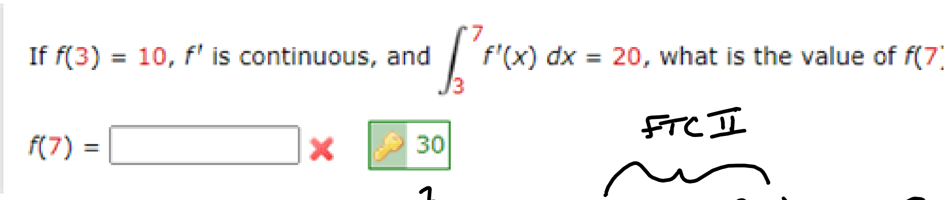


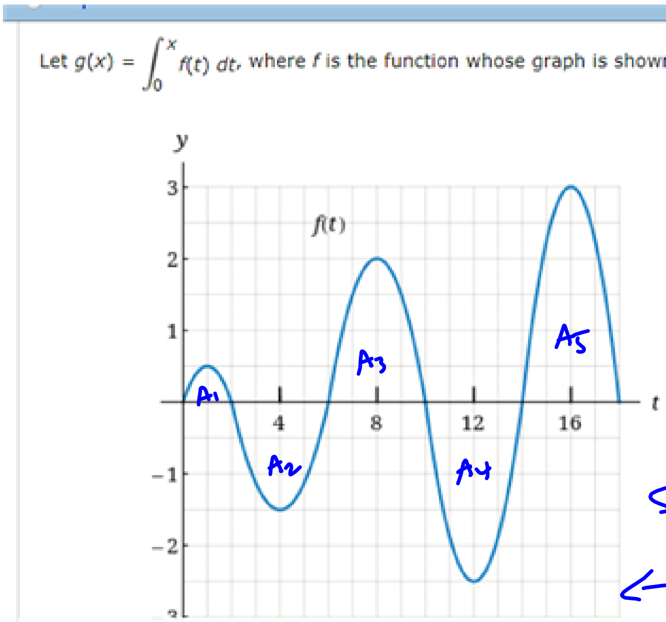
If $f(3) = 10$, f' is continuous, and $\int_3^7 f'(x)\,dx = 20$, what is the value of $f(7$

$f(7) =$ ☐ ✖ 30

$$\int_3^7 f'(x)dx = \overbrace{f(7) - f(3)}^{FTC\ II} = 20$$

$$\Longrightarrow f(7) - 10 = 20$$

$$\Longrightarrow f(7) = 30$$



Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

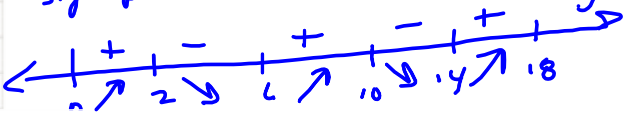
$$\int_0^{18} f(t)dt = \int_0^{18} f(x)dx$$

