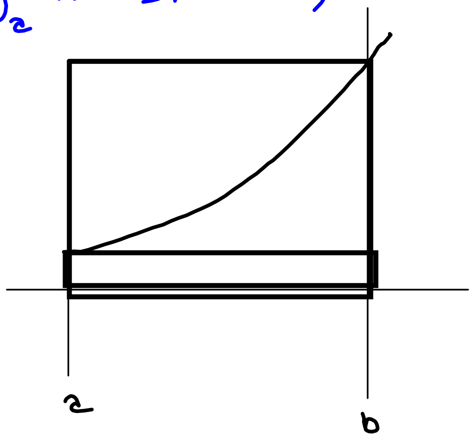
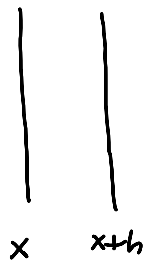
$$\frac{G(x+h)-G(x)}{h} = \frac{1}{h}\int_x^{x+h} f(t)\,dt \text{, so } m\,h \le \int_x^{x+h} f(t)\,dt \le M\,h$$

$$\xrightarrow{h\to 0} f(x) \le \frac{\int_x^{x+h} f(t)\,dt}{h} \le f(x)$$

Theorem:

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a) \text{, where } m=\min_{[a,b]}\{f(x)\},\ M=\max_{[a,b]}\{f(x)\}$$





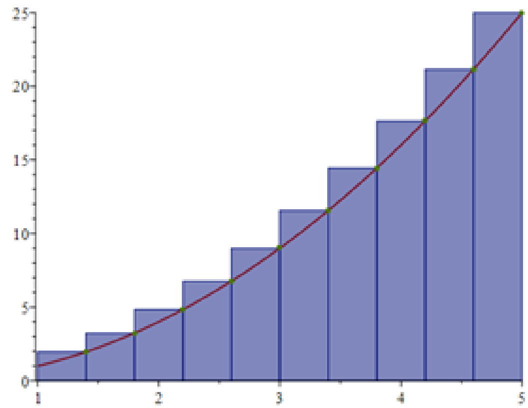
I think the video's better.

Find the area under $f(x)=x^2$ over $[a,b]=[1,5]$, by the definition.

$$\text{Area} = \lim_{n\to\infty} \frac{b-a}{n}\sum_{k=1}^{n} f\Big(\underbrace{a+k\left(\tfrac{b-a}{n}\right)}_{x_k^*}\Big)$$ Right-Endpoint

x_k^* taken from k^{th} interval.

$$\frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$$ = step size / increment



A right Riemann sum approximation of $\int_1^5 f(x)\,dx$, where $f(x)=x^2$ and the partition is uniform. The approximate value of the integral is 46.24000000. Number of subintervals used: 10.

$a=1$

$$a+k\left(\frac{b-a}{n}\right) = 1+\frac{4k}{n}$$

$$\frac{b-a}{n}\sum_{k=1}^{n} f(x_k) = \frac{4}{n}\sum_{k=1}^{n}\left(1+\frac{4k}{n}\right)^2$$

$$= \frac{4}{n}\sum_{k=1}^{n}\left(1+\frac{8k}{n}+\frac{16k^2}{n^2}\right)$$

$$= \frac{4}{n}\left[\sum_{k=1}^{n}1+\sum_{k=1}^{n}\frac{8k}{n}+\sum_{k=1}^{n}\frac{16k^2}{n^2}\right]$$

$$= \frac{4}{n}\left[n+\frac{8}{n}\sum_{k=1}^{n}k+\frac{16}{n^2}\sum_{k=1}^{n}k^2\right]$$

$$= \cancel{\frac{4}{n}\cdot n} + \frac{32}{n^2}\left[\frac{n^2+\cdots}{2}\right] + \frac{64}{n^3}\left[\frac{n^3+\cdots}{3}\right]$$

$$\xrightarrow{n\to\infty} 4+\frac{32}{2}+\frac{64}{3} = \frac{20}{1}\cdot\frac{3}{3}+\frac{64}{3} = \frac{124}{3}$$

$$\frac{n^3+\cdots}{n^3} \xrightarrow{n\to\infty} 1$$

Think "Horizontal Asymptote."

