3.6 #6

$f := x \to \frac{(x^3 + 5 \cdot x^2 + 1)}{x^4 + x^3 - x^2 + 2}$

$$f := x \mapsto \frac{x^3 + 5 \cdot x^2 + 1}{x^4 + x^3 - x^2 + 2} \tag{1.1}$$

$numer(f(x))$

$$x^3 + 5\,x^2 + 1 \tag{1.2}$$

$p := x \to x^3 + 5\,x^2 + 1$

$$p := x \mapsto x^3 + 5 \cdot x^2 + 1 \tag{1.3}$$

$q := x \to denom(f(x))$

$$q := x \mapsto denom(f(x)) \tag{1.4}$$

$q(x)$

$$x^4 + x^3 - x^2 + 2 \tag{1.5}$$

$solve(p(x) = 0)$

$$-\frac{(1108 + 12\sqrt{1581})^{1/3}}{6} - \frac{50}{3\,(1108 + 12\sqrt{1581})^{1/3}} - \frac{5}{3}, \frac{(1108 + 12\sqrt{1581})^{1/3}}{12} + \frac{25}{3\,(1108 + 12\sqrt{1581})^{1/3}} - \frac{5}{3} + \frac{I\sqrt{3}\left(-\frac{(1108 + 12\sqrt{1581})^{1/3}}{6} + \frac{50}{3\,(1108 + 12\sqrt{1581})^{1/3}}\right)}{2}, \frac{(1108 + 12\sqrt{1581})^{1/3}}{12} + \frac{25}{3\,(1108 + 12\sqrt{1581})^{1/3}} - \frac{5}{3} - \frac{I\sqrt{3}\left(-\frac{(1108 + 12\sqrt{1581})^{1/3}}{6} + \frac{50}{3\,(1108 + 12\sqrt{1581})^{1/3}}\right)}{2} \tag{1.6}$$

$evalf(\%)$

$$-5.039377329,\ 0.019688664 - 0.4450276076\ I,\ 0.019688664 + 0.4450276076\ I \tag{1.7}$$

$solve(q(x) = 0)$

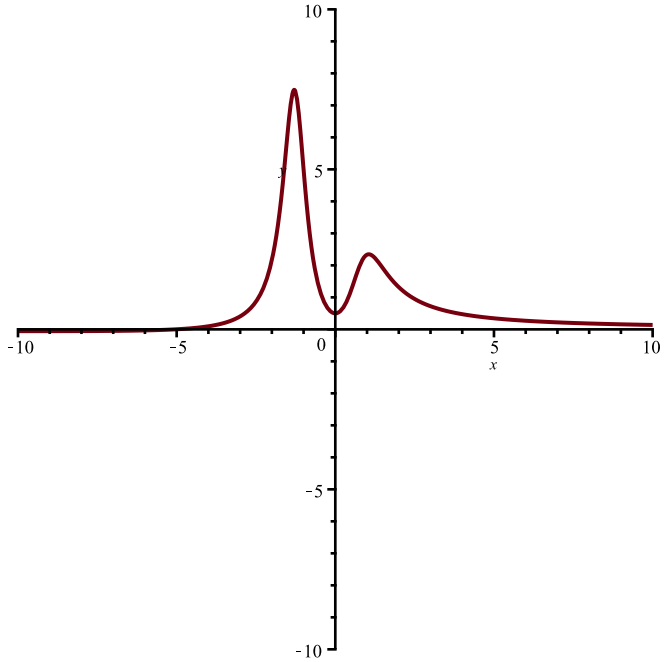
$$RootOf(\_Z^4 + \_Z^3 - \_Z^2 + 2, index = 1), RootOf(\_Z^4 + \_Z^3 - \_Z^2 + 2, index = 2), RootOf(\_Z^4 + \_Z^3 - \_Z^2 + 2, index = 3), RootOf(\_Z^4 + \_Z^3 - \_Z^2 + 2, index = 4) \tag{1.8}$$

