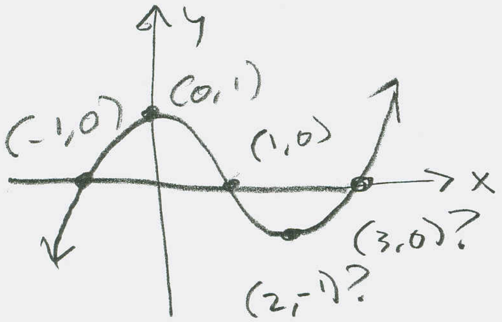
201 §1.3 II #s 28, 30, 33, 39, 43, 49, 51, 54, 63

(28) Use the graph of f to sketch the graph of $\frac{1}{f(x)}$. Which features are most important in sketching $y = \frac{1}{f(x)}$? Explain how they are used.



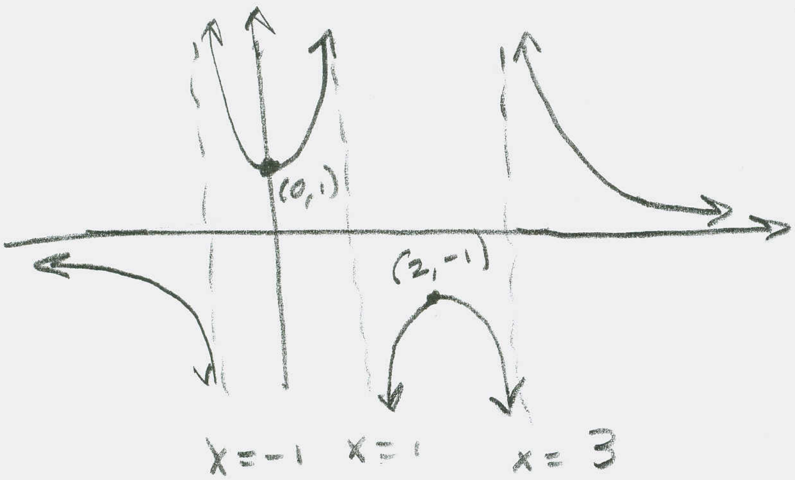
$f(x) = 0 \Rightarrow \frac{1}{f(x)} \nexists$

Those will be vertical asymptotes.

$f(0) = 1 \Rightarrow \frac{1}{f(0)} = 1$

$f(2) = -1 \Rightarrow \frac{1}{f(2)} = -1$

Behavior near asymptotes is controlled by the sign of f(x).



A sketch of $y = \frac{1}{f(x)}$

f(x) grows without bound as $x \to \pm\infty$, which means $\frac{1}{f(x)} \to 0$ as $x \to \pm\infty$

(0,1) is now a max.

(2,-1) is now a min.

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30 Find $f+g$, $f-g$, fg & $\frac{f}{g}$. State their domains.

$f(x) = \sqrt{3-x}$, $g(x) = \sqrt{x^2-1}$

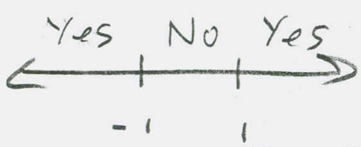
$\mathcal{D}(f) = \{x \mid 3-x \geq 0\} = \{x \mid x \leq 3\} = (-\infty, 3]$

$\mathcal{D}(g) = \{x \mid x^2-1 \geq 0\} = \{x \mid x \leq -1 \text{ OR } x \geq 1\}$

$= (-\infty, -1] \cup [1, \infty)$

$x^2-1 \geq 0$:

$(x-1)(x+1) \geq 0$

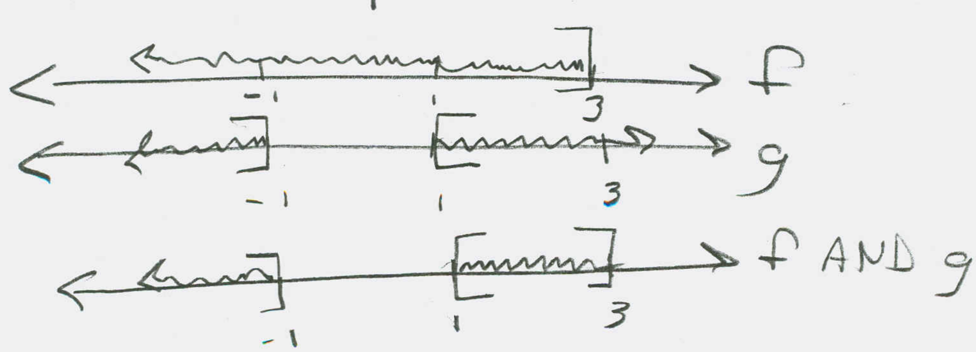


supports $\mathcal{D}(g)$, above.

$\mathcal{D}(f+g) = \mathcal{D}(f-g) = \mathcal{D}(fg) =$

$= \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g)\}$

$= \{x \mid x \in \mathcal{D}(f) \cap \mathcal{D}(g)\}$



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(30) Cnt'd. We take the intersection

$$\boxed{\mathcal{D} = (-\infty, -1] \cup [1, 3] \text{ for } f \pm g \text{ and } fg.}$$

For $\mathcal{D}\left(\frac{f}{g}\right)$, we need $g \neq 0$, also:

$g(x) = 0$

$\sqrt{x^2-1} = 0$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$ are bad.

