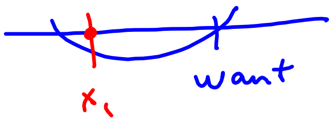
30. (a) Use Newton's method with $x_1 = 1$ to find the root of the equation $x^3 - x = 1$ correct to six decimal places.

(b) Solve the equation in part (a) using $x_1 = 0.6$ as the initial approximation.

(c) Solve the equation in part (a) using $x_1 = 0.57$. (You definitely need a programmable calculator for this part.)

(d) Graph $f(x) = x^3 - x - 1$ and its tangent lines at $x_1 = 1$, 0.6, and 0.57 to explain why Newton's method is so sensitive to the value of the initial approximation.

If it's on the wrong side of a local min, it can throw it off.



what if x_n is $\ni f'(x_n) = 0$?

It won't have an x-int.

The tangent line.

$\int \cos(3x)\,dx = 3\sin(x) + C$ Nope

$= \int f'(x)\,dx$ $\quad \frac{d}{dx}\left[3\sin(x)\right] = 3\cos(x)$

$= f(x) + C$ $\quad \frac{d}{dx}\left[a\sin(3x)\right] = a\cos(3x)\cdot 3 \overset{\text{want}}{=} \cos(3x)$

Find $f(x)$

$\Rightarrow a = \frac{1}{3}$

$\therefore \int \cos(3x)\,dx = \boxed{\frac{1}{3}\sin(3x) + C}$

$\frac{d}{dx}\left[\frac{1}{3}\sin(3x)\right] = \frac{1}{3}\cos(3x)\cdot 3 = \cos(3x)$ ✓

We don't know how to handle

$$\int \sqrt{6x^7 + 5x^2}\,dx$$

But if we can <u>see</u> it as a derivative:

$$\int \left(\sqrt{6x^7 + 5x^2}\right)\left(42x^6 + 10x\right)dx$$

$$= \int \left(6x^7 + 5x^2\right)^{\frac{1}{2}}\left(42x^6 + 10x\right)dx$$

$$= \int u^{\frac{1}{2}}\,du \doteq \int (u(x))^{\frac{1}{2}}\left(u'(x)\,dx\right) = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \left(\sqrt{6x^2 + 5x^2}\right)^3 + C$$

57. $a(t) = 10 \sin t + 3 \cos t$, $\boxed{s(0) = 0, \quad s(2\pi) = 12}$ → Boundary-Value problem

58. $a(t) = t^2 - 4t + 6$, $s(0) = 0$, $s(1) = 20$ → (Boundary conditions on s)

57) $a(t) = 10\sin(t) + 3\cos(t) = s''(t)$

$v(t) = \int a(t)\,dt = -10\cos(t) + 3\sin(t) + C = s'(t)$

$\Rightarrow s(t) = -10\sin(t) - 3\cos(t) + Ct + D$

$s(0) = 0 = -3 + D = 0 \Rightarrow D = 3$

$s(2\pi) = -3 + 2\pi C + 3 = 12$

$\Rightarrow 2\pi C = 12 \Rightarrow$

$C = \frac{12}{2\pi} = \frac{6}{\pi} \Rightarrow$

$$\boxed{s(t) = -10\sin(t) - 3\cos(t) + \frac{6}{\pi}t + 3}$$

37. $f''(\theta) = \sin\theta + \cos\theta, \quad f(0) = 3, \quad f'(0) = 4$ → Initial Value Problem (Initial conditions on f, f')

$\rightarrow f'(\theta) = -\cos\theta + \sin\theta + C \rightarrow = 5$

$f'(0) = -1 + C = 4 \rightarrow C = 5 \rightarrow$

$f(\theta) = -\sin\theta - \cos\theta + 5t + D$

$f(0) = -1 + +D = 3 \rightarrow$

$D = 4 \rightarrow$

$$\boxed{f(\theta) = -\sin\theta - \cos\theta + 5t + 4}$$

sine $\sin(\theta)$ $\sin\theta$

sin is something God doesn't like.

Questions 3.6–3.9?

$f(x) = 2\sin(x) + \cos(2x) = 2\sin(x) + (1 - 2\sin^2(x))$

$= -2\sin^2(x) + 2\sin(x) + 1$ SET 0

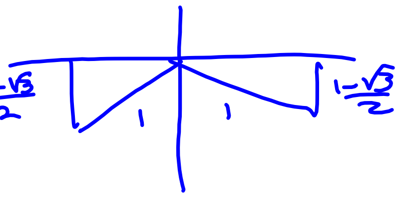
$\longrightarrow a = -2, b = 2, c = 1$

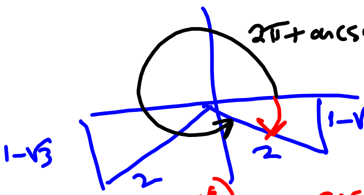
$b^2 - 4ac = 4 - 4(-2)(1) = 4 + 8 = 12$

$\sin(x) = \dfrac{-2 \pm \sqrt{12}}{2(-2)} = \dfrac{-2 \pm 2\sqrt{3}}{-4} = \dfrac{1 \pm \sqrt{3}}{2}$ $\nearrow \dfrac{1+\sqrt{3}}{2} \approx \dfrac{2.73}{2} > 1$ $\searrow \dfrac{1-\sqrt{3}}{2}$

$\sin(x) = \dfrac{1-\sqrt{3}}{2}$

$2\pi + \arcsin\left(\frac{1-\sqrt{3}}{2}\right)$





$\arcsin\left(\frac{1-\sqrt{3}}{2}\right) < 0$

so $2\pi + \arcsin\left(\frac{1-\sqrt{3}}{2}\right)$

$\pi - \arcsin\left(\frac{1-\sqrt{3}}{2}\right)$

$-2\sin^2\left(2\pi + \arcsin\left(\frac{1-\sqrt{3}}{2}\right)\right) + 2\sin\left(2\pi + \arcsin\left(\frac{1-\sqrt{3}}{2}\right)\right) + 1$

$= -2\sin^2\left(\arcsin\left(\frac{1-\sqrt{3}}{2}\right)\right) + 2\sin\left(\arcsin\left(\frac{1-\sqrt{3}}{2}\right)\right) + 1$

$= -2\left(\frac{1-\sqrt{3}}{2}\right)^2 + 2\left(\frac{1-\sqrt{3}}{2}\right) + 1$

$= 2\left(\dfrac{1 - 2\sqrt{3} + 3}{4}\right) + 1 - \sqrt{3} + 1$

$= \dfrac{4 - 2\sqrt{3}}{4} + 1 - \sqrt{3} + 1$

$= \dfrac{2-\sqrt{3}}{2} + \dfrac{2 - 2\sqrt{3}}{2} + \dfrac{2}{2} = \dfrac{4 - 3\sqrt{3} + 2}{2}$

$= \dfrac{6 - 3\sqrt{3}}{2}$