1. (a) $$\begin{aligned}\lim_{x\to 2}[f(x)+5g(x)] &= \lim_{x\to 2}f(x)+\lim_{x\to 2}[5g(x)] && \text{[Limit Law 1]}\\ &= \lim_{x\to 2}f(x)+5\lim_{x\to 2}g(x) && \text{[Limit Law 3]}\\ &= 4+5(-2)=-6\end{aligned}$$

(b) $$\begin{aligned}\lim_{x\to 2}[g(x)]^3 &= \left[\lim_{x\to 2}g(x)\right]^3 && \text{[Limit Law 6]}\\ &= (-2)^3=-8\end{aligned}$$

(c) $$\begin{aligned}\lim_{x\to 2}\sqrt{f(x)} &= \sqrt{\lim_{x\to 2}f(x)} && \text{[Limit Law 11]}\\ &= \sqrt{4}=2\end{aligned}$$

(d) $$\begin{aligned}\lim_{x\to 2}\frac{3f(x)}{g(x)} &= \frac{\lim_{x\to 2}[3f(x)]}{\lim_{x\to 2}g(x)} && \text{[Limit Law 5]}\\ &= \frac{3\lim_{x\to 2}f(x)}{\lim_{x\to 2}g(x)} && \text{[Limit Law 3]}\\ &= \frac{3(4)}{-2}=-6\end{aligned}$$

(e) Because the limit of the denominator is 0, we can't use Limit Law 5. The given limit, $\lim_{x\to 2}\frac{g(x)}{h(x)}$, does not exist because the denominator approaches 0 while the numerator approaches a nonzero number.

(f) $$\begin{aligned}\lim_{x\to 2}\frac{g(x)\,h(x)}{f(x)} &= \frac{\lim_{x\to 2}[g(x)\,h(x)]}{\lim_{x\to 2}f(x)} && \text{[Limit Law 5]}\\ &= \frac{\lim_{x\to 2}g(x)\cdot\lim_{x\to 2}h(x)}{\lim_{x\to 2}f(x)} && \text{[Limit Law 4]}\\ &= \frac{-2\cdot 0}{4}=0\end{aligned}$$

3. $$\begin{aligned}\lim_{x\to -2}(3x^4+2x^2-x+1) &= \lim_{x\to -2}3x^4+\lim_{x\to -2}2x^2-\lim_{x\to -2}x+\lim_{x\to -2}1 && \text{[Limit Laws 1 and 2]}\\ &= 3\lim_{x\to -2}x^4+2\lim_{x\to -2}x^2-\lim_{x\to -2}x+\lim_{x\to -2}1 && \text{[3]}\\ &= 3(-2)^4+2(-2)^2-(-2)+(1) && \text{[9, 8, and 7]}\\ &= 48+8+2+1=59\end{aligned}$$

8. $$\begin{aligned}\lim_{u\to -2}\sqrt{u^4+3u+6} &= \sqrt{\lim_{u\to -2}(u^4+3u+6)} && \text{[11]}\\ &= \sqrt{\lim_{u\to -2}u^4+3\lim_{u\to -2}u+\lim_{u\to -2}6} && \text{[1, 2, and 3]}\\ &= \sqrt{(-2)^4+3(-2)+6} && \text{[9, 8, and 7]}\\ &= \sqrt{16-6+6}=\sqrt{16}=4\end{aligned}$$

11. $\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2} \frac{(x+3)(x-2)}{x-2} = \lim_{x\to 2}(x+3) = 2+3 = 5$

17. $\lim_{h\to 0} \frac{(4+h)^2-16}{h} = \lim_{h\to 0} \frac{(16+8h+h^2)-16}{h} = \lim_{h\to 0} \frac{8h+h^2}{h} = \lim_{h\to 0} \frac{h(8+h)}{h} = \lim_{h\to 0}(8+h) = 8+0 = 8$

30.
$$\lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{x+4} = \lim_{x\to -4} \frac{\left(\sqrt{x^2+9}-5\right)\left(\sqrt{x^2+9}+5\right)}{(x+4)\left(\sqrt{x^2+9}+5\right)} = \lim_{x\to -4} \frac{(x^2+9)-25}{(x+4)\left(\sqrt{x^2+9}+5\right)}$$
$$= \lim_{x\to -4} \frac{x^2-16}{(x+4)\left(\sqrt{x^2+9}+5\right)} = \lim_{x\to -4} \frac{(x+4)(x-4)}{(x+4)\left(\sqrt{x^2+9}+5\right)}$$
$$= \lim_{x\to -4} \frac{x-4}{\sqrt{x^2+9}+5} = \frac{-4-4}{\sqrt{16+9}+5} = \frac{-8}{5+5} = -\frac{4}{5}$$