$\therefore$ Throw $x = \pm 1$ out of previous:

$$\boxed{\mathcal{D}(f/g) = (-\infty, -1) \cup (1, 3]}$$

Finally,

$$\boxed{\begin{aligned}(f+g)(x) &= \sqrt{3-x} + \sqrt{x^2-1}\\ (f-g)(x) &= \sqrt{3-x} - \sqrt{x^2-1}\\ (fg)(x) &= \sqrt{3-x}\sqrt{x^2-1}\end{aligned}}$$

, which you could write as $\sqrt{(3-x)(x^2-1)}$, etc.

$$\boxed{\left(\frac{f}{g}\right)(x) = \frac{\sqrt{3-x}}{\sqrt{x^2-1}}}$$

(33) Find (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, (d) $g \circ g$

Find domains of each.

$f(x) = 1 - 3x$, $\mathcal{D} = \mathbb{R}$

$g(x) = \cos(x)$, $\mathcal{D} = \mathbb{R}$

$\boxed{\text{So all these domains will be } \mathbb{R}.}$

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33 cont'd

(a) $(f \circ g)(x) = f(g(x)) = \boxed{1 - 3\cos x = (f \circ g)(x)}$

(b) $(g \circ f)(x) = g(f(x)) = \boxed{\cos(1 - 3x) = (g \circ f)(x)}$

(c) $(f \circ f)(x) = 1 - 3(1 - 3x) = 1 - 3 + 9x = \boxed{9x - 2 = (f \circ f)(x)}$

(d) $(g \circ g)(x) = g(g(x)) = \boxed{\cos(\cos x) = (g \circ g)(x)}$

39 Find $f \circ g \circ h$:

$f(x) = \sqrt{x-3}$, $g(x) = x^2$, $h(x) = x^3 + 2$

$\Rightarrow (f \circ g \circ h)(x) = f(g(h(x))) =$

$f((x^3+2)^2) = \boxed{\sqrt{(x^3+2)^2 - 3} = (f \circ g \circ h)(x)}$

43 Express in the form $f \circ g$, where

$F(x) = \dfrac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$. Let $f(x) = \dfrac{x}{1+x}$ and

let $g(x) = \sqrt[3]{x}$. Then $(f \circ g)(x) = F(x)$

201 §1.3 II #s 49, 51, 54, 63

(49) Express $H(x) = \sec^4(\sqrt{x})$ in the form $f \circ g \circ h$:

Let $f(x) = x^4$

$g(x) = \sec x$ Then $H(x) = (f \circ g \circ h)(x)$

$h(x) = \sqrt{x}$

(51) Use the graphs of f and g to evaluate each expression

(a) $f(g(2)) = f(5) = 4$

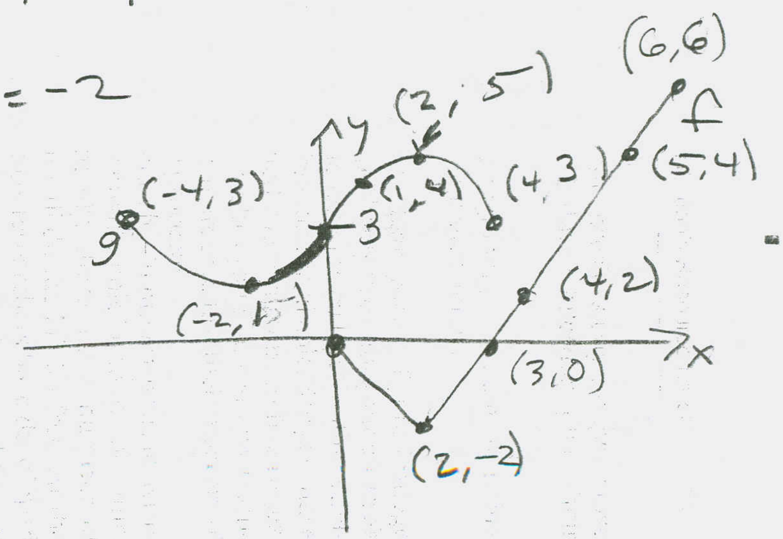
(b) $g(f(0)) = g(0) = 3$

(c) $(f \circ g)(0) = f(g(0)) = f(3) = 0$

(d) $(g \circ f)(6) = g(f(6)) = g(6) = \nexists$

(e) $(g \circ g)(-2) = g(1) = 4$

f) $(f \circ f)(4) = \underline{f(2)} = -2$



201 §1.3 II #s 54, 63

(54) a spherical balloon's being blown up and its radius is increasing at a rate of $2\,\frac{cm}{s}$.

(a) Express radius of balloon as a function of t = time in seconds.

$$r = 2t$$

(b) V = volume of balloon, in cm^3, as a function of r is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (2t)^3 = \frac{4}{3}\pi (2^3 t^3)$$

$$= \frac{32}{3}\pi t^3$$ gives volume as a function of time. Notice V increases in proportion to the <u>cube</u> of the time. So the guy must be blowing harder & harder, to keep the radius expanding at a constant rate.

201 §1.3 II #63

(63) If f & g are even functions.

(a) What can be said about $f+g$ & fg?

They are both even. Check:

$$(f+g)(-x) = f(-x)+g(-x) = f(x)+g(x)$$
$$= (f+g)(x) \checkmark$$

$$(fg)(-x) = f(-x)\,g(-x) = f(x)\,g(x) = (fg)(x) \checkmark$$

(b) If f & g are odd, then $f+g$ is odd and fg is EVEN! Check:

$$(f+g)(-x) = f(-x)+g(-x) = -f(x) + -g(x)$$
$$= -\big(f(x)+g(x)\big) = -(f+g)(x) \checkmark$$

$$(fg)(-x) = f(-x)\,g(-x) = \big(-f(x)\big)\big(-g(x)\big)$$
$$= +f(x)\,g(x) = (fg)(x) \checkmark$$