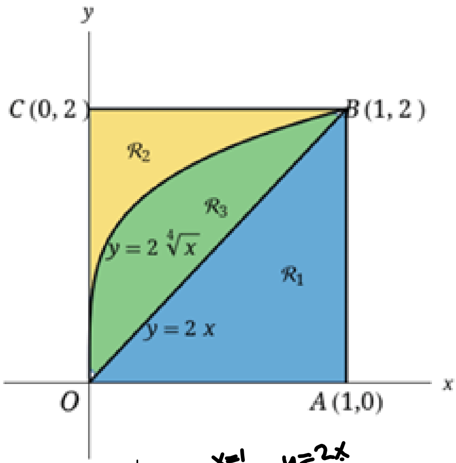
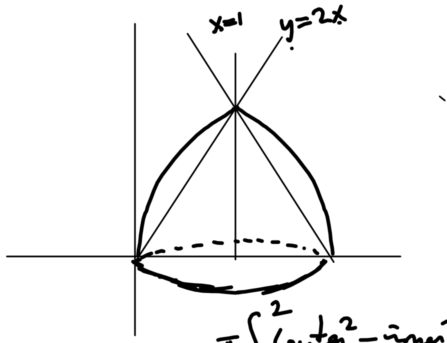
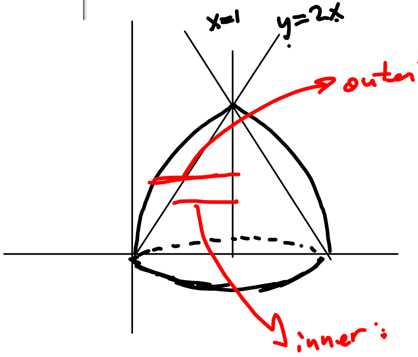
5.2 #11





$\pi\int_0^2 (\text{outer}^2 - \text{inner}^2)\, dy$



outer: $1-\frac{1}{16}y^4$

$y = 2\sqrt[4]{x} = y$

$\sqrt[4]{x} = \frac{1}{2}y$

$x = \left(\frac{1}{2}y\right)^4 = \frac{1}{16}y^4$

$= \pi\int_0^2 \left(\left(1-\frac{1}{16}y^4\right)^2 - \left(1-\frac{1}{2}y\right)^2\right) dy$

$= \frac{34\pi}{45}$

$y = 2x = y$

$x = \frac{1}{2}y$

inner: $= 1 - \frac{1}{2}y$

Jocelyn's Version $y = 4\sqrt[4]{x}$ & $y = 4x$ about $x = 1$ from $y=0$ to $y=4$

$x = \frac{1}{4}y$

$= \pi\int_0^4 \left(\left(1-\frac{1}{64}y^4\right)^2 - \left(1-\frac{1}{4}y\right)^2\right) dy = \frac{152\pi}{45}$

Looks like I rotated about the y-axis in my notes on harryzaims.com! Rotating about $x=1$ is what we want.

$(y^4)^2 = y^8$

$1 - \frac{1}{2}y + \frac{1}{16}y^2$

$\left(\frac{1}{4}y\right)^4 = \frac{1}{64}y^4$

$= \pi \int_0^4 \left(1^2 - \frac{1}{32}y^4 + \frac{1}{4096}y^8 - 1 + \frac{1}{2}y - \frac{1}{16}y^2\right) dy$

$64^4 = 4096$

$(4^4)^4 = 4^{16}$

$= \pi \left[-\frac{1}{32 \cdot 5} y^5 + \frac{1}{9 \cdot 4096} y^9 + \frac{1}{4}y^2 - \frac{1}{48}y^3 \right]_0^4$

$= \pi \left[-\frac{1}{32 \cdot 5}(2^{10}) + \frac{1}{9 \cdot 4096}(2^{18}) + \frac{1}{4}(4^2) - \frac{1}{3 \cdot 16}(4^3) \right]$

$= \pi \left[-\frac{32}{5} + \frac{1}{9 \cdot 2^{24}}(2^{18}) + 4 - \frac{4}{3} \right]$

$= \pi \left[-\frac{32}{5} + \frac{1}{9 \cdot 64} + 4 - \frac{4}{3} \right]$

$4096 = 64^4 = (8^2)^4 = (2^3)^8 = 2^{24}$

$= \pi \left[-\frac{32}{5} \cdot \frac{9 \cdot 64}{9 \cdot 64} + \frac{1}{9 \cdot 64} \cdot \frac{5}{5} + \frac{4}{1} \cdot \frac{9 \cdot 64 \cdot 5}{9 \cdot 64 \cdot 5} - \frac{4}{3} \cdot \frac{5 \cdot 3 \cdot 64}{5 \cdot 3 \cdot 64} \right]$

$= \pi \left[\frac{-18432 + 5 + 11520 - 3840}{5 \cdot 9 \cdot 64} \right]$

$$\begin{array}{r} 18432 \\ 3840 \\ \hline 22272 \\ -11525 \\ \hline 10747 \end{array}$$

Nope!