$evalf(\%)$

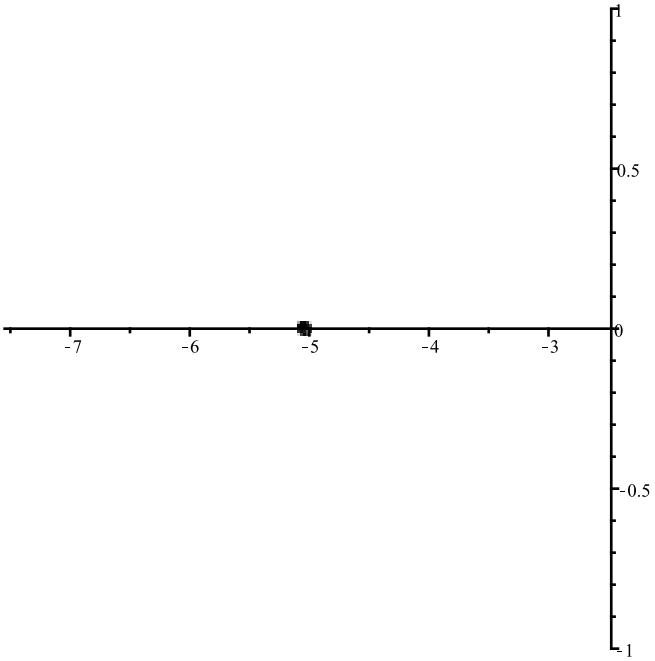
$$0.7706965750 + 0.7123224205\ I,\ -1.270696575 + 0.4485975544\ I,\ -1.270696575 \tag{1.9}$$

$-0.4485975544\,I,\ 0.7706965750 - 0.7123224205\,I$

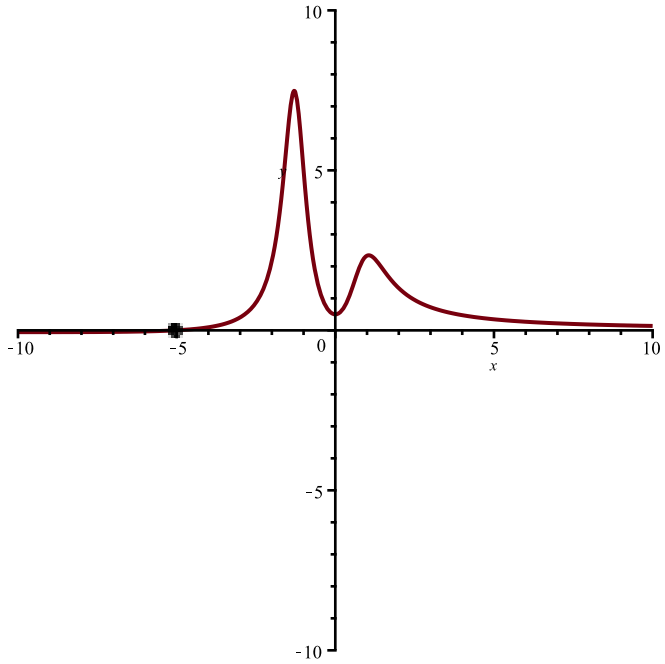
```
with(plots):
fplot := plot(f(x), x = -10..10, y = -10..10)
```




```
xint := pointplot([[-5.039377329, 0]], symbol = solidcircle, symbolsize = 15)
```




```
display([fplot, xint])
```



$factor(f(x))$

$$\frac{x^3 + 5x^2 + 1}{x^4 + x^3 - x^2 + 2} \tag{1.10}$$

$f(-6)$

$$-\frac{35}{1046} \tag{1.11}$$

$f(-5)$

$$\frac{1}{477} \tag{1.12}$$

$fp := D(f)$

$$fp := x \mapsto \frac{3 \cdot x^2 + 10 \cdot x}{x^4 + x^3 - x^2 + 2} - \frac{\left(x^3 + 5 \cdot x^2 + 1\right) \cdot \left(4 \cdot x^3 + 3 \cdot x^2 - 2 \cdot x\right)}{\left(x^4 + x^3 - x^2 + 2\right)^2} \tag{1.13}$$

