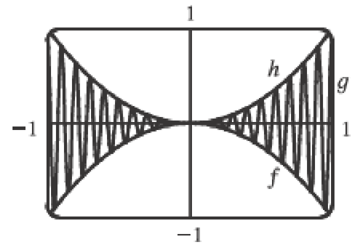
33. Let $f(x) = -x^2$, $g(x) = x^2 \cos 20\pi x$ and $h(x) = x^2$. Then

$-1 \le \cos 20\pi x \le 1 \quad\Rightarrow\quad -x^2 \le x^2 \cos 20\pi x \le x^2 \quad\Rightarrow\quad f(x) \le g(x) \le h(x)$.

So since $\lim_{x\to 0} f(x) = \lim_{x\to 0} h(x) = 0$, by the Squeeze Theorem we have

$\lim_{x\to 0} g(x) = 0$.



35. We have $\lim_{x\to 4}(4x-9) = 4(4) - 9 = 7$ and $\lim_{x\to 4}\left(x^2 - 4x + 7\right) = 4^2 - 4(4) + 7 = 7$. Since $4x - 9 \le f(x) \le x^2 - 4x + 7$ for $x \ge 0$, $\lim_{x\to 4} f(x) = 7$ by the Squeeze Theorem.

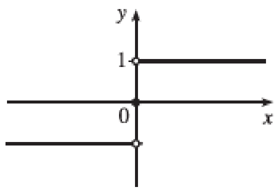
41. $\left|2x^3 - x^2\right| = \left|x^2(2x-1)\right| = \left|x^2\right| \cdot |2x-1| = x^2\,|2x-1|$

$$|2x-1| = \begin{cases} 2x-1 & \text{if } 2x-1 \ge 0 \\ -(2x-1) & \text{if } 2x-1 < 0 \end{cases} = \begin{cases} 2x-1 & \text{if } x \ge 0.5 \\ -(2x-1) & \text{if } x < 0.5 \end{cases}$$

So $\left|2x^3 - x^2\right| = x^2[-(2x-1)]$ for $x < 0.5$.

Thus, $\lim_{x\to 0.5^-} \dfrac{2x-1}{|2x^3 - x^2|} = \lim_{x\to 0.5^-} \dfrac{2x-1}{x^2[-(2x-1)]} = \lim_{x\to 0.5^-} \dfrac{-1}{x^2} = \dfrac{-1}{(0.5)^2} = \dfrac{-1}{0.25} = -4$.

45. (a)



(b) (i) Since $\operatorname{sgn} x = 1$ for $x > 0$, $\lim_{x\to 0^+} \operatorname{sgn} x = \lim_{x\to 0^+} 1 = 1$.

(ii) Since $\operatorname{sgn} x = -1$ for $x < 0$, $\lim_{x\to 0^-} \operatorname{sgn} x = \lim_{x\to 0^-} -1 = -1$.

(iii) Since $\lim_{x\to 0^-} \operatorname{sgn} x \ne \lim_{x\to 0^+} \operatorname{sgn} x$, $\lim_{x\to 0} \operatorname{sgn} x$ does not exist.

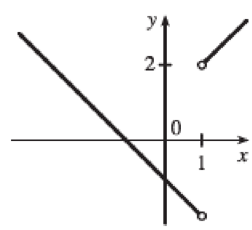
(iv) Since $|\operatorname{sgn} x| = 1$ for $x \ne 0$, $\lim_{x\to 0} |\operatorname{sgn} x| = \lim_{x\to 0} 1 = 1$.

47. (a) (i) $\lim_{x\to 1^+} F(x) = \lim_{x\to 1^+} \dfrac{x^2-1}{|x-1|} = \lim_{x\to 1^+} \dfrac{x^2-1}{x-1} = \lim_{x\to 1^+} (x+1) = 2$

(ii) $\lim_{x\to 1^-} F(x) = \lim_{x\to 1^-} \dfrac{x^2-1}{|x-1|} = \lim_{x\to 1^-} \dfrac{x^2-1}{-(x-1)} = \lim_{x\to 1^-} -(x+1) = -2$

(b) No, $\lim_{x\to 1} F(x)$ does not exist since $\lim_{x\to 1^+} F(x) \ne \lim_{x\to 1^-} F(x)$.

(c)



49. (a) (i) $[\![x]\!] = -2$ for $-2 \le x < -1$, so $\lim\limits_{x\to -2^+} [\![x]\!] = \lim\limits_{x\to -2^+} (-2) = -2$

(ii) $[\![x]\!] = -3$ for $-3 \le x < -2$, so $\lim\limits_{x\to -2^-} [\![x]\!] = \lim\limits_{x\to -2^-} (-3) = -3$.

The right and left limits are different, so $\lim\limits_{x\to -2} [\![x]\!]$ does not exist.

(iii) $[\![x]\!] = -3$ for $-3 \le x < -2$, so $\lim\limits_{x\to -2.4} [\![x]\!] = \lim\limits_{x\to -2.4} (-3) = -3$.

(b) (i) $[\![x]\!] = n-1$ for $n-1 \le x < n$, so $\lim\limits_{x\to n^-} [\![x]\!] = \lim\limits_{x\to n^-} (n-1) = n-1$.

(ii) $[\![x]\!] = n$ for $n \le x < n+1$, so $\lim\limits_{x\to n^+} [\![x]\!] = \lim\limits_{x\to n^+} n = n$.

(c) $\lim\limits_{x\to a} [\![x]\!]$ exists $\Leftrightarrow$ a is not an integer.

55. $\lim\limits_{x\to 1} [f(x) - 8] = \lim\limits_{x\to 1} \left[\dfrac{f(x)-8}{x-1} \cdot (x-1)\right] = \lim\limits_{x\to 1} \dfrac{f(x)-8}{x-1} \cdot \lim\limits_{x\to 1} (x-1) = 10 \cdot 0 = 0$.

Thus, $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \{[f(x) - 8] + 8\} = \lim\limits_{x\to 1} [f(x) - 8] + \lim\limits_{x\to 1} 8 = 0 + 8 = 8$.

Note: The value of $\lim\limits_{x\to 1} \dfrac{f(x)-8}{x-1}$ does not affect the answer since it's multiplied by 0. What's important is that $\lim\limits_{x\to 1} \dfrac{f(x)-8}{x-1}$ exists.