$\boxed{\text{FTC I}}$ The derivative of the integral is the integrand.

§4.3

$\int_a^b f(x)\,dx$

upper limit

integrand

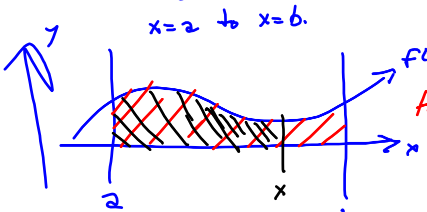
differential of the variable with respect to which we're integrating.

lower limit

Integral Sign

FTC I: $\int_a^x f(t)\,dt = G(x)$

Think of $\int_a^b f(t)\,dt$ as the area under $f(x)$ from $x=a$ to $x=b$.




Area $= \int_a^b f(t)\,dt = \int_a^b f(x)\,dx$

$\int_a^x f(t)\,dt = G(x)$ is a function of $x$!

$\sum_{k=1}^{n} k = 1+2+3+\cdots+(n-1)+n$ is a function of $n$!

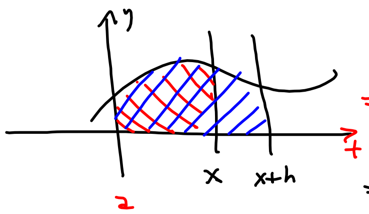
"$k$" is a "dummy variable."

"Hand-wave it."

$G'(x) = \frac{d}{dx}\left[\int_a^x f(t)\,dt\right]$

Difference Quotient $= \dfrac{G(x+h)-G(x)}{h}$

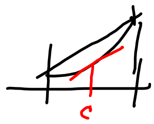
$= \frac{1}{h}\left[\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt\right] = \frac{1}{h}\int_x^{x+h} f(t)\,dt$




Red Area − Blue area

= the area between $x$ & $x+h$ under $f(t)$.

$= \int_x^{x+h} f(t)\,dt$




Recall: Mean Value Theorem

$f$ cont$^s$ on $[a,b]$ & dif$b^{le}$ on $(a,b) \Rightarrow$

$\exists\, c \in (a,b) \ni f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$\dfrac{G(x+h)-G(x)}{h} = G'(c)$ for some $c \in (x, x+h)$

That $c$ is between $x$ & $x+h$, but $h \to 0 \Rightarrow$

$x+h \to x$, so the $c$ we're talking about converges to $x$, itself, i.e.,

$G'(x) = f(c) = f(x)$

$\frac{1}{h}\int_x^{x+h} f(t)\,dt \xrightarrow{h\to 0} f(x)$

Apply it:

$$G(x) = \int_0^x \sin(t)\,dt \Longrightarrow G'(x) = \sin(x)$$

Tricky Part! Chain Rule:

$\frac{d}{dx}[x^2]$

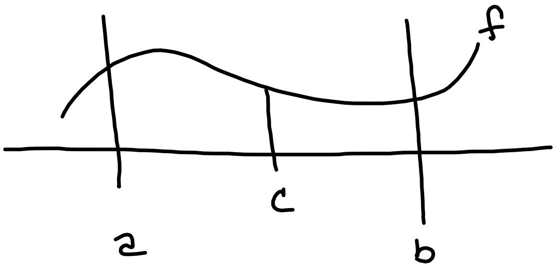
$$G(x^2) = \int_0^{x^2} \sin(t)\,dt \rightarrow \boxed{\frac{dG(x^2)}{dx} =} \sin(x^2)\cdot 2x$$

Don't leave this out of your narrative, Steve.

$$\frac{d}{dx}\left[G(x^2)\right] = \frac{dG}{d(x^2)} \cdot \frac{d(x^2)}{dx}$$

From Fall '17 Test 4

$$\frac{d}{dx}\int_0^{\cos(x)} \frac{\sin(3t)}{t^2+4}\,dt = \left(\frac{\sin(3(\cos(x)))}{(\cos(x))^2+4}\right)(-\sin(x))$$



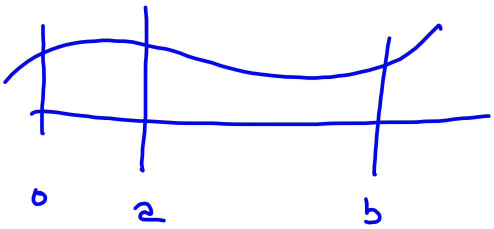

$$\int_a^b f = \int_a^c f + \int_c^b f$$

$$\int_b^a f = -\int_a^b f$$

Basically marching right to left makes the "dx" negative.

$$\int_a^b f = \int_a^0 f + \int_0^b f$$

Even if 0 is to the left!




$\int_a^0 f$ = negative of area from 0 to a

$\int_0^b f$ = positive area from 0 to b

$$\int_a^0 f + \int_0^b f = -\int_0^a f + \int_0^a f + \int_a^b f = \int_a^b f \;!$$

$$\frac{d}{dx}\int_{x^2}^{\cos(x)} \frac{\sin(3t)}{t^2+4}\,dt = \frac{d}{dx}\left[\int_{x^2}^{0} \sim\sim\sim + \int_{0}^{\cos(x)} \sim\sim\sim\right]$$

$$= \frac{d}{dx}\left[-\int_{0}^{x^2} \sim\sim\sim + \int_{0}^{\cos(x)} \sim\sim\sim\right]$$

$$= -\left(\frac{\sin(3x^2)}{(x^2)^2+4}\right)(2x) + \left(\frac{\sin(3\cos(x))}{\cos^2(x)+4}\right)(-\sin(x))$$

https://harryzaims.com/201/videos/chapter-04/4-3/videos/00a-FTC-I-and-Proof.mp4

See harryzaims.com for FTC I proof.

https://harryzaims.com/jams/160608-with-erik/b4-ya-cuze-me.mp4

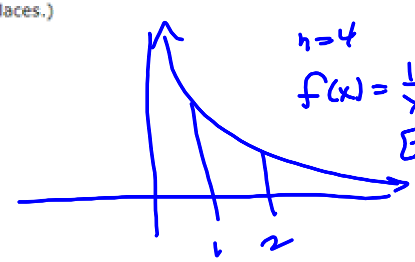
Unless specified, all approximating rectangles are assumed to have the same width.

