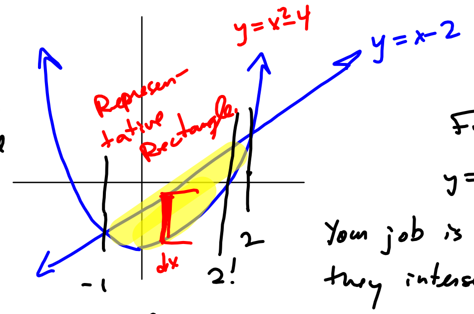
Bad picture! Take more care & time



Find the area bdd by $y=x^2-4$ & $y=x-2$

Your job is to find where they intersect

$x^2-4=x-2 \Rightarrow$

$x^2-x-2=(x-2)(x+1)=0 \Rightarrow x\in\{-1,2\}$

$$\int_a^b |f(x)-g(x)|\,dx = \int_{-1}^{2}(\text{upper}-\text{lower})\,dx = \int_{-1}^{2}\left(x-2-(x^2-4)\right)dx$$

$$= \int_{-1}^{2}(x-2-x^2+4)\,dx = \int_{-1}^{2}(-x^2+x+2)\,dx$$

$$= \left[-\frac{x^3}{3}+\frac{x^2}{2}+2x\right]_{-1}^{2} = -\frac{(2)^3}{3}+\frac{2^2}{2}+2(2)$$

$$-\left[-\frac{(-1)^3}{3}+\frac{(-1)^2}{2}+2(-1)\right]$$

$$= -\frac{8}{3}+2+4-\left[-\frac{-1}{3}+\frac{1}{2}-2\right] = -\frac{8}{3}+\frac{1}{3}+6-\frac{1}{2}+2 \qquad LCD=2\cdot 3$$

$$= -\frac{7}{3}+8-\frac{1}{2} = -\frac{7}{3}\cdot\frac{2}{2}+\boxed{\frac{8}{1}\cdot\frac{6}{6}-\frac{1}{2}\cdot\frac{3}{3}} = \cancel{\frac{-14+48-3}{6}}$$

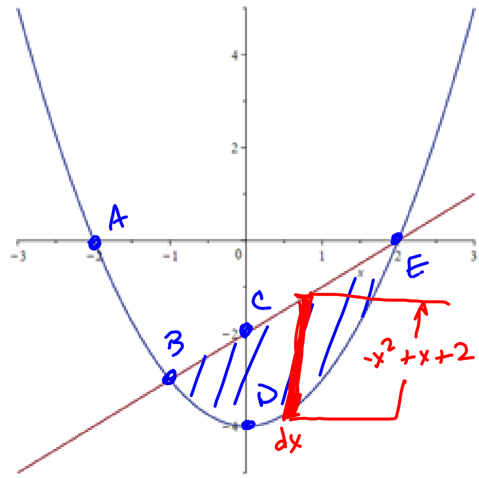
$\frac{9}{3}$

$= \frac{31}{6}$

$\frac{9}{2}$, idiot!

$$-\frac{3}{1}\cdot\frac{2}{2}+\frac{8}{1}\cdot\frac{2}{2}-\frac{1}{2}$$

$$= \frac{-6+16-1}{2} = \frac{9}{2}$$ ✓



Better!

$A=(-2,0)$

$B=(-1,-3)$

$C=(0,-2)$

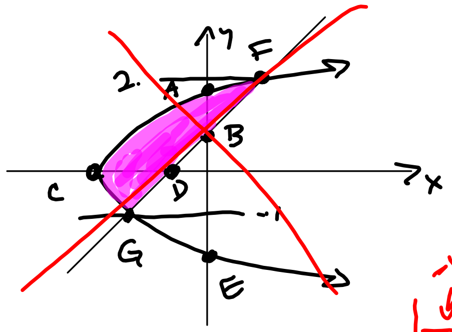
$D=(0,-4)$

$E=(2,0)$

Bad picture didn't hurt the calculation, b/c: upper & lower functions correct & their intersections were correct (actually all we needed was their x-coordinates).

