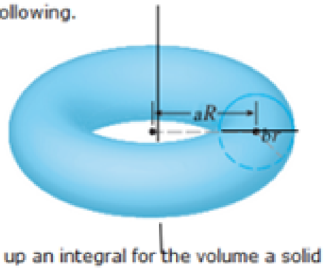
§5.2 #16

16. Question Details SCalc8 5.2.063. [3353997]

Consider the following.



Eq'n of circle:

$(x-aR)^2+(y^2)=(br)^2$

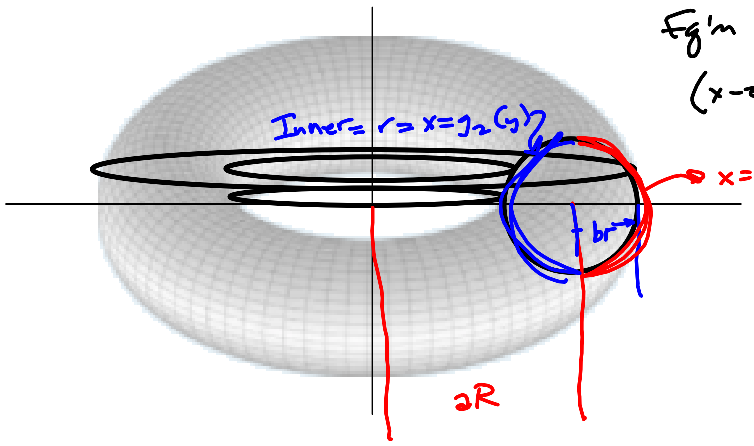
$y^2=b^2r^2-(x-aR)^2$

$y=\pm\sqrt{\sim\sim}$

we take the top half & Double it

(a) Set up an integral for the volume a solid *torus* (the donut-shaped solid shown in the figure) with radii *br* and *aR*. (Let *a* = 4 and *b* = 3.)

(b) By interpreting the integral as an area, find the volume *V* of the torus.



Eq'n of circle

$(x-aR)^2+y^2=(br)^2$

$(x-aR)^2+y^2=(br)^2$ Solve for x:

$(x-aR)^2=b^2r^2-y^2$

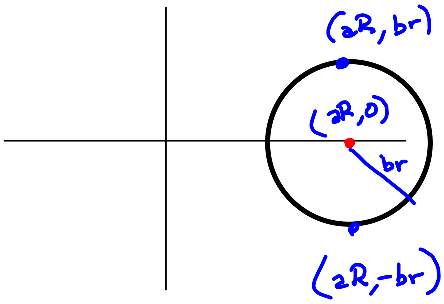
$x-aR=\pm\sqrt{b^2r^2-y^2}$

$x=aR\pm\sqrt{b^2r^2-y^2}$

outer: $aR+\sqrt{b^2r^2-y^2}$

inner: $aR-\sqrt{b^2r^2-y^2}$

Need limits of integration



$$\pi\int_{-br}^{br}(\text{outer}^2-\text{inner}^2)$$

$$=\pi\int_{-br}^{br}\left(aR+\sqrt{b^2r^2-y^2}\right)^2-\left(aR-\sqrt{b^2r^2-y^2}\right)^2$$

$$A^2 - B^2 = (A-B)(A+B)$$

$$= \pi \int_{-br}^{br} \left(2R + \sqrt{b^2r^2-y^2} - \left(2R - \sqrt{b^2r^2-y^2}\right)\right)\left(2R + \sqrt{b^2r^2-y^2} + 2R - \sqrt{b^2r^2-y^2}\right)$$

$$= \pi \int_{-br}^{br} \left(2\sqrt{b^2r^2-y^2}\right)(2 \cdot 2R)\,dy = \pi \int_{-br}^{br} 4 \cdot 2R\sqrt{b^2r^2-y^2}\,dy$$

$$= 2\pi \int_0^{br} 4 \cdot 2R\sqrt{b^2r^2-y^2}\,dy$$

$$= 8\pi \cdot 2R \int_0^{br} \sqrt{b^2r^2-y^2}\,dy$$ is correct formulation

My notes on harryzaims.com are trash.

I'm putting grades on WebAssign. The only thing it can't do is run scenarios with and without attendance.

Written assignments category scores aren't entered, yet. But they will.

Take-Home Test 3/Writing Project Chapter 3 will be its own category, because of the re-take.

Questions

C4, §5.1, 5.2, §6.1, 6.2 ?