FTC II: If f is cnts and $G'(x) = f(x)$, then

$$\int_a^b f(x)dx = G(b) - G(a)$$

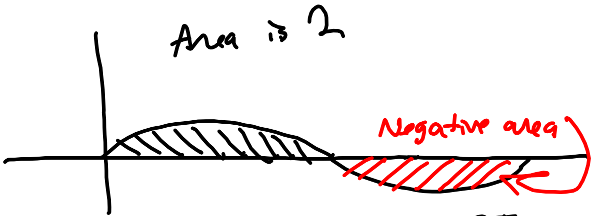
$$\int_1^5 x^2 dx = \left[\frac{x^3}{3}\right]_1^5 = \frac{5^3}{3} - \frac{1^3}{3} = \frac{125}{3} - \frac{1}{3} = \frac{124}{3}$$

Area under $\sin(x)$ from $x=0$ to $x=\pi$

$$\int_0^\pi \sin(x)\,dx = \left[-\cos(x)\right]_0^\pi = -\cos(\pi) - (-\cos(0))$$

$$= -(-1) - (-1) = 2$$

Area is 2



$$\int_0^{2\pi} \sin(x)\,dx = \left[-\cos(x)\right]_0^{2\pi} = -\cos(2\pi) - (-\cos(0))$$

$$= -1 - (-1) = 0$$

What's the area enclosed by sin(x) and the x-axis, between x = 0 and x = 2Pi?

$$\int_0^{2\pi} |\sin(x)|\,dx = \int_0^\pi \sin(x)dx + \int_\pi^{2\pi} -\sin(x)dx = 2 + -(-2) = 4$$

$$|\sin(x)| = \begin{cases} \sin(x) & \text{if } \sin(x) \geq 0 \\ -\sin(x) & \text{if } \quad '' \quad < 0 \end{cases}$$

$$= \begin{cases} \sin(x) & \text{if } 0 \leq x \leq \pi \\ -\sin(x) & \text{if } \pi < x < 2\pi \end{cases}$$

Chain Rule is your friend.

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

$$\int (x^5 + 3x^2)^{2/3} dx$$ No Clue!

$$\int (x^5+3x^2)^{2/3}(5x^4+6x)\,dx$$ is Do-Able!

Scratch Let $u = x^5+3x^2$. Then

$$du = \frac{du}{dx}dx = (5x^4+6x)\,dx$$

$$= \int u^{\frac{2}{3}} du = \frac{u^{\frac{5}{3}}}{\frac{5}{3}} + C = \frac{3}{5}(x^5+3x^2)^{\frac{5}{3}} + C$$

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \sin(\cos(x)) \cdot \sin(x)\,dx = -\int \sin(\cos(x))(-\sin(x))\,dx$$

$$= -\int \sin(u)\,du = -(-\cos(u) + C)$$

$$\int \sin(u)\,du = -\cos(u) + C$$

$u = \cos(x) \Longrightarrow$

$du = -\sin(x)\,dx$

$$= -(-\cos(\cos(x))) + C$$

$$= \cos(\cos(x)) + C$$

$$\int_0^4 \frac{x}{\sqrt{2x+1}}\,dx$$

$x=0 \Rightarrow u=1$
$x=4 \Rightarrow u=9$

I'm thinkin! "$u = 2x+1$" But I don't have "du"

$\Rightarrow 2x = u-1$ & $\quad$ we'll make $du = 2\,dx$

$x = \frac{u-1}{2}$ $\qquad dx = \frac{du}{2}$

$$\rightarrow \int \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\frac{du}{2} = \frac{1}{4}\int \frac{u-1}{u^{\frac{1}{2}}}\,du = \frac{1}{4}\int\left(\frac{u}{u^{\frac{1}{2}}} - \frac{1}{u^{\frac{1}{2}}}\right)du$$

$$= \frac{1}{4}\int\left(u^{1-\frac{1}{2}} - u^{-\frac{1}{2}}\right)du = \frac{1}{4}\int\left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du$$

$$= \frac{1}{4}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]_{u=0}^{u=9} + C$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right] + C = \frac{1}{4}\cdot\frac{2}{3}u^{\frac{3}{2}} - \frac{1}{4}\cdot 2\,u^{\frac{1}{2}} + C$$

$$= \frac{1}{6}u^{\frac{3}{2}} - \frac{1}{2}u^{\frac{1}{2}} + C$$

$$= \frac{1}{6}(2x+1)^{\frac{3}{2}} - \frac{1}{2}(2x+1)^{\frac{1}{2}} + C$$

What's the Next Level?