$simplify(fp(x))$

$$-\frac{x\left(x^5 + 10x^4 + 6x^3 + 4x^2 - 3x - 22\right)}{\left(x^4 + x^3 - x^2 + 2\right)^2} \tag{1.14}$$

$solve(fp(x) = 0)$

$$0, RootOf\left(\_Z^5 + 10\_Z^4 + 6\_Z^3 + 4\_Z^2 - 3\_Z - 22, index = 1\right), RootOf\left(\_Z^5 + 10\_Z^4 + 6\_Z^3 + 4\_Z^2 - 3\_Z - 22, index = 2\right), RootOf\left(\_Z^5 + 10\_Z^4 + 6\_Z^3 + 4\_Z^2 - 3\_Z - 22, index = 3\right), RootOf\left(\_Z^5 + 10\_Z^4 + 6\_Z^3 + 4\_Z^2 - 3\_Z - 22, index = 4\right), RootOf\left(\_Z^5 + 10\_Z^4 + 6\_Z^3 + 4\_Z^2 - 3\_Z - 22, index = 5\right) \tag{1.15}$$

$evalf(\%)$

$$0., 1.054646897, -0.1760997067 + 1.296987697\,I, -1.294201384, -9.408246099, -0.1760997067 - 1.296987697\,I \tag{1.16}$$

$[fp(-1), fp(1), fp(2)]$

$$\left[-12, \frac{4}{9}, -\frac{114}{121}\right] \tag{1.17}$$

$fpp := \mathrm{D}(fp)$

$$fpp := x \mapsto \frac{6\cdot x + 10}{x^4 + x^3 - x^2 + 2} - \frac{2\cdot\left(3\cdot x^2 + 10\cdot x\right)\cdot\left(4\cdot x^3 + 3\cdot x^2 - 2\cdot x\right)}{\left(x^4 + x^3 - x^2 + 2\right)^2} \tag{1.18}$$
$$+ \frac{2\cdot\left(x^3 + 5\cdot x^2 + 1\right)\cdot\left(4\cdot x^3 + 3\cdot x^2 - 2\cdot x\right)^2}{\left(x^4 + x^3 - x^2 + 2\right)^3} - \frac{\left(x^3 + 5\cdot x^2 + 1\right)\cdot\left(12\cdot x^2 + 6\cdot x - 2\right)}{\left(x^4 + x^3 - x^2 + 2\right)^2}$$

$simplify(fpp(x))$

$$\frac{2\,x^9 + 30\,x^8 + 36\,x^7 + 42\,x^6 - 18\,x^5 - 270\,x^4 - 152\,x^3 + 42\,x^2 + 12\,x + 44}{\left(x^4 + x^3 - x^2 + 2\right)^3} \tag{1.19}$$