Same thing for $x = f(y)$

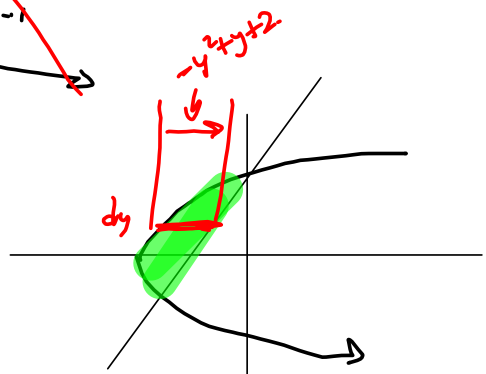
$x = y^2 - 4, \quad x = y - 2$



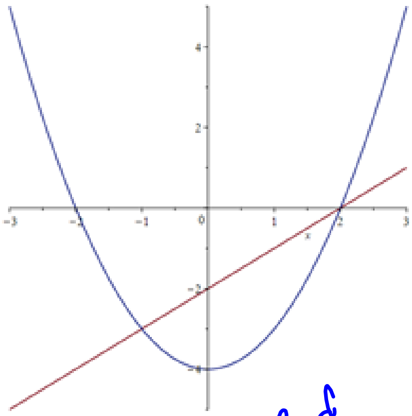
$$\int_{-1}^{2} (\text{Right} - \text{left})\,dy = \frac{9}{2}$$

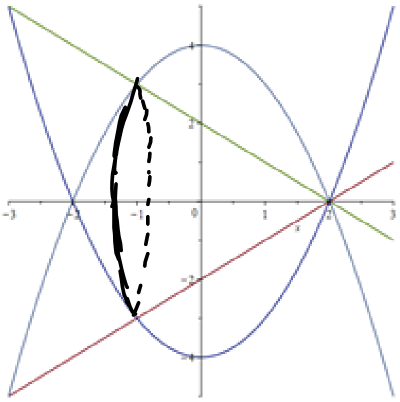
A = (0,

B =



§5.2 – Take the region in 1st example & rotate it about the x-axis

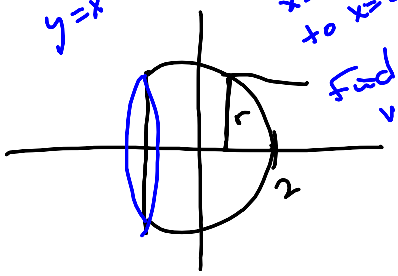




A bowl laying on its side!

$y = x^2-4$ Revolved from $x=-1$ to $x=2$.

Find volume!

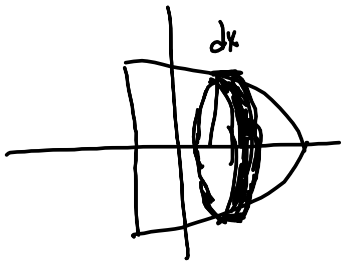


$x=-1$

radius $= r = x^2-4$

Revolving about x-axis

$\pi \int_a^b f(x)^2\,dx$



Volume of the disc is

$(\pi r^2)\,dx$

$= \pi f(x)^2\,dx$

$\int_{-1}^{2} \pi (x^2-4)^2\,dx$ = volume!

So for $g(x) = x-2$, we have

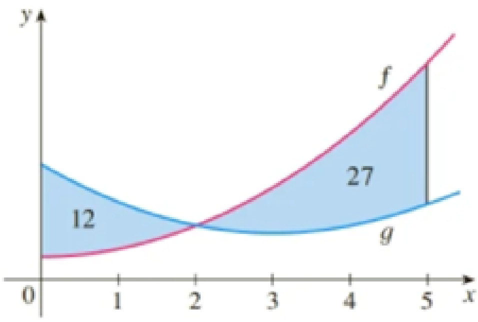
$$\pi \int_{-1}^{2} (x-2)^2\,dx$$

And for the volume of the solid obtained by rotating the region between $f(x)$ & $g(x)$, it's

$$\pi \int_a^b f(x)^2\,dx - \pi \int_a^b g(x)^2\,dx = \pi \int_a^b \left(f(x)^2 - g(x)^2\right) dx$$

$$= \pi \int_{-1}^{2} \left((x^2-4)^2 - (x-2)^2\right) dx$$

29. The graphs of two functions are shown with the areas of the regions between the curves indicated.
(a) What is the total area between the curves for $0 \leqslant x \leqslant 5$?
(b) What is the value of $\int_0^5 [f(x) - g(x)]\,dx$?



a) $\boxed{A = 39}$

b) $\int_0^5 (f(x) - g(x))\,dx$

$= \int_0^2 + \int_2^5$

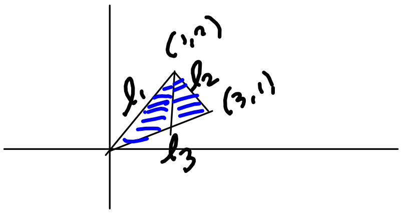
$= -12 + 27 = \boxed{15 = A}$

33–34 Use calculus to find the area of the triangle with the given vertices.

