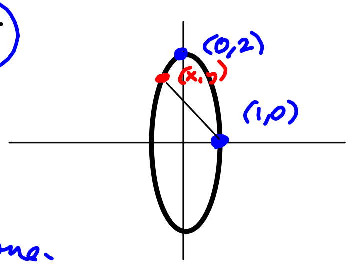
23. Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.

$$x^2 + \frac{y^2}{4} = 1$$

$$y^2 = 4 - 4x^2$$

$$y = \pm 2\sqrt{1-x^2} = \boxed{\pm\sqrt{4-4x^2}}$$ Explicitly

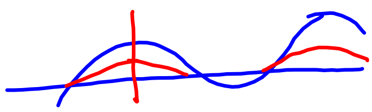


By symmetry, we'll find the top one & then change the sign for the lower one.

$$y = 2\sqrt{1-x^2}$$

Now, distance from point (x,y) on the ellipse to $(1,0)$ is $\sqrt{(x-1)^2 + (y-0)^2} = d$

$= \sqrt{(x-1)^2 + (2\sqrt{1-x^2})^2}$ &, since $\sqrt{\ }$ is increasing function, maximizing "$\sqrt{\text{stuff}}$" is the same as maximizing "stuff"



Maximize

$f(x) = d^2 = (x-1)^2 + 4(1-x^2)$ to be maximized

$= x^2 - 2x + 1 + 4 - 4x^2 = -3x^2 - 2x + 5 \longrightarrow$

$\Longrightarrow f'(x) = 6x - 2 \overset{\text{SET}}{=} 0 \Longrightarrow x = \frac{1}{3}$

$-$

Mario sez: Shouldn't it be $-\frac{1}{3}$, you idiot?

So $-6x - 2 = 0 \longrightarrow$

$x = -\frac{1}{3} \longrightarrow$

$y = \pm\sqrt{4-4x^2} = \sqrt{4 - 4\left(\frac{1}{3}\right)^2}$

$= \pm\sqrt{\frac{36-4}{9}} = \pm\sqrt{\frac{32}{9}} = \pm\frac{4\sqrt{2}}{3}$

$$\boxed{\left(-\frac{1}{3}, \pm\frac{4\sqrt{2}}{3}\right)}$$

Find 2 #s whose sum is 23 & whose product is max:

Let $x = 1^{st}$ # &
$y = 2^{nd}$ " . Then

$x + y = 23 \Rightarrow y = 23 - x$. We maximize

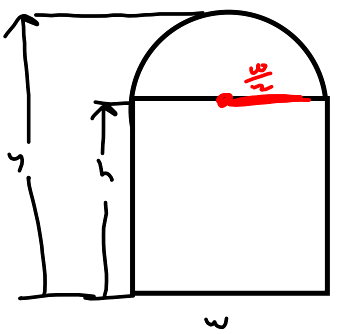
$xy = x(23 - x) = 23x - x^2 = 23 - 2x \overset{\text{SET}}{=} 0 \Rightarrow$

$23 = 2x$

$\boxed{\frac{23}{2} = 11.5 = x}$ & $23 - x = 23 - 11.5 = \boxed{11.5 = y}$

34. A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle. See Exercise 1.1.62.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.

Perimeter =



Let w = width of window (in ft)
y = height " " "
h = " " " "
P = perimeter (in feet)
A = area (in ft^2).

#34 We maximize the area of a Norman Window (see pic.) that has a perimeter of 30ft.

We know P = perimeter (in ft) = $\boxed{w+2h+\pi w = 30}$ Nice auxiliary eq'n

And area in square feet (ft^2) = $hw + \frac{1}{2}\pi\left(\frac{w}{2}\right)^2$ to be maximized.

$w + \pi w - 30 = -h \rightarrow -2h$ $\quad \boxed{\frac{30-w-\pi w}{2} = h}$

$h = 30 - w - \pi w \longrightarrow$ $\quad \left(\frac{30-w-\pi w}{2}\right)w$

$A = (30 - w - \pi w)(w) + \frac{\pi w^2}{8}$ $\quad \frac{8}{8}\cdot\frac{-\pi w^2}{1} + \frac{\pi w^2}{8} = -\frac{1\pi w^2}{8}$

$= 30w - w^2 - \pi w^2 + \frac{\pi}{8}w^2$ $\quad \frac{30w - w^2 - \pi w^2}{2}$

$= 30w - w^2 - \frac{7}{8}\pi w^2$ $\quad 15w - \frac{w^2}{2} - \frac{\pi w^2}{2}\cdot\frac{4}{4} + \frac{\pi}{8}w^2$

$= 30w - (1 + \frac{7}{8}\pi)w^2 = A(w)$ $\quad = 15w - \frac{w^2}{2} - \frac{3\pi w^2}{8} = A$

Nope! $\Longrightarrow \frac{dA}{dw} = 30 - 2(1 + \frac{7}{8}\pi)w \overset{\text{SET}}{=} 0$

$\Longrightarrow 2(1 + \frac{7\pi}{8})w = 30$

$w = \frac{30}{2(1 + \frac{7\pi}{8})} \approx$

$A(w) = 15w - \left(\frac{1}{2} + \frac{3\pi}{8}\right)w^2 \Longrightarrow$

$A'(w) = 15 - 2\left(\frac{1}{2} + \frac{3\pi}{8}\right)w \overset{\text{SET}}{=} 0$

$\Rightarrow w = \dfrac{15}{2\left(\frac{1}{2}+\frac{3\pi}{8}\right)} \approx$

$$evalf\left(\frac{15}{2\cdot\left(\frac{1}{2}+\frac{3\cdot \mathrm{Pi}}{8}\right)}\right)$$

Solve for w

4.469347664

$$\frac{(30 - \% - \mathrm{Pi}\cdot\%)}{2}$$

Find h

5.744891273

$4.469347664 + 2\cdot\% + \mathrm{Pi}\cdot 4.469347664$

checked perimeter = 30 ✓

30.00000000

$\text{Area} = hw + \dfrac{\pi\left(\frac{w}{2}\right)^2}{2} = hw + \dfrac{\pi w^2}{8}$

$5.744891273\cdot 4.469347664\,\mathrm{Pi} + \mathrm{Pi}\cdot 4.469347664^2$

$\boxed{143.4167989\ ft^2}$ Area.

§ 3.9 Antiderivatives

3.9 says: Find the function whose derivative is $f(x) = 2x + \cos(x)$

$\int (2x + \cos(x))\,dx$ is notation for antiderivative

Integral sign stands for "SUM" just like "Σ"

$$\frac{d}{dx}[x^2+7] = \frac{d}{dx}[x^2-19]$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int 2x = 2\int x\,dx = 2\left[\frac{x^2}{2}\right] + C = x^2 + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

Check:

$$\frac{d}{dx}[\sin(x) + C] = \cos(x) \checkmark$$

"$\int$ ~ dx" is Linear, just like $\frac{d}{dx}(\text{~})$

$$\int (af(x) + bg(x))\,dx = a\int f(x)\,dx + b\int g(x)\,dx$$

$\int (af + bg) = a\int f + b\int g$, where f, g functions & a & b are numbers (constants)

$$\int (x^3 + \sin(x))\,dx = \frac{x^4}{4} - \cos(x) + C$$