

① $f(x) = 2\sin(x) + \cos(2x)$

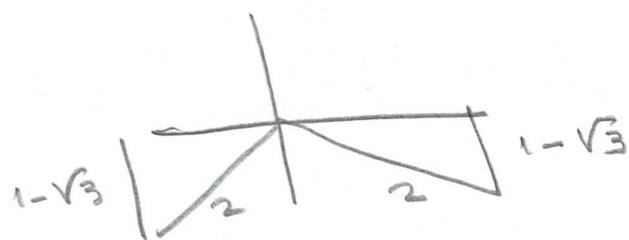
$$= 2\sin(x) + (1 - 2\sin^2(x)) = -2\sin^2(x) + 2\sin(x) + 1$$

$$\underline{SEI} 0 \Rightarrow a = -2, b = 2, c = 1 \Rightarrow$$

$$b^2 - 4ac = 2^2 - 4(-2)(1) = 4 + 8 = 12 \rightarrow$$

$$\sin(x) = \frac{-2 \pm \sqrt{12}}{2(-2)} = \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

$$\Rightarrow \sin(x) = \frac{1 - \sqrt{3}}{2}, \text{ b/c } \frac{1 + \sqrt{3}}{2} > \frac{2.73}{2} > 1$$



$$2\pi + \arcsin\left(\frac{1 - \sqrt{3}}{2}\right)$$

$$\pi - \arcsin\left(\frac{1 - \sqrt{3}}{2}\right)$$

$$x\text{-int: } \left(2\pi + \arcsin\left(\frac{1 - \sqrt{3}}{2}\right), 0\right) \approx (5.908450874, 0)$$

$$\left(\pi - \arcsin\left(\frac{1 - \sqrt{3}}{2}\right), 0\right) \approx (3.516327086, 0)$$

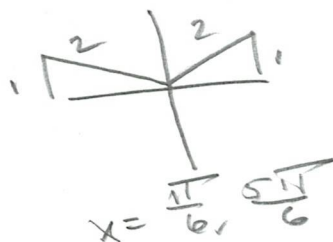
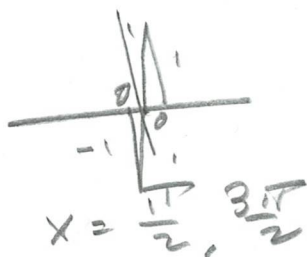
$$\boxed{y\text{-int: } (0, 1)}$$

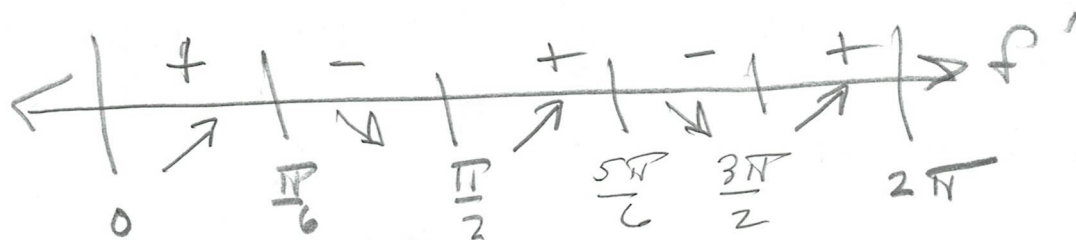
$$f'(x) = -4\sin(x)\cos(x) + 2\cos(x)$$

$$= -2\cos(x)[2\sin(x) - 1] \quad \underline{SEI} 0 \rightarrow$$

$$\cos(x) = 0$$

$$\sin(x) = \frac{1}{2}$$





Test: $\frac{\pi}{12} = 15^\circ$ $f'(\frac{\pi}{6}) \approx .93185$ + $(0, \frac{\pi}{6})$
 $\frac{\pi}{3} = 60^\circ$ $f'(\frac{\pi}{3}) \approx -.73205$ - $(\frac{\pi}{6}, \frac{\pi}{2})$
 $\frac{2\pi}{3} = 120^\circ$ $f'(\frac{2\pi}{3}) \approx .73205$ + $(\frac{\pi}{2}, \frac{5\pi}{6})$
 $\frac{4\pi}{3} = 240^\circ$ $f'(\frac{4\pi}{3}) \approx -2.73205$ - $(\frac{5\pi}{6}, \frac{3\pi}{2})$
 $\frac{5\pi}{3} = 300^\circ$ $f'(\frac{5\pi}{3}) \approx 2.73205$ + $(\frac{3\pi}{2}, 2\pi)$