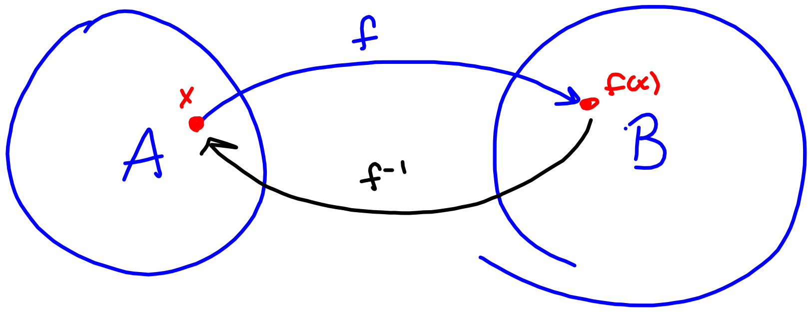
Chapter 6 Good $\int \tan(x)dx = \int \frac{\sin(x)}{\cos(x)}dx = -\int \frac{-\sin(x)\,dx}{\cos(x)}$

So, $u = \cos(x) \Rightarrow du = -\sin(x)dx$

$$= -\int \frac{du}{u} = -1\ln|u| + C = \ln(|u|^{-1}) + C$$

$$= \ln(|\cos(x)|^{-1}) + C = \ln\left|\frac{1}{\cos(x)}\right| + C = \ln|\sec(x)| + C$$

More on inverses

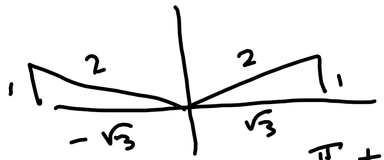


$\sin(x) = \frac{1}{2}$

$\sin^{-1}(\sin(x)) = \sin^{-1}(\frac{1}{2})$ Not strictly true, but useful

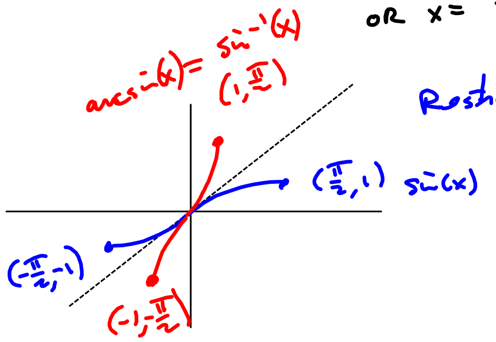
$\cancel{\Rightarrow}$ $x = \sin^{-1}(\frac{1}{2})$, provided x is in the restricted domain.

$\sin(x) = \frac{1}{2}$



$x = \frac{\pi}{6} + 2n\pi,\ n \in \mathbb{Z}$

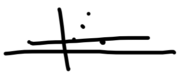
OR $x = \frac{5\pi}{6} + 2n\pi,\ n \in \mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$



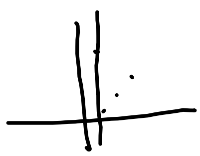
Restricted sine.

That's why $\sin^{-1}(x)$ only has range of $[-\frac{\pi}{2}, \frac{\pi}{2}]$. We restrict the domain of sine to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ to keep it 1-to-1.

$f = \{(1,2), (3,4), (5,6), (7,2)\}$ Why?



$f^{-1} = \{(2,1), (4,3), (6,5), (2,7)\}$ Flunks vertical line test



A function is a rule that assigns each x in some domain (set) to a y in some other set (domain, range)

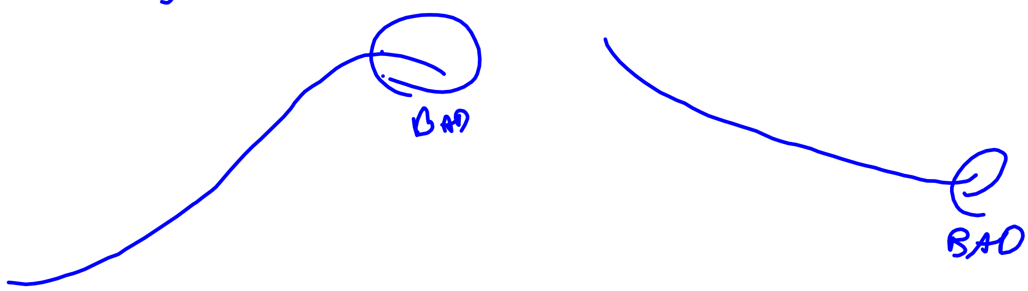
A 1-to-1 function is a function that assigns to each x a unique y.