Let $f(x) = \frac{3}{x}$.

(a) Estimate the area under the graph of $f$, the $x$-axis, and the lines $x = 1$ and $x = 2$ using four approximating rectangles and right endpoints. (Round your answer to four decimal places.)

1.9036

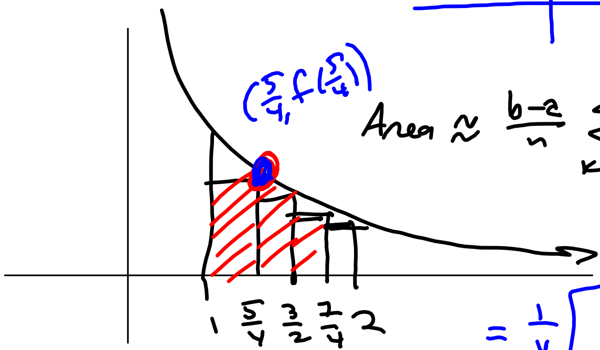
$$\Delta x = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$$




$n=4$

$f(x) = \frac{1}{x}$

$[a,b] = [1,2]$




$$\text{Area} \approx \frac{b-a}{n} \sum_{k=1}^{n} f(x_k) = \frac{1}{4}\left[f\left(\frac{5}{4}\right) + f\left(\frac{3}{2}\right) + f\left(\frac{7}{4}\right) + f(2)\right]$$

$$= \frac{1}{4}\left[\frac{1}{\frac{5}{4}} + \frac{1}{\frac{3}{2}} + \frac{1}{\frac{7}{4}} + \frac{1}{2}\right]$$

$$= \frac{1}{4}\left[\frac{4}{5} + \frac{2}{3} + \frac{4}{7} + \frac{1}{2}\right]$$

$$= \frac{1}{4}\left[\frac{4\cdot 2\cdot 3\cdot 7 + 2\cdot 5\cdot 7\cdot 2 + 4\cdot 5\cdot 3\cdot 2 + 1\cdot 5\cdot 3\cdot 7}{2\cdot 3\cdot 5\cdot 7}\right]$$

$$= \frac{1}{4}\left[\frac{168 + 140 + 120 + 105}{210}\right]$$

$$= \frac{1}{4}\left[\frac{531}{210}\right] = \frac{533}{840} \approx 0.6345238095$$

$\begin{array}{r} 35 \\ \times\ 6 \\ \hline 210 \end{array}$ $\begin{array}{r} 21 \\ \times\ 8 \\ \hline 168 \end{array}$

1.903571429 is correct (actually 1.9036 to 4 places), because I did $\frac{1}{x}$ instead of $\frac{3}{x}$.

Integrating by the definition.

Area $\approx$ Right Riemann $n$-Sum:

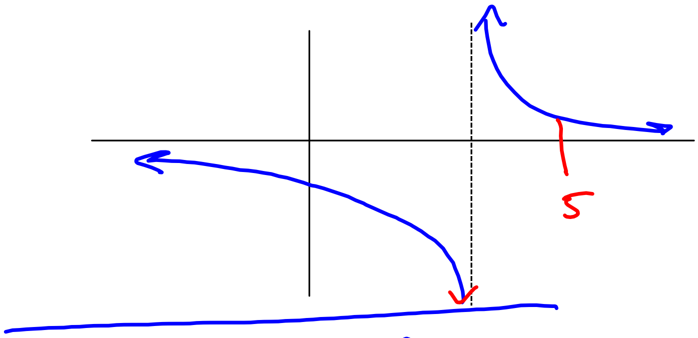
$$\frac{b-a}{n}\sum_{k=1}^{n} f\left(a+\frac{b-a}{n}k\right) \xrightarrow{n\to\infty} \text{EXACT AREA,}$$

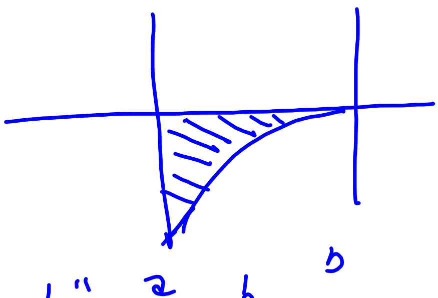
provided $f(x)$ is continuous on $[a,b]$.

What DOESN'T THIS WORK FOR?

For starters, functions that Blow Up!

$$f(x)=\frac{1}{x-3} \quad \text{on } [a,b]=[0,5]$$




Something I hid from you:

This definition of area is "signed."

$\int_a^b f(x)\,dx$ is negative!

https://www.youtube.com/watch?v=WUvTyaaNkzM&list=RDCMUCYO_jab_esuFRV4b17AJtAw&start_radio=1&rv=WUvTyaaNkzM&t=0&t=0