$$f''(x) = -4\cos^2(x) + 4\sin^2(x) - 2\sin(x)$$

$$= -4(1 - \sin^2(x)) + 4\sin^2(x) - 2\sin(x)$$

$$= -4 + 4\sin^2(x) + 4\sin^2(x) - 2\sin(x)$$

$$= 8\sin^2(x) - 2\sin(x) - 4 \stackrel{\text{SET } 0}{\Rightarrow}$$

$$4\sin^2(x) - \sin(x) - 2 = 0$$

$$a=4, b=-1, c=-2 \rightarrow b^2 - 4ac = 1 - 4(4)(-2)$$

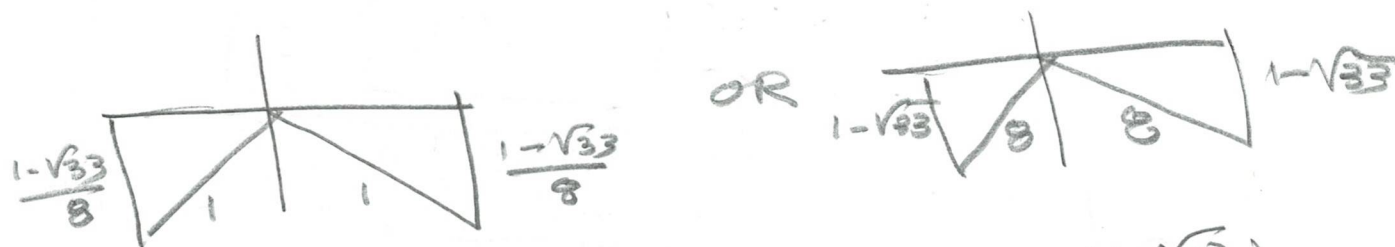
$$= 1 + 32 = 33 \rightarrow$$

$$\sin(x) = \frac{1 \pm \sqrt{33}}{2(4)} = \frac{1 \pm \sqrt{33}}{8}$$



$$x = \arcsin\left(\frac{1+\sqrt{33}}{8}\right), \pi - \arcsin\left(\frac{1+\sqrt{33}}{8}\right)$$

$$x \approx 1.002966954, 2.1386257 \quad f'' = 0$$



$$x = 2\pi + \arcsin\left(\frac{1-\sqrt{33}}{8}\right), \quad \pi - \arcsin\left(\frac{1-\sqrt{33}}{8}\right)$$

$$x \approx 5.648318436, 3.776459525 \quad f'' = 0$$

Zeros of f' digital: (Left to Right)

$$0.5235987758, 1.570796327, 2.617993878,$$

$$4.712388981$$

Zeros of f'' digital: (Left to Right)

$$1.002966954, 2.1386257, 3.776459525,$$

$$5.648318436$$

Scratch for $f\left(2\pi + \arcsin\left(\frac{1-\sqrt{33}}{8}\right)\right) =$

$$-2\sin^2\left(2\pi + \arcsin\left(\frac{1-\sqrt{33}}{8}\right)\right) + 2\sin\left(2\pi + \arcsin\left(\frac{1-\sqrt{33}}{8}\right)\right)$$

$$+ 1$$

$$= -2\left(\frac{1-\sqrt{33}}{8}\right)^2 + 2\left(\frac{1-\sqrt{33}}{8}\right) + 1$$

$$= -2\left(\frac{1-2\sqrt{33}+33}{64}\right) + \frac{2-2\sqrt{33}}{8} + 1$$

$$= -\left(\frac{1-2\sqrt{33}+33}{32}\right) + \frac{2-2\sqrt{33}}{8} \cdot \frac{4}{4} + \frac{32}{32}$$