$solve(fpp(x) = 0)$

$$RootOf\left(\_Z^9 + 15\,\_Z^8 + 18\,\_Z^7 + 21\,\_Z^6 - 9\,\_Z^5 - 135\,\_Z^4 - 76\,\_Z^3 + 21\,\_Z^2 + 6\,\_Z + 22, index = 1\right), RootOf\left(\_Z^9 + 15\,\_Z^8 + 18\,\_Z^7 + 21\,\_Z^6 - 9\,\_Z^5 - 135\,\_Z^4 - 76\,\_Z^3 + 21\,\_Z^2 + 6\,\_Z + 22, index = 2\right), RootOf\left(\_Z^9 + 15\,\_Z^8 + 18\,\_Z^7 + 21\,\_Z^6 - 9\,\_Z^5 - 135\,\_Z^4 - 76\,\_Z^3 + 21\,\_Z^2 + 6\,\_Z + 22, index = 3\right), RootOf\left(\_Z^9 + 15\,\_Z^8 + 18\,\_Z^7 + 21\,\_Z^6 - 9\,\_Z^5 - 135\,\_Z^4 - 76\,\_Z^3 + 21\,\_Z^2 + 6\,\_Z + 22, index = 4\right), RootOf\left(\_Z^9 + 15\,\_Z^8 + 18\,\_Z^7 + 21\,\_Z^6 - 9\,\_Z^5 - 135\,\_Z^4 - 76\,\_Z^3 + 21\,\_Z^2 + 6\,\_Z + 22, index = 5\right), RootOf\left(\_Z^9 + 15\,\_Z^8 + 18\,\_Z^7 + 21\,\_Z^6 - 9\,\_Z^5 - 135\,\_Z^4 - 76\,\_Z^3 + 21\,\_Z^2 + 6\,\_Z + 22, index = 6\right), RootOf\left(\_Z^9 + 15\,\_Z^8 + 18\,\_Z^7 + 21\,\_Z^6 - 9\,\_Z^5 - 135\,\_Z^4 - 76\,\_Z^3 + 21\,\_Z^2 + 6\,\_Z + 22, index = 7\right), RootOf\left(\_Z^9 + 15\,\_Z^8 + 18\,\_Z^7 + 21\,\_Z^6 - 9\,\_Z^5 - 135\,\_Z^4 - 76\,\_Z^3 + 21\,\_Z^2 + 6\,\_Z + 22, index = 8\right), RootOf\left(\_Z^9 + 15\,\_Z^8 + 18\,\_Z^7 + 21\,\_Z^6 - 9\,\_Z^5 - 135\,\_Z^4 - 76\,\_Z^3 + 21\,\_Z^2 + 6\,\_Z + 22, index = 9\right) \tag{1.20}$$

$evalf(\%)$

$$0.6018981518, 1.480577630, -0.2241830772 + 1.895136155\,I, -0.1227549252 + 0.5401897616\,I, -1.032097636, -1.550227639, -13.80627450, -0.1227549252 - 0.5401897616\,I, -0.2241830772 - 1.895136155\,I \tag{1.21}$$

$[fpp(0), fpp(1), fpp(2)]$

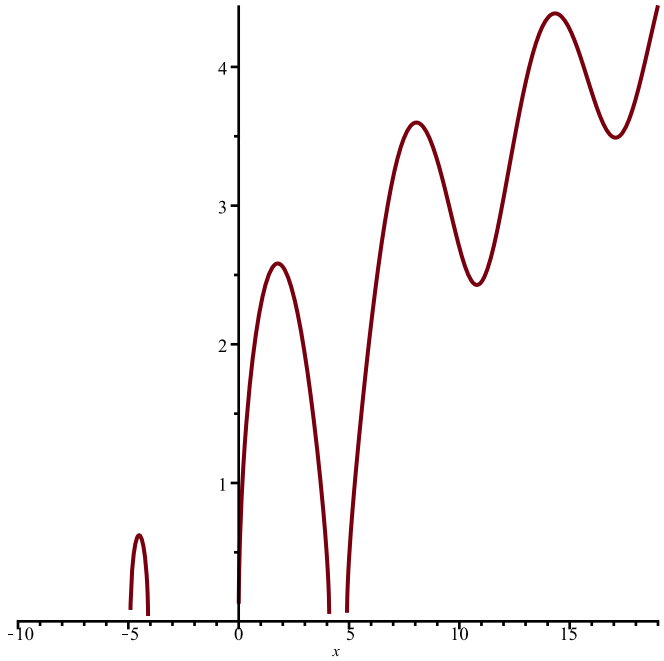
$$\left[\frac{11}{2}, -\frac{232}{27}, \frac{2531}{2662}\right] \tag{1.22}$$

3.6 #8

$f := x \to \mathrm{sqrt}(x + 5\cdot\sin(x))$

$$f := x \mapsto \sqrt{x + 5\cdot\sin(x)} \tag{2.1}$$

$plot(f(x), x = -10..19)$



$fp := D(f)$

$$fp := x \mapsto \frac{1 + 5\cdot\cos(x)}{2\cdot\sqrt{x + 5\cdot\sin(x)}} \tag{2.2}$$

$fpp := D(fp)$

$$fpp := x \mapsto -\frac{5\cdot\sin(x)}{2\cdot\sqrt{x + 5\cdot\sin(x)}} - \frac{(1 + 5\cdot\cos(x))^2}{4\cdot(x + 5\cdot\sin(x))^{3/2}} \tag{2.3}$$

