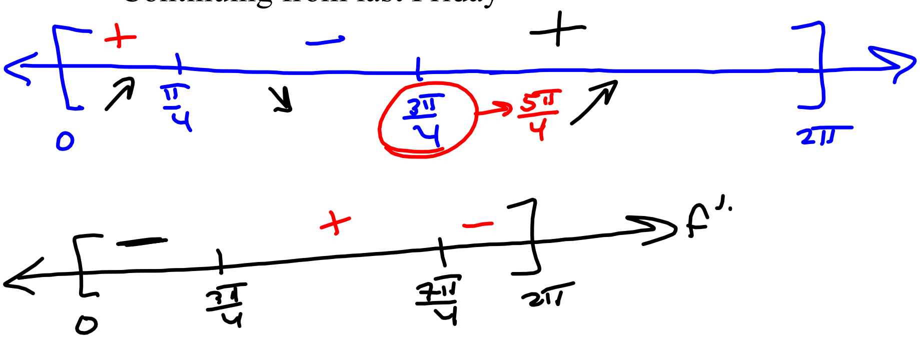
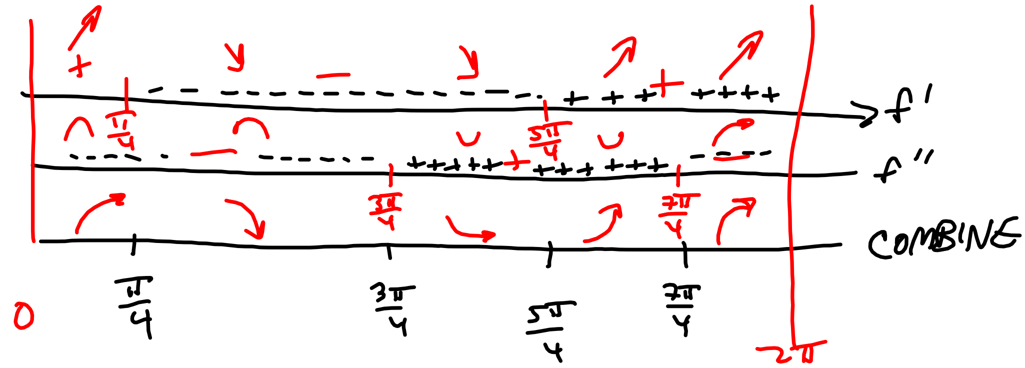
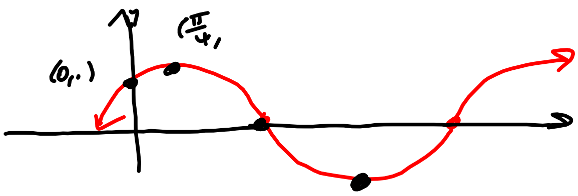
Remind me to hit "record!"

Today: 3.3 and 3.4 discussion.

Continuing from last Friday







Need: $f(0), f(\frac{\pi}{4}), f(\frac{3\pi}{4}), f(\frac{5\pi}{4}), f(\frac{7\pi}{4}), f(2\pi)$ to get the relative heights correct.

$f(0) = \sin(0) + \cos(0) = 1$

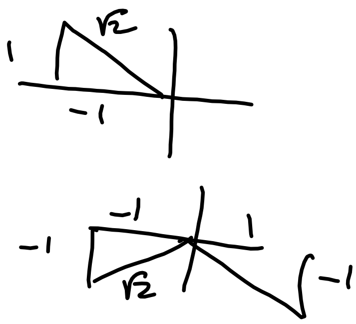
$f(\frac{\pi}{4}) = \sqrt{2}$

$f(\frac{3\pi}{4}) = 0$

$f(\frac{5\pi}{4}) = -\sqrt{2}$

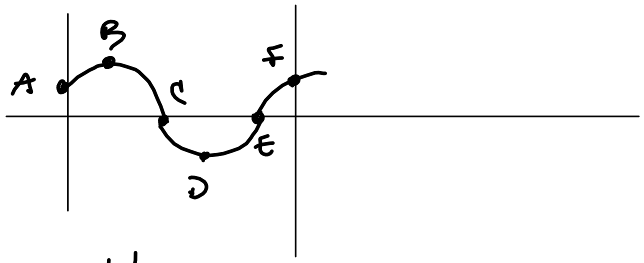
$f(\frac{7\pi}{4}) = 0$

$f(2\pi) = f(0) = 1$



Finished Product for $f(x) = \sin(x) + \cos(x)$

1st of all, show all work



$A = (0, 1)$ Endpt

$B = (\frac{\pi}{4}, \sqrt{2}) \approx (.7854, 1.4142)$ Local & Global max

$C = (\frac{3\pi}{4}, 0) \approx (2.3562, 0)$ Inflection Pt

$D = (\frac{5\pi}{4}, -\sqrt{2}) \approx (3.9270, 1.4142)$ Local & Global min.