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T3T14

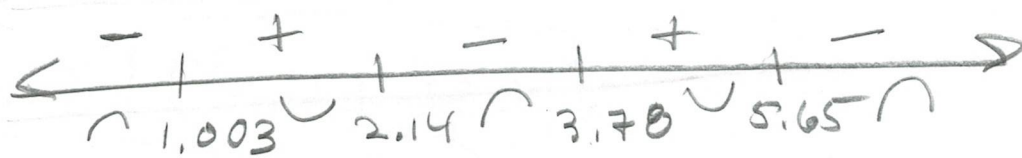
(4)

$$= \frac{-1 + 2\sqrt{33} - 33 + 8 - 8\sqrt{33} + 32}{32}$$

$$= \frac{6 - 6\sqrt{33}}{32} = \frac{3 - 3\sqrt{33}}{16}$$

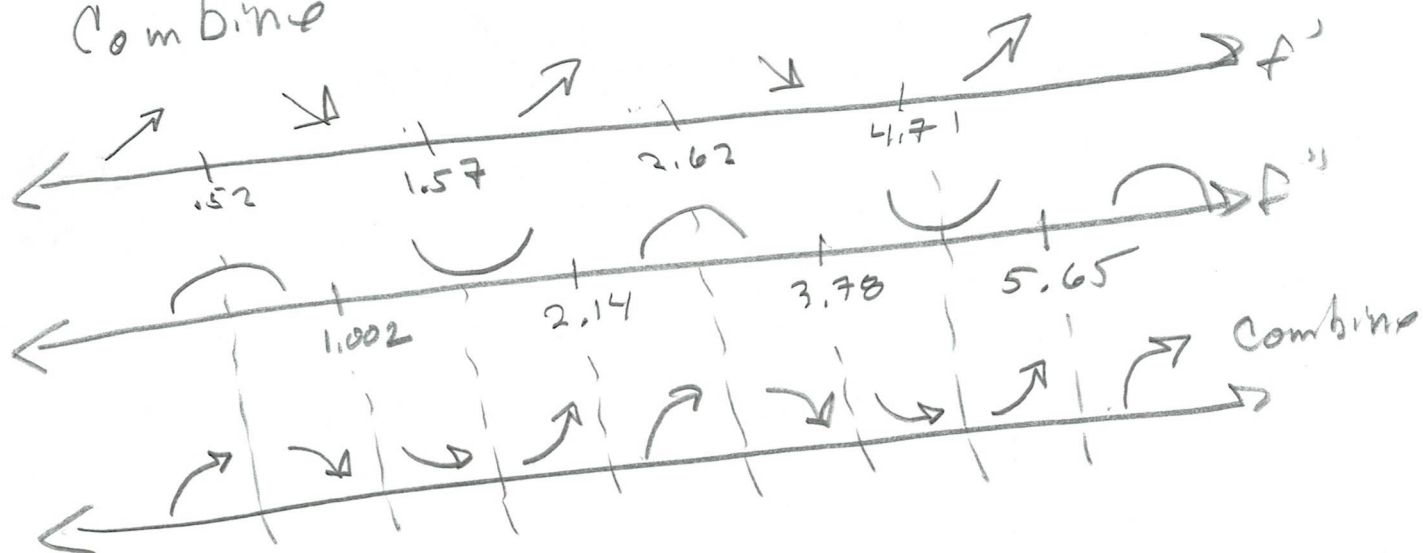
I cheated and used MAPLE to get the rest.

