A word on Integrals by the definition.

Right-endpoints is all we'll worry about, here. First, the theory...

Claim: $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

We prove by induction.

First, it works for $n=1$

$\sum_{k=1}^{1} k^2 = 1^2 = 1$ &

$\frac{1(1+1)(2(1)+1)}{6} = \frac{2(3)}{6} = \frac{6}{6} = 1 \rightarrow$ Holds for $n=1$.

Want to show that if it holds for some $n \geq 1$, then it holds for $n+1$.

$P(1)$ holds. Assume $P(n)$ holds for $n \geq 1$ & show that implies $P(n+1)$ holds.

$P(n+1)$ means $\sum_{k=1}^{n+1} k^2 = \frac{(n+1)(n+1+1)(2(n+1)+1)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$

We assume $P(n)$ holds, i.e., $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

Then $\sum_{k=1}^{n+1} k^2 = \underbrace{1^2+2^2+3^2+\cdots+(n-1)^2+n^2}_{\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}} + (n+1)^2$

$= \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{n(2n^2+3n+1)}{6} + \frac{6(n^2+2n+1)}{6}$

$= \frac{2n^3+3n^2+n+6n^2+12n+6}{6} = \frac{2n^3+9n^2+13n+6}{6}$

(Scratch

$$\begin{array}{r|rrrr} -1 & 2 & 9 & 13 & 6 \\ & & -2 & -7 & -6 \\ \hline & 2 & 7 & 6 & 0 \\ & n^2 & n & c & r \end{array}$$

) $= \frac{(n+1)(n+2)(2n+3)}{6}$

$= \frac{(n+1)(2n^2+7n+6)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$, i.e., $P(n+1)$ holds!

$\therefore$ $P(n)$ holds $\forall n \in \mathbb{N}$, by Principle of Mathematical Induction.

Victoria's Secret Research Confirms...

Use right-endpoint definition of the definite integral to evaluate

$$\int_1^3 (2x^2 - x)\,dx$$

$x_k = a + k\left(\frac{b-a}{n}\right) = a + k\Delta x$

$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$

$x_k = 1 + k\left(\frac{2}{n}\right) = 1 + \left(\frac{2}{n}\right)k = 1 + \frac{2k}{n}$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\left(\Delta x \sum_{k=1}^n f(x_k)\right)$$

So, $\frac{2}{n}\sum_{k=1}^n (2x_k^2 - x_k) = \frac{2}{n}\left(\sum_{k=1}^n 2x_k^2 - \sum_{k=1}^n x_k\right) = \frac{2}{n}(A - B)$

Now,

$\frac{2}{n}A = \frac{2}{n}\sum_{k=1}^n 2x_k^2 = \frac{4}{n}\sum_{k=1}^n x_k^2 = \frac{4}{n}\sum_{k=1}^n \left(1 + \frac{2k}{n}\right)^2$ $\qquad \left(\frac{2k}{n}\right)^2 = \frac{4k^2}{n^2}$

$= \frac{4}{n}\sum_{k=1}^n \left(1 + \frac{4k}{n} + \frac{4k^2}{n^2}\right) = \frac{4}{n}\sum_{k=1}^n 1 + \frac{4}{n}\sum_{k=1}^n \frac{4k}{n} + \frac{4}{n}\sum_{k=1}^n \frac{4k^2}{n^2}$

$= \frac{4}{n}\cdot n + \frac{16}{n^2}\sum_{k=1}^n k + \frac{16}{n^3}\sum_{k=1}^n k^2$

$= 4 + \frac{16}{n^2}\left(\frac{n^2 + \text{~~}}{2}\right) + \frac{16}{n^3}\left(\frac{n^3 + \text{~~}}{3}\right)$ $\qquad \left(1 + \frac{2k}{n}\right)^2 = 1^2 + 2(1)\left(\frac{2k}{n}\right) + \left(\frac{2k}{n}\right)^2$

$\xrightarrow{n\to\infty} 4 + \frac{16}{2} + \frac{16}{3}$ $\qquad \sum_{k=1}^n k = \frac{n(n+1)}{2} = \frac{n^2+n}{2} = \frac{n^2 + \text{~~}}{2}$

"~~" means "lower-degree terms"

$= \frac{24 + 48 + 32}{6} = \frac{104}{6} = \boxed{\frac{52}{3} = \lim_{n\to\infty}\frac{2}{n}A}$

$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3 + \text{~~}}{6} = \frac{n^3 + \text{~~}}{3}$

I'll leave the $-\frac{2}{n}B$ part to you, but to check your final answer, use FTC II

$$\int_1^3 (2x^2 - x)\,dx = \left(\frac{2x^3}{3} - \frac{x^2}{2}\right)\Big|_1^3 = \frac{54}{3} - \frac{9}{2} - \left(\frac{2}{3} - \frac{1}{2}\right) =$$

$$= \frac{52}{3} - 4 = \frac{52}{3} - \frac{12}{3} = \boxed{\frac{40}{3}}$$

Find the derivative of the function. 2.5

$$G(y) = \left(\frac{y^2}{y+4}\right)^5$$

Don't be thrown by the fact that the independent variable is y. This is not a chain-rule question. It's a straight-up "Differentiate w.r.t y" question.