$= \pi \left[\frac{-22272 + 11525}{5 \cdot 9 \cdot 64} \right] =$

Got in too much of a hurry with my arithmetic.

Sol'n for Jocelyn's version: $\frac{152\pi}{45}$ which I'm not gonna get on my 1st attempt!

My notes are bad.

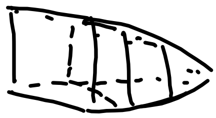
$$\left(1-\tfrac{1}{64}y\right)^2 = \left(\tfrac{1}{64}y^4-1\right)^2 = \left(\tfrac{1}{64}y^4\right)^2 - 2\left(\tfrac{1}{64}y^4\right) + 1$$

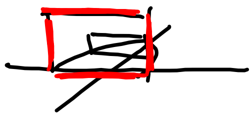
15. 0/1 points SCalc9 5.2.07

Find the volume V of the described solid S.

The base of a solid S is the region enclosed by the parabola $y = 4 - x^2$ and the x-axis. Cross-sections perpendicular to the y-axis are squares.

Looking Down:





$$y = 4-x^2$$
$$x^2 = 4-y$$
$$x = \pm\sqrt{4-y}$$

width will be right − left $= \sqrt{4-y} - \left(-\sqrt{4-y}\right)$

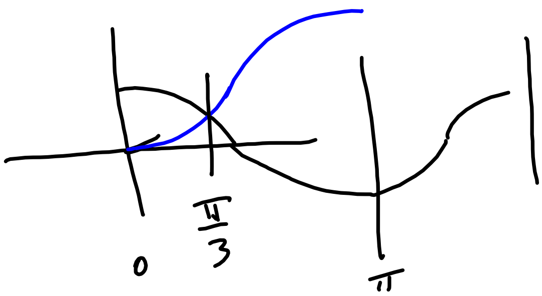
$= 2\sqrt{4-y}$ = width = height

$V = \left(2\sqrt{4-y}\right)\left(2\sqrt{4-y}\right)dy$ for representative square

Total $V = \int_0^4 4(4-y)\,dy = 4\left[4y - \tfrac{1}{2}y^2\right]_0^4 = 4\left[4(4) - \tfrac{1}{2}(16)\right]$

$= 4\left[16-8\right] = 4\left[8\right] = 32$

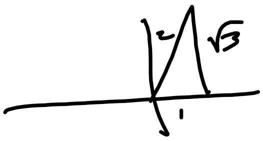
Area between $2\cos x$ & $2-2\cos(x)$ on $[0,\pi]$



$2\cos(x) = 2-2\cos(x)$

$4\cos(x) = 2$

$\cos(x) = \frac{1}{2}$



$$\int_0^{\pi} \left| 4\cos(x) - 2 \right| dx$$

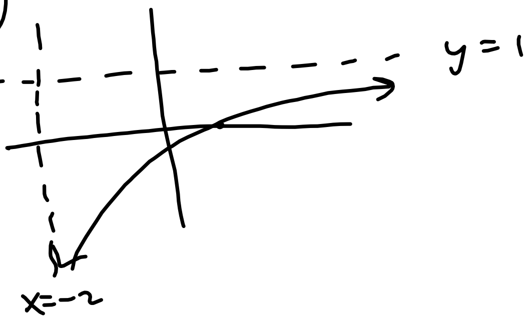
$$= \int_0^{\frac{\pi}{3}} \left(4\cos(x) - 2\right) dx - \int_{\frac{\pi}{3}}^{\pi} \left(4\cos(x) - 2\right) dx$$

§6.1

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$ = Derivative of the inverse shortcut

Find the inverse:

$$f(x) = y = \frac{x-1}{x+2} = x - \frac{3}{x+2}$$



$$\begin{array}{r|rr} -2 & 1 & -1 \\ & & -2 \\ \hline & 1 & -3 \end{array}$$

Swap x & y, solve for y:

$$\frac{y-1}{y+2} = x$$

$$y-1 = x(y+2) = xy + 2x$$

$$y - xy = 2x + 1$$

$$y(1-x) = 2x+1$$

$$y = \boxed{f^{-1}(x) = \frac{2x+1}{1-x}}$$