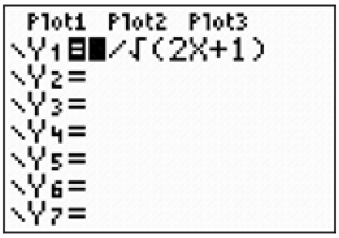
$$\int_0^4 \frac{x}{\sqrt{2x+1}}\,dx = \left[\frac{1}{6}(2x+1)^{\frac{3}{2}} - \frac{1}{2}(2x+1)^{\frac{1}{2}}\right]_{0=x}^{x=4} = \left[\frac{1}{6}u^{\frac{3}{2}} - \frac{1}{2}u^{\frac{1}{2}}\right]_{0=u}^{9=u}$$

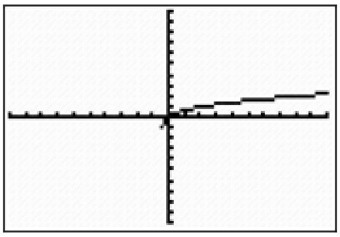
$$= \frac{1}{6}(2(4)+1)^{\frac{3}{2}} - \frac{1}{2}(2(4)+1)^{\frac{1}{2}} - \left[\frac{1}{6}(2(0)+1)^{\frac{3}{2}} - \frac{1}{2}(2(0)+1)^{\frac{1}{2}}\right]$$

$$= \frac{1}{6}(9)^{\frac{3}{2}} - \frac{1}{2}(9)^{\frac{1}{2}} - \left[\frac{1}{6} - \frac{1}{2}\right]$$

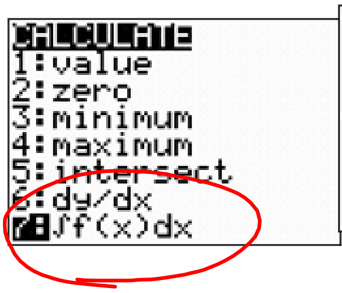
$$= \frac{1}{6}(3^3) - \frac{1}{2}(3) - \left[\frac{1-3}{6}\right] = \frac{27}{6} - \frac{3}{2} - \left(-\frac{2}{6}\right)$$

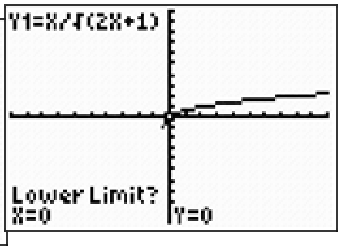
$$= \frac{9}{2} - \frac{3}{2} + \frac{1}{3} = \frac{6}{2} + \frac{1}{3} = \frac{18+2}{6} = \frac{20}{6} = \frac{10}{3}$$

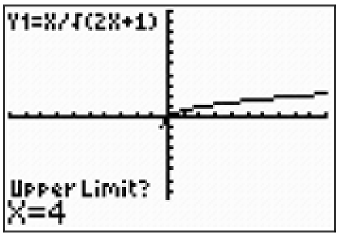

Plot1 Plot2 Plot3
\Y1=X/√(2X+1)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

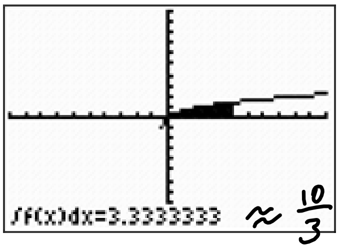



STAT PLOT F1
TBLSET F2
FORMAT F3
CALC F4
TABLE F5
Y=
WINDOW
ZOOM
TRACE
GRAPH
2ND
QUIT
MODE
INS
DEL
ALPHA
LINK
X,T,θ,n
LIST
STAT


CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx


Y1=X/√(2X+1)
Lower Limit?
X=0
Y=0


Y1=X/√(2X+1)
Upper Limit?
X=4


∫f(x)dx=3.3333333
≈ 10/3