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① A curve has eq'n $y = f(x)$

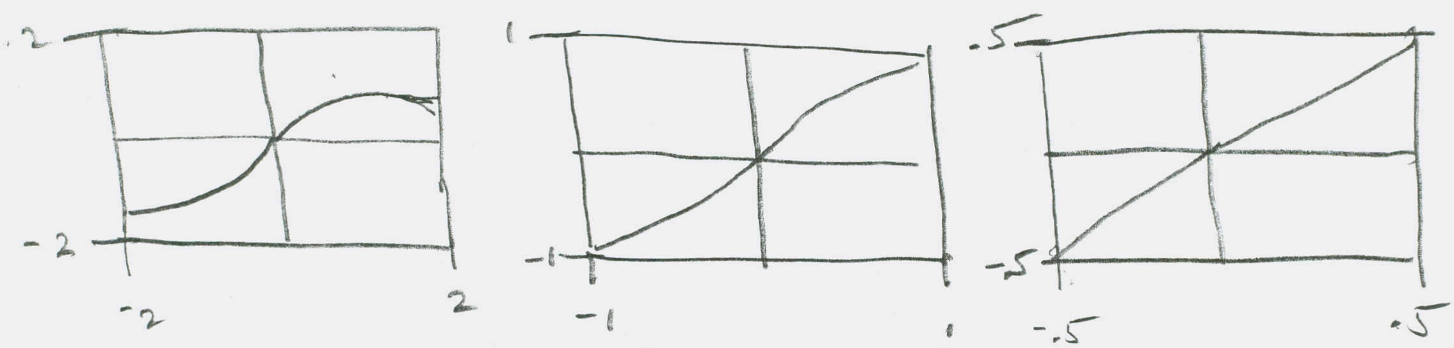
(a) Write expression for m_{sec} = slope of secant line between $P(3, f(3))$ & $Q(x, f(x))$

$$m_{PQ} = \boxed{\frac{f(x) - f(3)}{x - 3}}$$

(b) Write expression for m_{tan} = slope of tangent line @ P :

$$m_{tan}\Big|_{(3, f(3))} = \boxed{\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}}$$

② Graph $y = \sin x$ in the rectangle $[-2, 2] \times [-2, 2]$, $[-1, 1] \times [-1, 1]$, and $[-.5, .5] \times [-.5, .5]$



As we zoom in, it starts to "straighten." This is "local linearity" at work.

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3 (a) Find the slope of the tangent line to the parabola $y = 4x - x^2$ @ $(1, 3)$

(i) Using Definition 1:

$$m_{tan} = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{4x - x^2 - 3}{x - 1} =$$

$$= \lim_{x \to 1} \frac{-(x^2 - 4x + 3)}{x - 1} = \lim_{x \to 1} \frac{-(x-3)(x-1)}{x-1}$$

$$= \lim_{x \to 1} -(x - 3) = -(1 - 3) = -(-2) = \boxed{2 = m_{tan}}$$

(ii) Using Eq'n 2:

$$m_{tan} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{4(1+h) - (1+h)^2 - 3}{h}$$

$$= \lim_{h \to 0} \frac{4 + 4h - 1 - 2h - h^2 - 3}{h} = \lim_{h \to 0} (2 + h) = \boxed{2 = m_{tan}}$$

(b) An equation of the tangent line is

$$\boxed{y = 2(x - 1) + 3} = 2x - 2 + 3 = 2x + 1$$

OR $\boxed{y = 2x + 1}$

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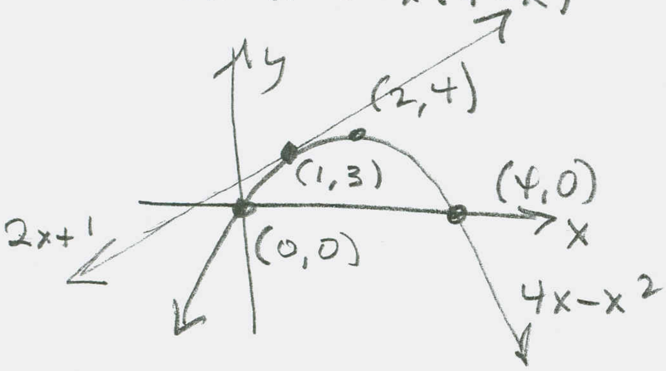
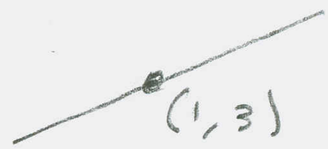
(3) (a) Graph the parabola and the tangent line. As a check, zoom in until the parabola & tangent line are indistinguishable.

