MAT 201 §2.1 #s 1, 3, 4, 9

① A tank holds 1000 gal. of H_2O, which drains from the bottom in $\frac{1}{2}$ - hour. The table shows V = Volume (in gallons) as a function of t = time (min)

t (minutes)	5	10	15	20	25	30
V (gal)	694	444	250	111	28	0

(a) P = (15, 250) on graph. Find m_{sec} for $\overline{PQ}$, when Q is point corresponding to t = 5, 10, 20, 25, 30

$(t_1, V_1) = (15, 250)$

$(t_2, V_2) = (5, 694) \longrightarrow m_{PQ} = \dfrac{V_2 - V_1}{t_2 - t_1} = \dfrac{694 - 250}{5 - 15} \approx -44.4$

$(t_2, V_2) = (10, 444) \Longrightarrow m_{SEC} = \dfrac{444 - 250}{10 - 15} = -38.8$

$(t_2, V_2) = (20, 111) \Longrightarrow m_{SEC} = \dfrac{111 - 250}{20 - 15} = -27.8$

$(t_2, V_2) = (25, 28) \Longrightarrow m_{sec} = \dfrac{28 - 250}{25 - 15} = -22.2$

$(t_2, V_2) = (30, 0) \longrightarrow m_{sec} = \dfrac{-250}{30 - 15} = -16.\overline{66} \approx -16.67$

(b) Estimate m_{tan} @ P, by taking an average of two slopes:

$\dfrac{-22.2 + (-16.\overline{6})}{2} = -19.4\overline{3} \approx -19.4$

(c) Estimate the tangent @ P from a graph. (Not too keen on this one)

MAT 201 §2.1 #s 3, 4, 9

(3) $P(1, \frac{1}{2})$ lies on $y = \frac{x}{1+x}$

(a) Let $Q = (x, \frac{x}{1+x})$. Find m_{PQ} (to 5 places) for these values:

(i) $x = .5 \Rightarrow m = .\overline{3}$

(ii) $x = .9 \Rightarrow m \approx .26316$

(iii) $x = .99 \Rightarrow m \approx .25126$

(iv) $x = .999 \Rightarrow m \approx .25013$

(v) $x = 1.5 \Rightarrow m \approx .2$

(vi) $x = 1.1 \Rightarrow m \approx .2381$

(vii) $x = 1.01 \Rightarrow m \approx .24876$

(viii) $x = 1.001 \Rightarrow m \approx .24988$

(b) From part (a), we estimate $m_{tan} = .25$ @ $x = 1$.

(c) From part (b), we build an eq'n for the tangent line to $y = \frac{x}{1+x}$ @ $x = 1$:

$y = m(x - x_1) + y_1$

$\boxed{y = .25(x-1) + \frac{1}{2}} = \frac{1}{4}x - \frac{1}{4} + \frac{1}{2} = \frac{1}{4}x + \frac{1}{4}$

MAT 201 §2.1 #s 4, 9

(4) $P(3,1)$ is on $y = \sqrt{x-2}$

(a) Let $Q = (x, \sqrt{x-2})$. Find $m_{\overline{PQ}}$ $\forall$ of

(i) $x = 2.5 \Rightarrow m \approx .58579$

(ii) $x = 2.9 \Rightarrow m \approx .51317$

(iii) $x = 2.99 \Rightarrow m \approx .50126$

(iv) $x = 2.999 \Rightarrow m \approx .50013$

(v) $x = 3.5 \Rightarrow m \approx .44949$

(vi) $x = 3.1 \Rightarrow m \approx .48809$

(vii) $x = 3.01 \Rightarrow m \approx .49876$

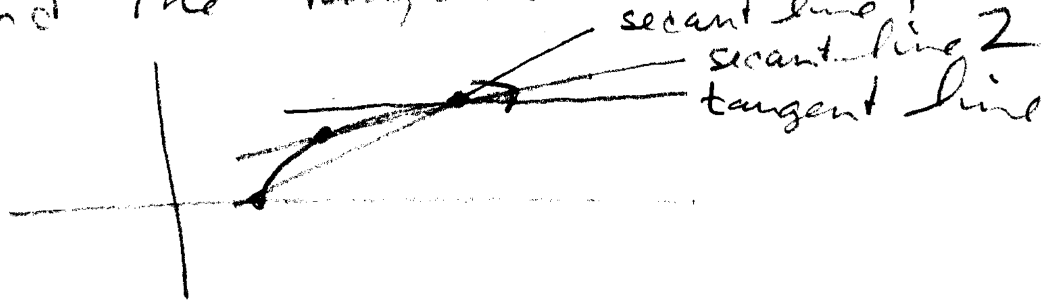
(iv) $x = 3.001 \Rightarrow m \approx .49988$

(b) From (a) we estimate m_{tan} @ $x = 3$ is $m \approx .5$

(c) Tangent line Equation:

$\boxed{y = .5(x-3)+1} = .5x - 1.5 + 1 = .5x - .5$

(d) We sketch the curve & show two secant lines and the tangent line



MAT 201 §2.1 I #9

(9) $P(1,0)$ lies on $f(x) = \sin\left(\frac{10\pi}{x}\right)$

(a) Q is $\left(x, \sin\left(\frac{10\pi}{x}\right)\right)$ Find m_{PQ} for

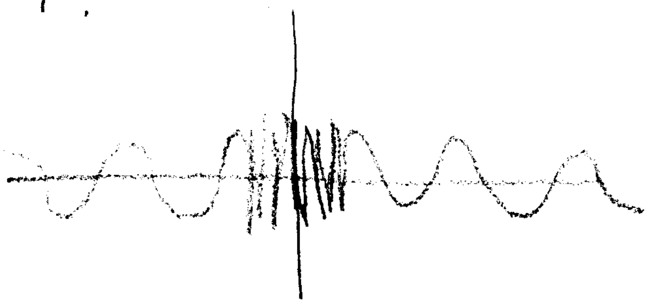
$x = 2, 1.5, 1.4, 1.3, 1.2, 1.1, 0.5, 0.6, 0.7, 0.8, 0.9$

Do the slopes appear to approach a limit?

x	m_{PQ}	x	m_{PQ}
2	0	.5	0
1.5	1.7321	.6	-2.165
1.4	-1.085	.7	-2.606
1.3	-2.743	.8	-5
1.2	4.3301	.9	3.4202
1.1	-2.817		

They don't appear to be approaching a limit

(b) Use a graph to explain why the slopes in (a) are not close to the slope of the tangent line @ P.



The graph is oscillating much too rapidly. Our 2nd points are all over the place.

(c)

To use a numerical method, we'd need to get *much* closer to $x=1$.

Using $x = 1.00001$ I get $\boxed{-31.42 \approx m_{tan}}$

" $x = .9999$, I get $\boxed{-31.42 \approx m_{tan}}$.

#1 b: The solutions should have used two points closer to (15, 250), for instance, (10, 444), and (20, 111). This (in principle) should give a better estimate for the slope of the tangent. Using *these* points as the 2nd point in the secant slope calculation and then taking the average gives:

$\frac{-28.8+(-27.8)}{2}=\frac{-66.6}{2}=-33.3\approx m_{\tan}$. The original solutions gave -19.4, approximately.