33. (0, 0), (3, 1), (1, 2)

34. (2, 0), (0, 2), (−1, 1)

$$\int_0^1 (l_1 - l_3) + \int_1^3 (l_2 - l_3)$$



l_1: (0,0), (1,2)

$y = \frac{2-0}{1-0}(x-0)+0 = 2x = l_1(x)$

$\boxed{y = m(x-x_1)+y}$

l_2: (1,2), (3,1)

$= \frac{1-2}{3-1}(x-1)+2$

$= -\frac{1}{2}(x-1)+2$

$= -\frac{1}{2}x + \frac{1}{2} + 2 = -\frac{1}{2}x + \frac{5}{2} = l_2(x)$

l_3: (0,0), (3,1)

$y = \frac{1}{3}x$

$\frac{1-0}{3-0}(x-0)+0$

$= \frac{1}{3}x$

$$\int_0^1 \left(2x - \frac{1}{3}x\right) dx$$

$$+ \int_1^3 \left(-\frac{1}{2}x + \frac{5}{2} - \frac{1}{3}x\right) dx$$

$$= \int_0^1 \frac{5}{3}x\, dx + \int_1^3 \left(-\frac{5}{6}x + \frac{5}{2}\right) dx$$

$$= \left[\frac{5}{3}\cdot\frac{x^2}{2}\right]_0^1 + \left[-\frac{5}{6}\cdot\frac{x^2}{2} + \frac{5}{2}x\right]_1^3$$

$$= \frac{5(1)^2}{6} - 0 + \frac{-5(3)^2}{12} + \frac{5}{2}(3) - \left(\frac{-5(1)^2}{12} + \frac{5}{2}(1)\right)$$

$$= \frac{5}{6} - \frac{45}{12} + \frac{15}{2} - \left(-\frac{5}{12} + \frac{5}{2}\right) =$$

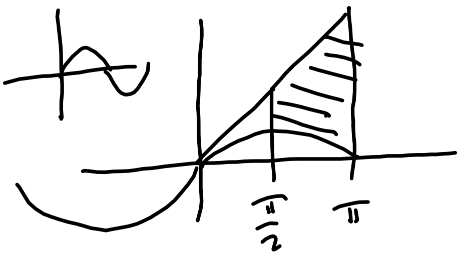
$$= \frac{5}{6} - \frac{45}{12} + \frac{15}{2} + \frac{5}{12} - \frac{5}{2} = \frac{5}{6} - \frac{40}{12} + 5$$

$$= \frac{5}{6} - \frac{20}{6} + \frac{30}{6} = \boxed{\frac{15}{6} = \text{Area}}$$

3. 0/2 points

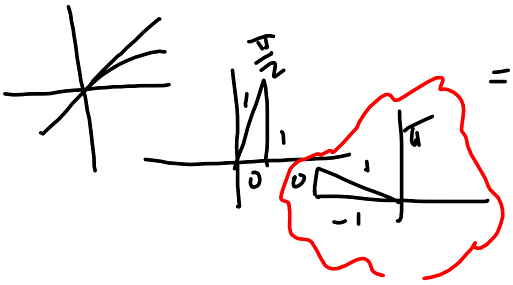
Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle.

$y = \sin(x)$, $y = x$, $x = \pi/2$, $x = \pi$



$$\int_{\frac{\pi}{2}}^{\pi} \text{upper} - \text{lower} =$$

$$\int_{\frac{\pi}{2}}^{\pi} (x - \sin(x))\,dx = \left[\frac{x^2}{2} - (-\cos(x))\right]_{\frac{\pi}{2}}^{\pi}$$



$$= \frac{\pi^2}{2} - \cos(\pi) - \left(\frac{\left(\frac{\pi}{2}\right)^2}{2} - \cos\left(\frac{\pi}{2}\right)\right)$$

$$= \frac{\pi^2}{2} - (-1) - \left(\frac{\pi^2}{8} - 0\right)$$

$$= \frac{\pi^2}{2}\cdot\frac{4}{4} + 1 - \frac{\pi^2}{8} = \boxed{\frac{3\pi^2}{8} + 1}$$

$+$ $+$ $+$ $-$ -1

Schueib kicks teacher's teacher's butt.