7 = m_{tan}$$
$(x \neq 2)$

$$\boxed{y = 7(x-2)+2}$$

#s 25-30 Find $f'(a)$

(25) $f(x) = 3-2x+4x^2$

$$\frac{f(a+h)-f(a)}{h} = \frac{3-2(a+h)+4(a+h)^2-(3-2a+4a^2)}{h}$$

$$= \frac{3-2a-2h+4a^2+8ah+4h^2-3+2a-4a^2}{h}$$

201 §3.1 II #s 25, 31, 32, 44

25 cnt'd

$$= \frac{-2h + 8ah + 4h^2}{h} = \frac{6ah + 4h^2}{h} = \frac{h(6a + 4h)}{h}$$

$$= 6a + 4h \xrightarrow{h \to 0} \boxed{6a = f'(a)}$$
$(h \neq 0)$

#s 31-36 Each limit represents the derivative of some function f @ some # a. State f @ a:

(31) $\lim_{h \to 0} \frac{(1+h)^{10} - 1}{h} = \boxed{f'(1) \text{ for } f(x) = x^{10}}$

(32) $\lim_{x \to 5} \frac{2^x - 32}{x - 5} = \boxed{f'(5) \text{ for } f(x) = 2^x}$

(44) A cylindrical tank holds 100,000 gallons of water, which can be drained from the bottom in an hour. Torricelli's Law gives the volume of water remaining after t minutes as

$$V(t) = 100{,}000\left(1 - \frac{t}{60}\right)^2$$

201 §3.1 II # 44

$\boxed{\text{44 cnt'd}}$ Find the rate at which the water is flowing out as a function of t. What are its units? Find flow rate of remaining volume for $t = 0, 10, 20, 30, 40, 50, 60$ minutes. Summarize results. At what time is flow rate the greatest? The least?

$$100000\left(1-\frac{t}{60}\right)^2 = 100000\left(1 - \frac{t}{30} + \frac{t^2}{3600}\right)$$

$$= 100000 - \frac{10000}{3}t + \frac{250}{9}t^2$$

$$= \frac{250}{9}t^2 - \frac{10000}{3}t + 100000 = at^2 + bt + c$$

where $a = \frac{250}{9}$, $b = -\frac{10000}{3}$, $c = 100000$

$$\frac{V(t+h)-V(t)}{h} = \frac{a(t+h)^2 + b(t+h) + c - (at^2+bt+c)}{h}$$

$$= \frac{a(t^2+2th+h^2) + bt + bh + c - at^2 - bt - c}{h}$$

$$= \frac{at^2 + 2ath + ah^2 + bt + bh + c - at^2 - bt - c}{h}$$

$$= \frac{2ath + ah^2 + bh}{h} = \frac{h(2at + ah + b)}{h}$$

$$= \underset{(h \neq 0)}{2at + ah + b} \xrightarrow{h \to 0} 2at + b = V'(t)$$

201 §3.1 II #44

$\boxed{\text{44 cnt'd}}$ $\boxed{v'(t) = 2at + b = \frac{500}{9}t - \frac{10000}{3}}$

t	$V(t)$	$V'(t)$
0	100,000	-3333.33
10	69,444.44	-2,777.78
20	44,444.44	-2,222.22
30	25,000	-1,666.67
40	11,111.11	-1,111.11
50	2,777.78	-555.56
60	0	0

Flow rate is greatest @ the beginning. It steadily drops to zero. This makes sense. More water $\Longrightarrow$ more flow / force.