$R(x) = \frac{x^2+3x-3}{x-2}$ will be the re-do #2.

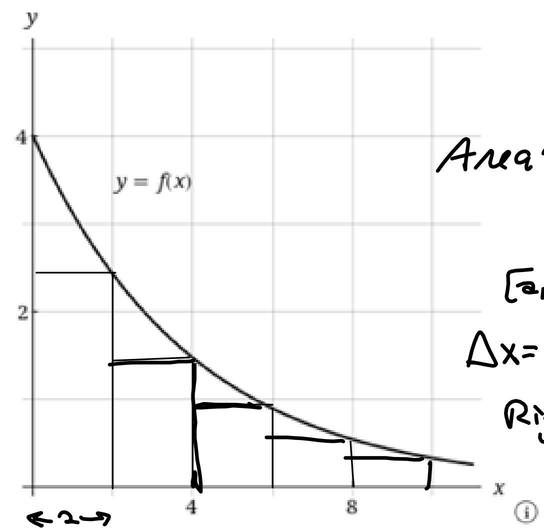
Power Rule for all $n \neq -1$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\frac{d}{dx}\left[\frac{x^{n+1}}{n+1} + C\right] = (n+1)\frac{x^n}{n+1} + C = x^n$$

$$\frac{x^3 + 27x^2 - 18x + 900}{n^3} = \frac{x^3 + \text{smaller}}{n^3} = \frac{x^3 + \cdots}{n^3}$$

$\xrightarrow{n \to \infty}$



Lower Estimate, $n=5$

$$\text{Area} \approx f(2)\cdot 2 + f(4)\cdot 2 + f(6)\cdot 2 + f(8)\cdot 2 + f(10)\cdot 2$$

$$[a,b] = [0,10] = [a,b]$$

$$\Delta x = \text{width} = \frac{b-a}{n} = \frac{10-0}{5} = 2 = \Delta x$$

Right endpoints

$$a + \Delta x = x_1$$

$$x_1 + \Delta x = x_2 = a + 2\Delta x$$

$$\vdots$$

$$x_k = a + k\Delta x = a + k\left(\frac{b-a}{n}\right)$$

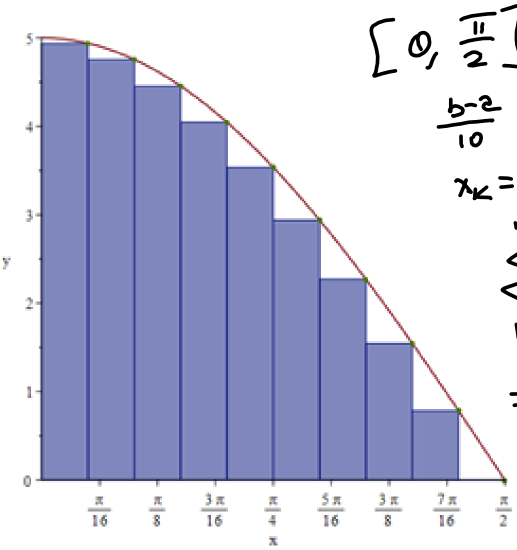
$$= a + \left(\frac{b-a}{n}\right)k$$

$$= 0 + 2k,\ k = 1, \dots, 5$$

$$2, 4, 6, 8, 10$$

With a programmable calculator (or a computer), it is possible to evaluate the expressions for the sums of areas of approximating rectangles, even for large values of n, using looping. (On a TI use the Is > command or a For-EndFor loop, on a Casio use Isz, on an HP or in BASIC use a FOR-NEXT loop.) Compute the sum of the areas of approximating rectangles using equal subintervals and right endpoints for n = 10, 30, 50, and 100.

the region under $y = 5\cos(x)$, the x-axis, and the lines $x = 0$ and $x = \frac{\pi}{2}$



$$\left[0, \frac{\pi}{2}\right] = [a, b]$$

$$\frac{b-a}{10} = \frac{\pi}{20} = \Delta x$$

$$x_k = a + k\Delta x = 0 + \frac{\pi}{20}k = \frac{\pi}{20}k = \frac{\pi k}{20}$$

$$\sum_{k=1}^{10}\left(\cos(x_k)\Delta x\right) = \sum_{k=1}^{n} \cos\left(\frac{\pi}{20}k\right)\cdot\frac{\pi}{20}$$