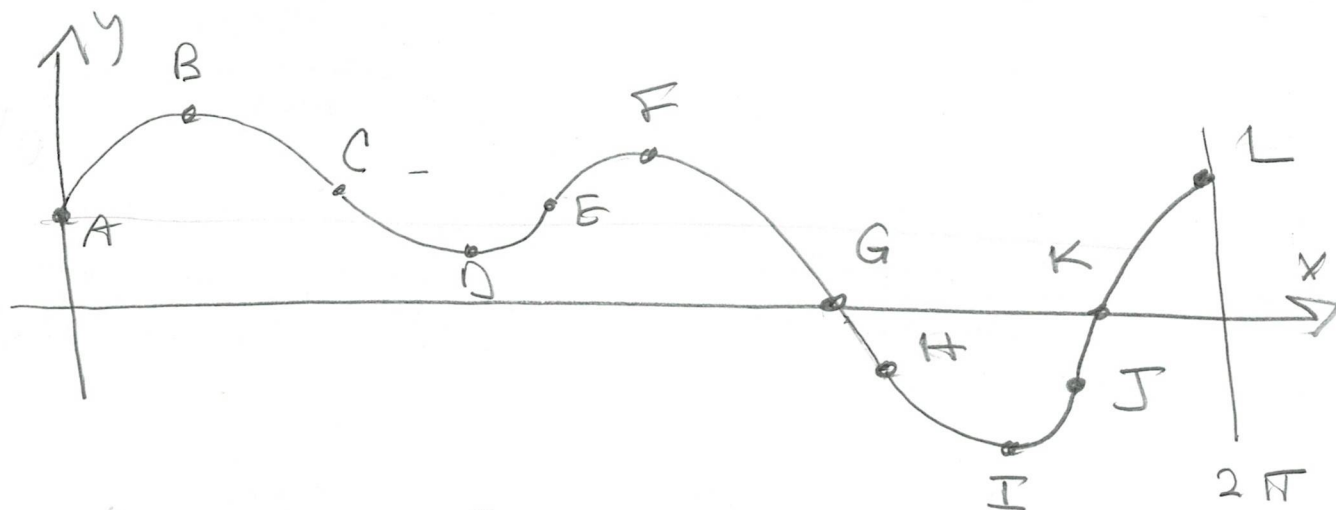
Now, do sign pattern for f''



Test: 2	$f''(2) \approx 0.796$	+
3	$f''(3) \approx -4.12$	-
4	$f''(4) \approx 2.096$	+
6	$f''(6) \approx -2.82$	-
1	$f''(1) \approx -0.018$	-

Combine





$$A = (0, 1) \text{ END POINT}$$

$$B = \left(\frac{\pi}{6}, \frac{3}{2}\right) \approx (.5326, 1.5000) \text{ MAX}$$

$$C = \left(\arcsin\left(\frac{1+\sqrt{33}}{8}\right), \frac{3+3\sqrt{33}}{16}\right) \text{ IP}$$

$$\approx (1.0030, 1.2646)$$

$$D = \left(\frac{\pi}{2}, 1\right) \approx (1.5708, 1.0000) \text{ MIN}$$

$$E = \left(\pi - \arcsin\left(\frac{1+\sqrt{33}}{8}\right), \frac{3+3\sqrt{33}}{16}\right)$$

$$\approx (2.1386, 1.2646)$$

$$F = \left(\frac{5\pi}{6}, \frac{3}{2}\right) \approx (2.6180, 1.5000)$$

$$G = \left(\pi - \arcsin\left(\frac{1-\sqrt{3}}{2}\right), 0\right) \approx (3.5163, 0)$$

$$H = \left(\pi - \arcsin\left(\frac{1-\sqrt{33}}{8}\right), \frac{3-3\sqrt{33}}{16}\right)$$

$$\approx (3.7765, -.8896)$$

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T3TH

(6)

$$I = \left(\frac{3\pi}{2}, -3 \right) \approx (4.7124, 3.0000) \text{ MIN}$$

$$J = \left(2\pi + \arcsin\left(\frac{1-\sqrt{33}}{2}\right), \frac{3-3\sqrt{33}}{16} \right)$$

$$\approx (5.6483, -0.8896) \quad \text{IP}$$

$$K = \left(2\pi + \arcsin\left(\frac{1-\sqrt{3}}{2}\right), 0 \right) \approx (5.9095, 0)$$