$G(x) = \left(\frac{x^2}{x+4}\right)^5$ Find $G'(x)$

Find the linearization $L_a(x)$ for $f(x) = \sin(x)$ @ $a = \frac{\pi}{4}$ §2.9

$f'(x) = \cos(x)$

$m = f'\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$y = m(x - x_1) + y_1$ Need $y_1 = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$$\boxed{L_{\frac{\pi}{4}}(x) = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) + \frac{\sqrt{2}}{2}}$$

$y =$

Compute Δy & dy for the given values of x & $dx = \Delta x$.

$3x^2 + 6x$, $x = 2$, $\Delta x = .5$

a) $x = 2$, $\Delta x = dx = .5$, we have

$dy = f'(x)\,dx = (6x+6)\,dx$

$dy = (18)(.5) = 9$

$\Delta y = f(x+\Delta x) - f(x) = f(2.5) - f(2)$

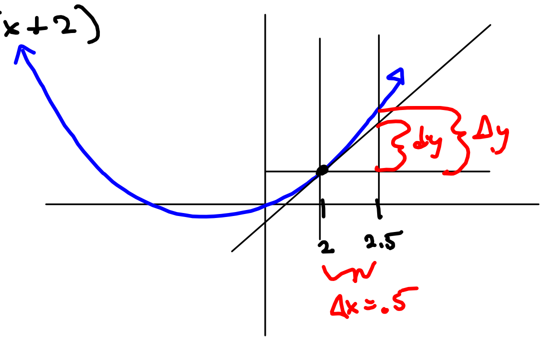
$= 3\left(\frac{5}{2}\right)^2 + (6)\left(\frac{5}{2}\right) - \left(3(2)^2 + 6(2)\right)$

$= \frac{3(25)}{4} + \frac{30}{2} \cdot \frac{2}{2} - (12 + 12)$

$= \frac{75 + 60 - 4(24)}{4} = \frac{135 - 96}{4} = \boxed{\frac{49}{4} = \Delta y}$

Draw a graph showing line segments w/ lengths dx, dy & Δy

$3x^2 + 6x = 3x(x+2)$

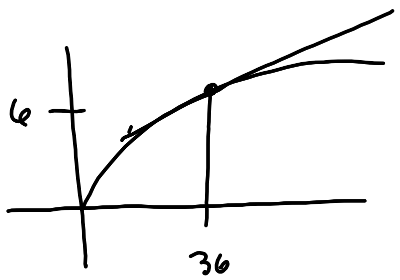


Use a linear approximation (or differentials) to estimate the given number. Round your answer to 5 decimal places. $\sqrt{35}$

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$

$x_1 = 36$

$\Delta x = dx = -1$



$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f'(36) = \frac{1}{2\sqrt{36}} = \frac{1}{12}$

$f(36) = 6 = y_1$

$L_{36}(x) = m(x - x_1) + y_1$

$= \frac{1}{12}(x-36) + 6$

we want $L_{36}(35) = \frac{1}{12}(35-36) + 6$

$= \frac{1}{12}(-1) + 6$

$= 6 - \frac{1}{12} = \frac{71}{12} \approx \sqrt{35}$

$\frac{71}{12} = 5.91\overline{6} \approx \boxed{5.91667}$

Differential Version

$f(x+\Delta x) - f(x) = \Delta y$

$f(x+\Delta x) = f(x) + \Delta y \approx f(x) + dy$

$= \sqrt{36} + f'(36)\,dx$

$= 6 + \left(\frac{1}{2\sqrt{36}}\right)(-1)$

$= 6 - \frac{1}{12}$

0/3 points SCalc9 3.2.019. [4709269]

Find the number c that satisfies the conclusion of the Mean Value Theorem on the given interval. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

