1. From Definition 1, $\lim_{x\to 4} f(x) = f(4)$.

3. (a) The following are the numbers at which f is discontinuous and the type of discontinuity at that number: -4 (removable), -2 (jump), 2 (jump), 4 (infinite).

(b) f is continuous from the left at -2 since $\lim_{x\to -2^-} f(x) = f(-2)$. f is continuous from the right at 2 and 4 since $\lim_{x\to 2^+} f(x) = f(2)$ and $\lim_{x\to 4^+} f(x) = f(4)$. It is continuous from neither side at -4 since $f(-4)$ is undefined.

9. Since f and g are continuous functions,

$$\begin{aligned}\lim_{x\to 3}[2f(x)-g(x)] &= 2\lim_{x\to 3} f(x) - \lim_{x\to 3} g(x) && \text{[by Limit Laws 2 and 3]}\\ &= 2f(3)-g(3) && \text{[by continuity of } f \text{ and } g \text{ at } x=3]\\ &= 2\cdot 5 - g(3) = 10 - g(3)\end{aligned}$$

Since it is given that $\lim_{x\to 3}[2f(x)-g(x)] = 4$, we have $10 - g(3) = 4$, so $g(3) = 6$.

11. $\lim_{x\to -1} f(x) = \lim_{x\to -1}\left(x+2x^3\right)^4 = \left(\lim_{x\to -1} x + 2\lim_{x\to -1} x^3\right)^4 = \left[-1+2(-1)^3\right]^4 = (-3)^4 = 81 = f(-1)$.

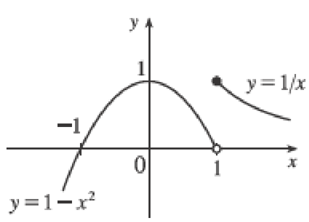
By the definition of continuity, f is continuous at $a = -1$.

13. For $a > 2$, we have

$$\begin{aligned}\lim_{x\to a} f(x) = \lim_{x\to a}\frac{2x+3}{x-2} &= \frac{\lim_{x\to a}(2x+3)}{\lim_{x\to a}(x-2)} && \text{[Limit Law 5]}\\ &= \frac{2\lim_{x\to a} x + \lim_{x\to a} 3}{\lim_{x\to a} x - \lim_{x\to a} 2} && \text{[1, 2, and 3]}\\ &= \frac{2a+3}{a-2} && \text{[7 and 8]}\\ &= f(a)\end{aligned}$$

Thus, f is continuous at $x = a$ for every a in $(2,\infty)$; that is, f is continuous on $(2,\infty)$.

17. $f(x) = \begin{cases} 1-x^2 & \text{if } x < 1 \\ 1/x & \text{if } x \geq 1\end{cases}$



The left-hand limit of f at $a = 1$ is $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(1-x^2) = 0$. The right-hand limit of f at $a = 1$ is $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(1/x) = 1$. Since these limits are not equal, $\lim_{x\to 1} f(x)$ does not exist and f is discontinuous at 1.

21. $F(x) = \dfrac{x}{x^2 + 5x + 6}$ is a rational function. So by Theorem 5 (or Theorem 7), F is continuous at every number in its domain, $\{x \mid x^2 + 5x + 6 \neq 0\} = \{x \mid (x+3)(x+2) \neq 0\} = \{x \mid x \neq -3, -2\}$ or $(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$.