$simplify(fpp(x))$

$$\frac{-51 + 25\cos(x)^2 - 10\sin(x)\,x - 10\cos(x)}{4\,(x + 5\sin(x))^{3/2}} \tag{2.4}$$

$solve(fp(x) = 0)$

$$-\arctan\left(2\sqrt{6}\right) + \pi,\ \arctan\left(2\sqrt{6}\right) - \pi \tag{2.5}$$

$solve(fpp(x) = 0)$

$$RootOf\left(51 - 25\cos(\_Z)^2 + 10\sin(\_Z)\,\_Z + 10\cos(\_Z)\right) \tag{2.6}$$

$evalf(\%)$

$$0. - 0.9520360524\,I \tag{2.7}$$

$solve(f(x) = 0)$

$$RootOf(\_Z + 5\sin(\_Z)) \tag{2.8}$$

$evalf(\%)$

$$0. \tag{2.9}$$

$evalf\left(\arccos\left(-\frac{1}{5}\right)\right)$

$$1.772154248 \tag{2.10}$$

$\% + 2\cdot Pi$

$$8.055339556 \quad \textbf{(2.11)}$$

$\% + 2 \cdot \mathrm{Pi}$

$$14.33852486 \quad \textbf{(2.12)}$$

$\% + 2 \cdot \mathrm{Pi}$

$$20.62171017 \quad \textbf{(2.13)}$$

$evalf\left(2 \cdot \mathrm{Pi} - \arccos\left(-\frac{1}{5}\right)\right)$

$$4.511031060 \quad \textbf{(2.14)}$$

$f(4.511031060)$

$$0.6228550602\ \mathrm{I} \quad \textbf{(2.15)}$$

$\% + 2 \cdot \mathrm{Pi}$

$$10.79421637 \quad \textbf{(2.16)}$$

$\% + 2 \cdot \mathrm{Pi}$

$$17.07740168 \quad \textbf{(2.17)}$$

$evalf\left(-2 \cdot \mathrm{Pi} + \arccos\left(-\frac{1}{5}\right)\right)$

$$-4.511031060 \quad \textbf{(2.18)}$$

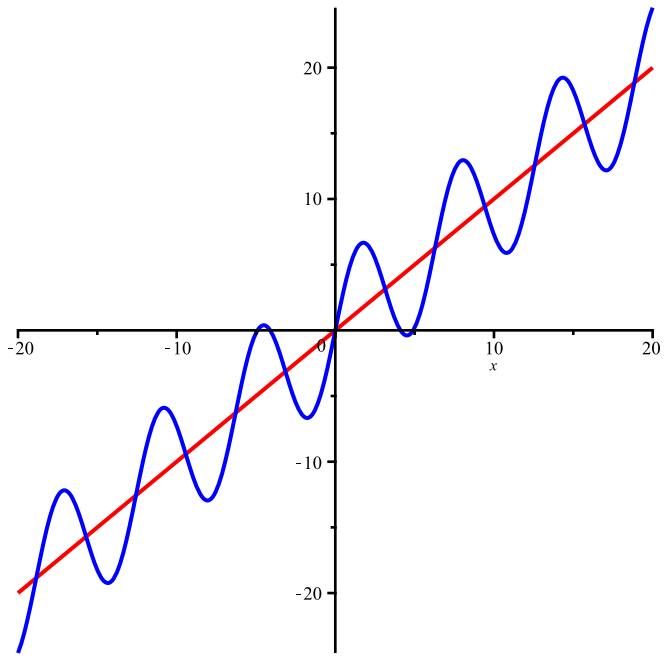
$solve(x + 5 \cdot \sin(x) = 0)$

$$RootOf(\_Z + 5\sin(\_Z)) \quad \textbf{(2.19)}$$

$evalf(\%)$

$$0. \quad \textbf{(2.20)}$$

$plot([x, x + 5 \cdot \sin(x)], x = -20..20, color = [red, blue])$



$plot(x - 5 \cdot \sin(x))$