Section 3.2

$f(x) = \sqrt{x}, \quad [0, 9]$

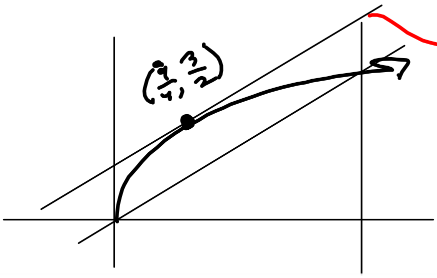
$$m_{sec} = \frac{f(9) - f(0)}{9 - 0} = \frac{\sqrt{9} - \sqrt{0}}{9 - 0} = \frac{3}{9} = \frac{1}{3}$$

$$f'(x) = \frac{1}{2\sqrt{x}} \overset{SET}{=} \frac{1}{3} \Longrightarrow 2\sqrt{x} = 3$$

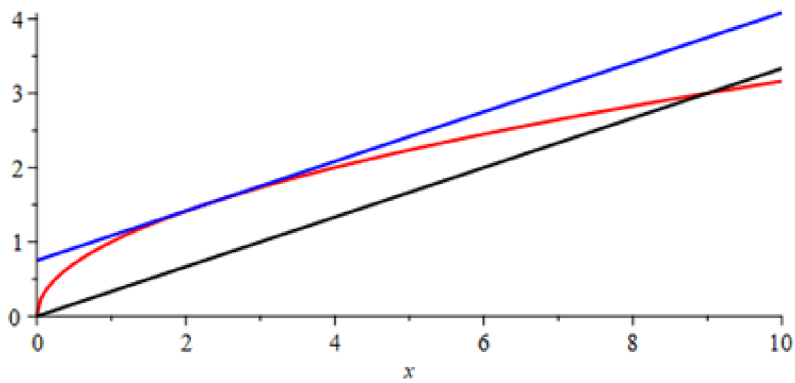
$$\sqrt{x} = \frac{3}{2}$$

$$x = \boxed{\frac{9}{4} = c}$$

Graph the function, the secant line through the endpoints, and the tangent line at $(c, f(c))$.



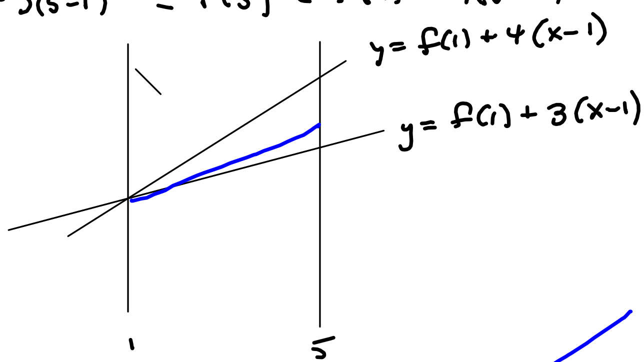
$\Rightarrow y = \frac{1}{3}\left(x - \frac{9}{4}\right) + \frac{3}{2}$

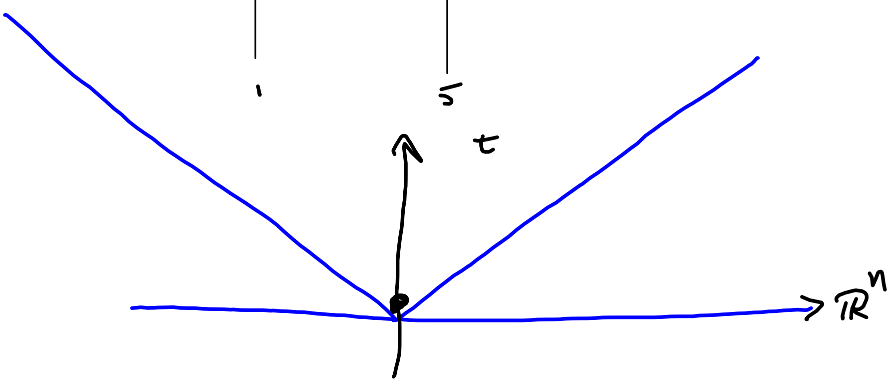


Suppose that $3 \le f'(x) \le 4$ for all values of x. What are the minimum and maximum possible values of $f(5) - f(1)$?