$$L = (2\pi, 1) \approx (6.2832, 0)$$

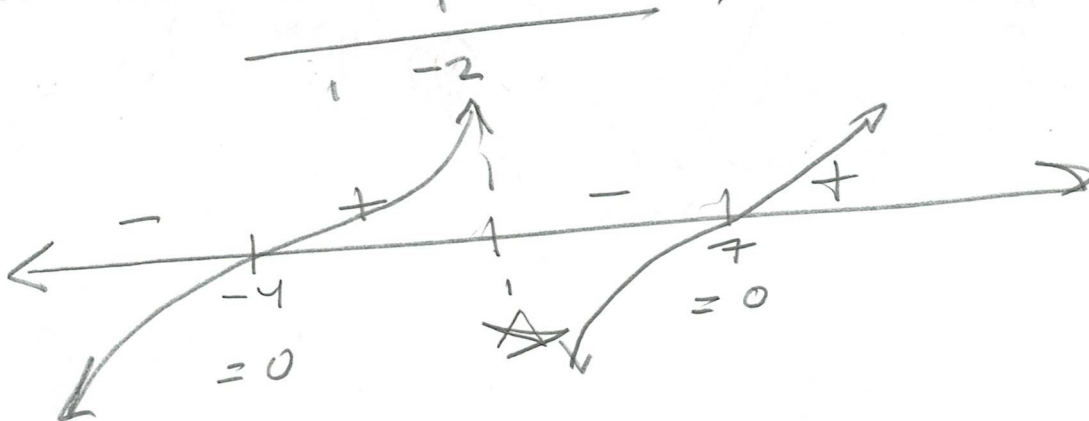
#2 $R(x) = \frac{x^2 - 3x - 28}{x - 1} = \frac{(x-7)(x+4)}{x-1}$

$$D = \mathbb{R} \setminus \{1\} \rightarrow \boxed{\text{V.A.: } x=1}$$

$$\begin{aligned} x\text{-int: } & (-4, 0), (7, 0) \\ y\text{-int: } & (0, 28) \end{aligned}$$

$$\text{O.A.: } \begin{array}{c} 1 \quad -3 \quad -28 \\ \hline -2 \end{array}$$

$$y = x - 2 \quad \text{O.A.}$$



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$$R'(x) = \frac{(2x-3)(x-1) - (x^2-3x-28)(1)}{(x-1)^2}$$

$$= \frac{2x^2 - 5x + 3 - x^2 + 3x + 28}{(x-1)^2} = \frac{x^2 - 2x + 31}{(x-1)^2}$$

$$\text{Set } 0 \Rightarrow x^2 - 2x + 31 = 0$$

$$\Rightarrow x^2 - 2x + 1^2 - 1 + 31 = (x-1)^2 + 30 = 0 \Rightarrow$$

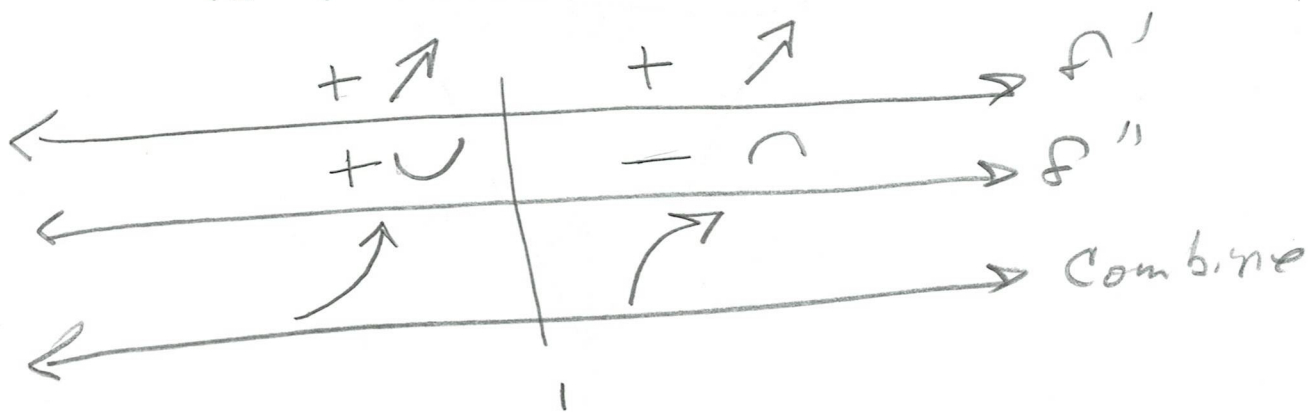
No real solns $\Rightarrow$ No max/min.