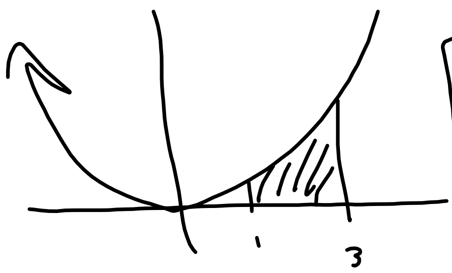
$$= \frac{\pi}{20}\sum_{k=1}^{10}\cos\left(\frac{\pi k}{20}\right)$$

$$R(n,k) = \frac{\pi}{2n}\sum_{k=1}^{n}\cos\left(\frac{\pi}{2n}k\right)$$

$$\Delta x = \frac{\frac{\pi}{2}}{n} = \frac{\pi}{2n}$$

$$x_k = a + \frac{b-a}{n}k = 0 + \frac{\frac{\pi}{2}}{n}k = \frac{\pi}{2n}k = \frac{\pi k}{2n}$$

Find the area under $\boxed{f(x) = x^2}$ from $x=1$ to $x=3$,



$\sum_{k=1}^{n} f(x_k)\Delta x$ $\quad \Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$

$x_k = a + k\Delta x = 1 + \frac{2k}{n} = \frac{n+2k}{n} = x_k$

$\Delta x \sum_{k=1}^{n} f(x_k) = \frac{2}{n}\sum_{k=1}^{n} x_k^2$

$$= \frac{2}{n}\sum_{k=1}^{n}\left(\frac{n+2}{n}\right)^2 = \frac{2}{n}\sum_{k=1}^{n}\frac{n^2+4n+4}{n^2}$$

$$= \frac{2}{n^3}\sum_{k=1}^{n}\left(n^2+4n+4\right) = \frac{2}{n^3}\sum n$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$= \frac{2n^3 + \text{smaller degree}}{6} = \frac{n^3 + \text{smaller}}{3}$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

PROOF Let $n=1$. Then $\sum_{k=1}^{1} k = 1 = \frac{1(1+1)}{2}$, so it works for $n=1$.

Suppose it works for some $n \geq 1$.

Then $\sum_{k=1}^{n+1} k = \underbrace{1+2+3+4+\cdots+n}_{} + n+1$

$$= \frac{n(n+1)}{2} + n+1$$

$$= \frac{n^2+n}{2} + \frac{2n+2}{2} = \frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2}$$

$$= \frac{(n+1)((n+1)+1)}{2}$$, i.e., it works for $n+1$!

$\Longrightarrow$ ■

Meh

$$\frac{2}{n}\sum_{k=1}^{n}\left(\frac{n+2k}{n}\right)^2 = \frac{2}{n}\sum_{k=1}^{n}\left(\frac{n^2+4kn+4k^2}{n^2}\right)$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$= \frac{2}{n^3}\left[\sum_{k=1}^{n} n^2+4kn+4k^2\right] =$$

$$= \frac{2}{n^3}\sum_{k=1}^{n} n^2 + \frac{2}{n^3}\sum_{k=1}^{n} 4kn + \frac{2}{n^3}\sum_{k=1}^{n} 4k^2$$

$$= \frac{2n^2}{n^3}\sum_{k=1}^{n} 1 + \frac{8n}{n^3}\sum_{k=1}^{n} k + \frac{8}{n^3}\sum_{k=1}^{n} k^2$$

$$= \frac{2}{n}\cdot n + \frac{8}{n^2}\cdot\frac{n^2+\cdots}{2} + \frac{8}{n^3}\cdot\frac{n^3+\cdots}{3}$$

$$= 2 + 4\,\frac{n^2+\cdots}{n^2} + \frac{8}{3}\cdot\frac{n^3+\cdots}{n^3}$$

$$\xrightarrow{n\to\infty} 2+4+\frac{8}{3} = 6+\frac{8}{3} = \frac{18+8}{3} = \boxed{\frac{26}{3} = \text{AREA}}$$

$$\int_1^3 x^2\,dx = \left[\frac{x^3}{3}\right]_1^3 = \frac{3^3}{3} - \frac{1^3}{3} = \frac{27}{3} - \frac{1}{3} = \frac{26}{3} \;!!!!$$

FTC II: If $F'(x) = f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

FTC I: $G(x) = \int_a^x f(t)\,dt \Rightarrow$

$$G'(x) = f(x)$$