$3 \le f'(x) \le 4 \longrightarrow$

$f(1) + 3(5-1) \le f(5) \le f(1) + 4(5-1)$





Sketch the curve

$$f(x) = \frac{x}{\sqrt{x^2-9}} = \frac{x}{(x^2-9)^{1/2}} \quad \frac{\text{stuff}}{0} \text{ Bad}$$

$\sqrt{\text{money}}$

$$\mathcal{D} = \{ x \mid x^2-9 \geq 0 \ \& \ \sqrt{x^2-9} \neq 0 \}$$

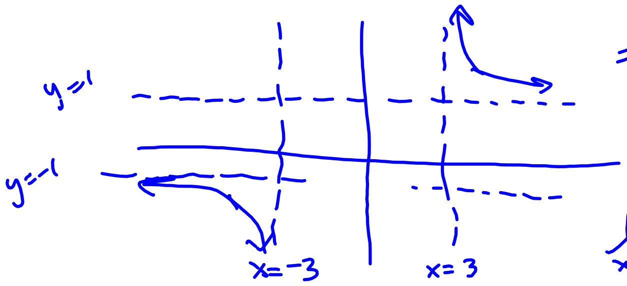
$$= \{ x \mid x^2-9 > 0 \} \quad \text{Need } x^2-9 = (x+3)(x-3) > 0$$

$+$ $=0$ $-$ $=0$ $+$

-3 3 x

Y N N N

$$\mathcal{D} = (-\infty, -3) \cup (3, \infty)$$



$$\lim_{x \to \infty} \frac{x}{\sqrt{x^2-9}} = \lim_{x \to \infty} \frac{x}{|x|\sqrt{1-9/x^2}} = \lim_{x \to \infty} \frac{x}{x\sqrt{1-9/x^2}} = \lim_{x \to \infty} \frac{1}{\sqrt{1-\frac{9}{x^2}}} = 1$$

$$\lim_{x \to -\infty} \frac{x}{|x|\sqrt{1-9/x^2}} = \lim_{x \to -\infty} \frac{x}{-x\sqrt{1-9/x^2}} = -1$$

$$f(x) = \frac{x}{\sqrt{x^2-9}}$$

$$f'(x) = \frac{1\left(\sqrt{x^2-9}\right) - x\left(\frac{1}{2}(x^2-9)^{-\frac{1}{2}}(2x)\right)}{\sqrt{x^2-9}^{\,2}}$$

$$= \frac{\sqrt{x^2-9} - \frac{x^2}{\sqrt{x^2-9}}}{x^2-9} = \frac{\frac{x^2-9-x^2}{\sqrt{x^2-9}}}{x^2-9} =$$

$$= \frac{-9}{(x^2-9)^{3/2}} < 0 \text{ on its domain!}$$

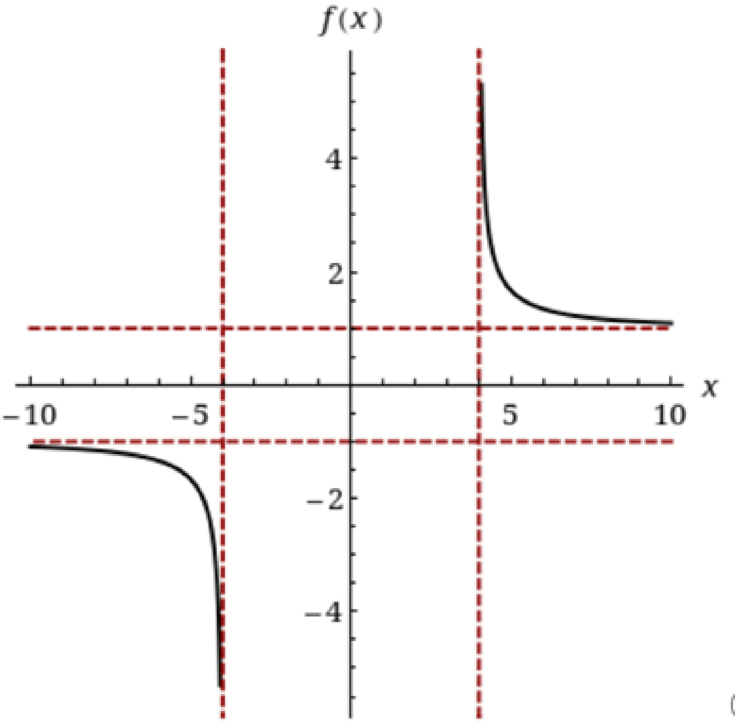
$$(x^2-9)^{\frac{1}{2}}(x^2-9) = (x^2-9)^{\frac{3}{2}} = \sqrt{(x^2-9)^3}$$

$$= -9(x^2-9)^{-3/2} \longrightarrow$$

$$f''(x) = \frac{27}{2}(x^2-9)^{-5/2}(2x) = \frac{27(2)x}{(x^2-9)^{5/2}}$$

$f'' > 0 \quad x > 0$

$f'' < 0 \quad x < 0$


f(x)
4
2
−2
−4
x
−10
−5
5
10

Find the point on the line $y = 7x - 2$ that's closest to the origin.

§13.7