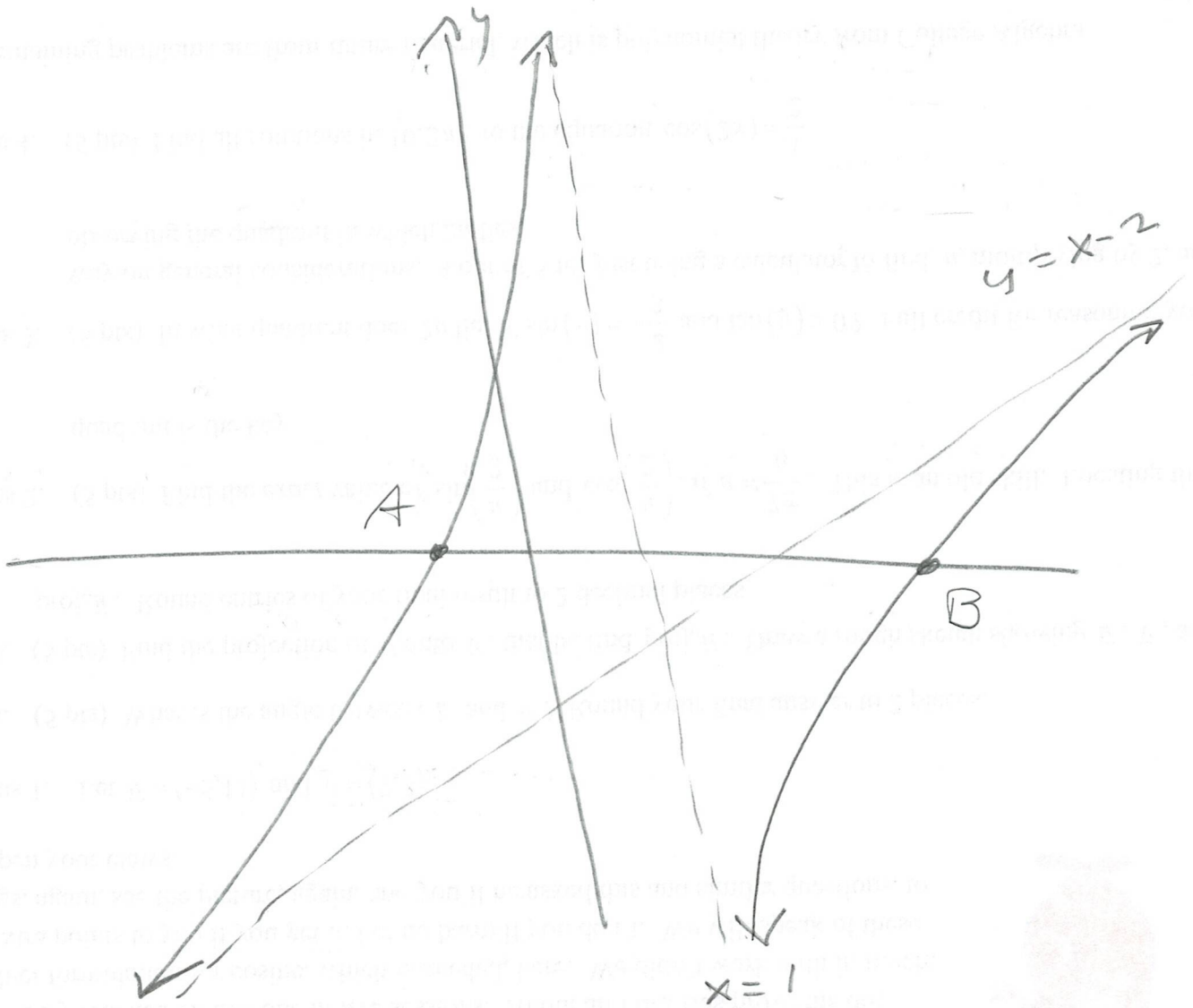
$$R''(x) = \frac{(2x-2)(x-1)^2 - (x^2-2x+31)(2(x-1))}{(x-1)^4}$$

$$= \frac{2(x-1)^2 - 2(x^2-2x+31)}{(x-1)^3}$$

$$= \frac{2(x^2-2x+1) - 2x^2 + 4x - 62}{(x-1)^3}$$

$$= \frac{2x^2 - 4x + 2 - 2x^2 + 4x - 62}{(x-1)^3} = \frac{-60}{(x-1)^3}$$





$$A = (-4, 0)$$

$$B = (7, 0)$$