$f(x) = 4x - x^2 = x(4-x)$



zoom =



#s 5–8 Find an eq'n of the tangent line to the curve @ the given point.

(5) $y = \dfrac{x-1}{x-2}$ @ $(3,2)$

$$\frac{f(x)-f(3)}{x-3} = \frac{1}{x-3}\left[\frac{x-1}{x-2} - 2\right] = \frac{1}{x-3}\left[f(x) - f(3)\right]$$

$$= \frac{1}{x-3}\left[\frac{x-1-2(x-2)}{x-2}\right] = \frac{1}{x-3}\left[\frac{x-1-2x+4}{x-2}\right]$$

$$= \frac{1}{x-3}\left[\frac{-x+3}{x-2}\right] = \frac{1}{\cancel{x-3}}\left[\frac{-\cancel{(x-3)}}{x-2}\right] = \frac{-1}{x-2} \xrightarrow{x \to 3} \boxed{-1 = m}$$

$(x \neq 3)$

$$y = m(x - x_1) + y_1 = \underbrace{\boxed{-1(x-3) + 2 = y}}_{\text{IS GOOD}} = \underbrace{\boxed{-x + 6 = y}}_{\text{Is also GOOD}}$$

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7 $y = \sqrt{x}$, $(1,1)$

$$\frac{f(x)-f(1)}{x-1} = \frac{\sqrt{x}-1}{x-1} = \left(\frac{\sqrt{x}-1}{x-1}\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}+1}\right)$$

$$= \frac{x-1}{(x-1)(\sqrt{x}+1)} = \frac{1}{\sqrt{x}+1} \xrightarrow{x\to 1} \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

$(x \neq 1)$

$\boxed{y = \frac{1}{2}(x-1)+1}$ OR $y = \frac{1}{2}x + \frac{1}{2}$

14 Rock is thrown upward @ 10 m/s. Its height (in m) is given after time t (in s) by $H(t) = 10t - 1.86t^2$

(a) Find velocity after 1 second.

I'm doing (b) first & plugging in $t=1$ to find (a).

$v(1) = H'(1) = 10 - 3.72(1) = \boxed{6.28 \text{ m/sec}}$

Find velocity when $t = a$

(b) $$\frac{H(t+h)-H(t)}{h} = \frac{10(t+h) - 1.86(t+h)^2 - (10t - 1.86t^2)}{h}$$

$$= \frac{10t + 10h - 1.86t^2 - 3.72th - 1.86h^2 - 10t + 1.86t^2}{h}$$

$$= \frac{10h - 3.72th - 1.86h^2}{h} = 10 - 3.72t - 1.86h \xrightarrow{h\to 0} \boxed{10 - 3.72t}$$

$(h \neq 0)$

$\boxed{v(a) = 10 - 3.72a}$

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(14) (c) when will rock hit surface?

$H(t) = 10t - 1.86t^2 \overset{SET}{=} 0 \Longrightarrow$

$t(10 - 1.86t) = 0 \Longrightarrow$

$t = 0$ OR $10 - 1.86t = 0$

$10 = 1.86t$

$\frac{10}{1.86} = \boxed{t \approx 5.376344086 \text{ s}}$

(d) It will hit the surface @ the same SPEED it left (by physics), but let's see what velocity @ $a = \frac{10}{1.86}$ is:

$V\left(\frac{10}{1.86}\right) = 10 - 3.72\left(\frac{10}{1.86}\right) = 10 - 20 = \boxed{-10 \frac{m}{s}}$

Yup. Same speed, opposite direction.

(18) (a) Find an eq'n of the tangent line to $y = g(x)$ @ $x = 5$, if $g(5) = -3$ & $g'(5) = 4$

$y = m(x - x_1) + y_1$

$\boxed{y = 4(x - 5) - 3} = g'(5)(x - 5) + g(5)$

(b) If tangent ~~to~~ y = f(x) @ (4, 3) if it passes thru (0, 2). Find f(4) & f'(4)

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(18) conclusion for (b):

Given (4,3) & (0,2) are on tangent line,

$$m = \frac{3-2}{4-0} = \boxed{\frac{1}{4} = f'(4)}$$

and, we're Given $\boxed{f(4) = 3}$, for free,

from (4,3) being handed to us.