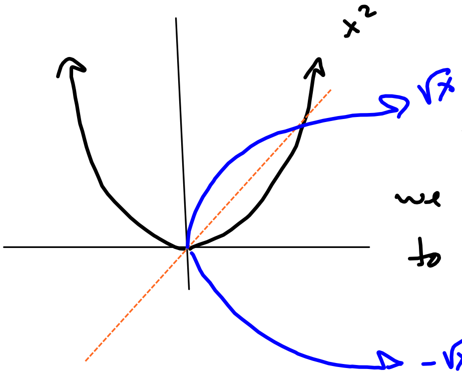
1-to-1 means

$f(x_1) = f(x_2)$, then $x_1 = x_2$

$x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$

For continuous functions, this means increasing everywhere (or decreasing everywhere)

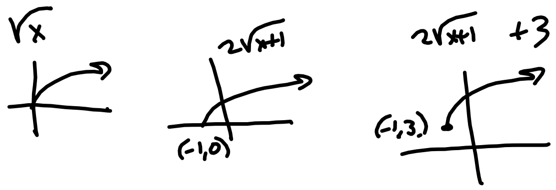




$f(x) = x^2 \longrightarrow f^{-1}(x) = \sqrt{x}$, but we need to restrict the domain to $[0, \infty)$

$x^2 = 2$ has 2 sol'ns

Let $f(x) = 3 + 2\sqrt{x+1}$. Find f^{-1}. Give its $\mathcal{D}$ & $\mathcal{R}$.



$\mathcal{D}(f) = [-1, \infty) = \mathcal{R}(f^{-1})$

$\mathcal{R}(f) = [3, \infty) = \mathcal{D}(f^{-1})$

$y = 3 + 2\sqrt{x+1}$

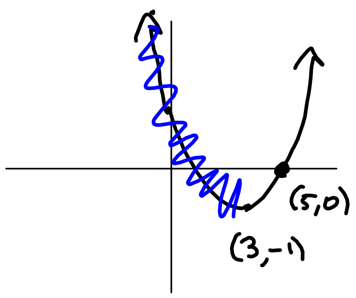
$x = 3 + 2\sqrt{y+1} = x$

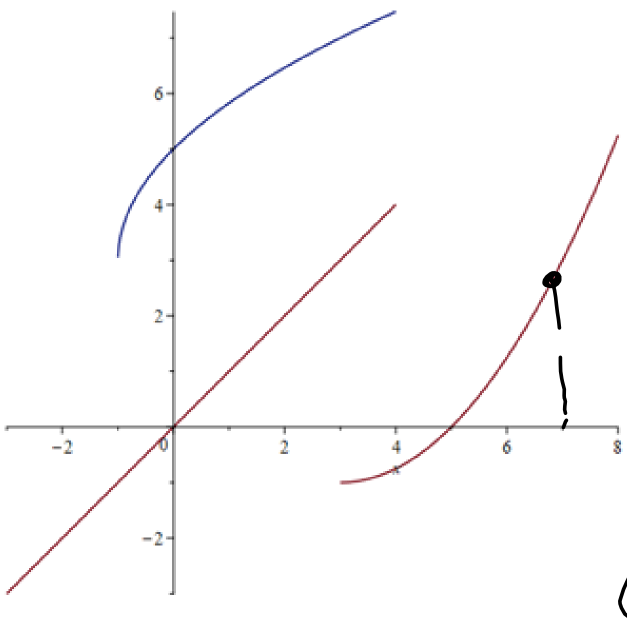
$2\sqrt{y+1} = -3 + x$

$\sqrt{y+1} = \frac{1}{2}(-3+x)$

$y + 1 = \frac{1}{4}(x-3)^2$

$y = \frac{1}{4}(x-3)^2 - 1$





$f'(x) = 2\left(\frac{1}{2}\right)(x+1)^{-\frac{1}{2}} = \frac{1}{\sqrt{x+1}}$

$f(3) = 3 + 2\sqrt{3+1}$

$= 3 + 2\sqrt{4} = 7$

$(3, f(3)) = (3, 7)$

what's $(f^{-1})'(7)$?

$f^{-1}(x) = \frac{1}{4}(x-3)^2 - 1$

$(f^{-1})'(x) = \frac{1}{2}(x-3)$

$(f^{-1})'(7) = \frac{1}{2}(7-3) = \boxed{2 = (f^{-1})'(7)}$

Now, the BOOK says

$$(f^{-1})'(7) = \frac{1}{f'(f^{-1}(7))} = \frac{1}{\frac{1}{\sqrt{3+1}}} = \frac{1}{\frac{1}{2}} = 2!$$

Confirms this formula.

Often, $f^{-1}(7)$ is easier to do/find than $f^{-1}(x)$, in general.

$f(x) = e^x$, $f^{-1}(x) = \ln(x)$

derivatives & antiderivatives & applications of those for the rest of the semester.

Big App:

Logarithmic Differentiation.

We can't handle $\frac{d}{dx}\left[f(x)^{g(x)} \right]$ is un-possible

$y = \sin(x)^{\cos(x)}$ Find y'

$\ln(y) = \ln\left(\sin(x)^{\cos(x)}\right) = \cos(x) \ln(\sin(x))$, which you can differentiate.