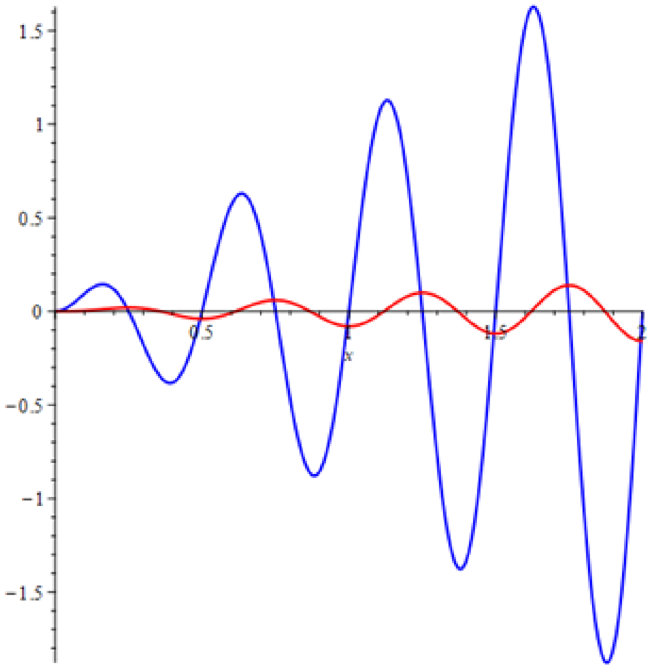
$$= A_1 - A_2 + A_3 - A_4 + A_5$$

$g(x)$ is increasing on $(0,2) \cup (6,10) \cup (14,18)$

$g(x)$ is decreasing $\forall\, x \in (2,6) \cup (10,14)$

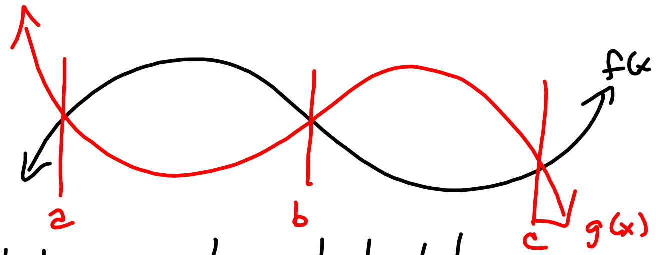
Sign pattern for g': (i.e. for f!)





$f(x) = x\sin(\pi x)$

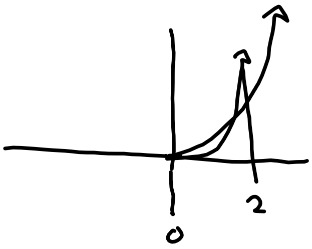
$g(x) = \int_0^x f(x)\,dx$



Find the area bounded between the 2 functions.

$$\text{Area} = \int_a^c |f(x)-g(x)|\,dx = \int_a^b (f(x)-g(x))\,dx + \int_b^c (g(x)-f(x))\,dx$$

Find the area bounded by $x=0,\ x=2,\ y=x^2, y=x^3$



$$\text{Area} = \int_0^2 |x^2-x^3|\,dy$$

$$= \int_0^1 (x^2-x^3)\,dx + \int_1^2 (x^3-x^2)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^3}{3}\right]_1^2$$

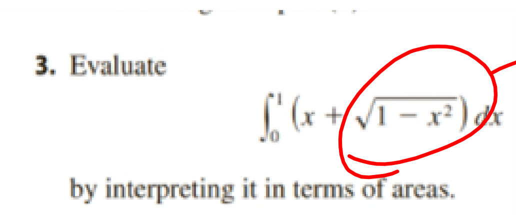
$$= \left[\frac{1}{3} - \frac{1}{4}\right] + \left[\left(\frac{16}{4} - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right)\right]$$

$$= \frac{4-3}{12} + \frac{48-32}{12} - \left(\frac{3-4}{12}\right)$$

$$= \frac{1}{12} + \frac{16}{12} + \frac{1}{12} = \frac{18}{12} = \frac{3}{2}$$

We WANT to back off at the end.

Let other classes accelerate down the stretch.



3. Evaluate

$$\int_0^1 \left(x + \sqrt{1-x^2}\right) dx$$

by interpreting it in terms of areas.

You have no idea!

$y = \sqrt{1-x^2}$ is top half of circle of radius $r = 1$

$x^2+y^2=1$

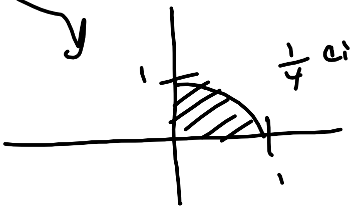
$y^2 = 1-x^2$

$y = \pm\sqrt{1-x^2}$

$= \int_0^1 x\,dx + \int_0^1 \sqrt{1-x^2}\,dy$

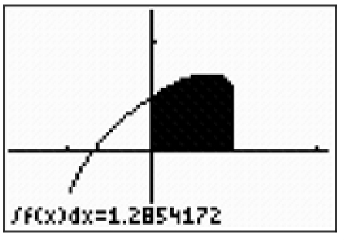
$= \left[\frac{x^2}{2}\right]_0^1 + \frac{\pi}{4}$

$= \boxed{\frac{\pi}{4} + \frac{1}{2}}$



$\frac{1}{4}$ circle!

$\frac{\pi r^2}{4} = \frac{\pi}{4}$!



```
∫f(x)dx=1.2854172
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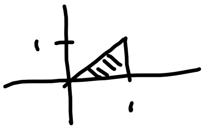
$\int_0^1 \left(x + \sqrt{1-x^2}\right) dx$

$= \int_0^1 x\,dx + \int_0^1 \sqrt{1-x^2}\,dx$

=  + 

$= \frac{1}{2}(1)(1) + \frac{1}{4}(\pi(1)^2)$

$= \frac{1}{2} + \frac{\pi}{4}$