$E = (\frac{7\pi}{4}, 0) \approx (5.4978, 0)$ I.P.

$F = (2\pi, 1) \approx (6.2832, 1)$ Endpoint.

When I say "exact," I mean "exact."

Suppose I'm graphing a trig. polynomial on $[0, 2\pi]$ & either $f(x) = 0$, $f'(x) = 0$ or $f''(x) = 0$, gives rise to

$$\sin(x) = \frac{1 \pm \sqrt{2}}{2} \begin{cases} \frac{1+\sqrt{2}}{2} > 1 \quad \times \\ \frac{1-\sqrt{2}}{2} \end{cases}$$

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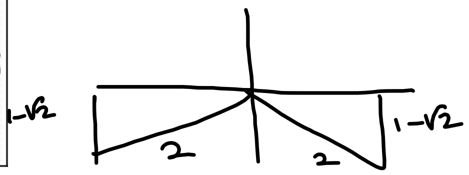
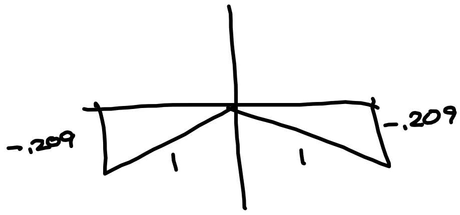
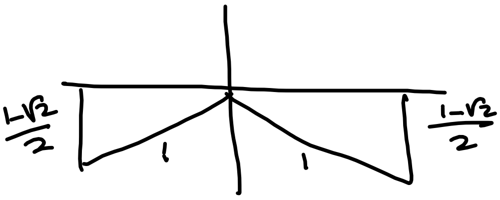
sin⁻¹((1+√(2))/2)
sin⁻¹((1-√(2))/2)
        -.2086166903
```

un-possible!







This $-.2086166903 \approx \arcsin\left(\frac{1-\sqrt{2}}{2}\right)$

The EXACT way to express the 2 solutis is

$2\pi + \arcsin\left(\frac{1-\sqrt{2}}{2}\right)$, $\pi - \arcsin\left(\frac{1-\sqrt{2}}{2}\right)$

That's what I'll be looking for for #1

$$2\sin(x)^2 - 6\sin(x) + 3$$

Find the roots of $f(x) = 2\sin^2(x) - 6\sin(x) + 3$

$2u^2 - 6u + 3 \overset{SET}{=} 0 \Rightarrow$

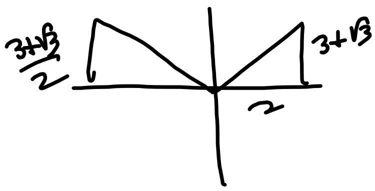
$u^2 - 3u + \frac{3}{2} = 0 \Rightarrow$

$u^2 - 3u + \left(\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{6}{4} = \left(u - \frac{3}{2}\right)^2 - \frac{3}{4} \overset{SET}{=} 0 \Rightarrow$

$\left(u - \frac{3}{2}\right)^2 = \frac{3}{4} \Rightarrow$

$u - \frac{3}{2} = \pm \frac{\sqrt{3}}{2} \Rightarrow$

$$u = \frac{3 \pm \sqrt{3}}{2} = \sin(x) \begin{cases} \frac{3+\sqrt{3}}{2} > 1 \quad \times \\ \frac{3-\sqrt{3}}{2} \text{ is ok} \end{cases}$$



$x = \arcsin\left(\frac{3-\sqrt{3}}{2}\right)$, $\pi - \arcsin\left(\frac{3-\sqrt{3}}{2}\right)$ are the 2 exact solutions

$R(x) = \dfrac{2x^2 - x - 15}{x^2 - 4x - 21}$. we graph $R(x)$ using MAT 121 techniques.

$\mathcal{D}$: $x^2 - 4x - 21 = (x-7)(x+3) \overset{\text{SET}}{=} 0 \rightarrow x \in \{-3, 7\} \Rightarrow \mathcal{D} = \mathbb{R} \setminus \{-3, 7\}$

Vertical Asymptotes : $\boxed{x = -3,\ x = 7}$ V.A.

x-int: $2x^2 - x - 15 = (2x+5)(x-3) \overset{\text{SET}}{=} 0 \rightarrow$

$x \in \{-\frac{5}{2}, 3\} \rightsquigarrow \boxed{(-\frac{5}{2}, 0),\ (3, 0) \text{ XINTS}}$

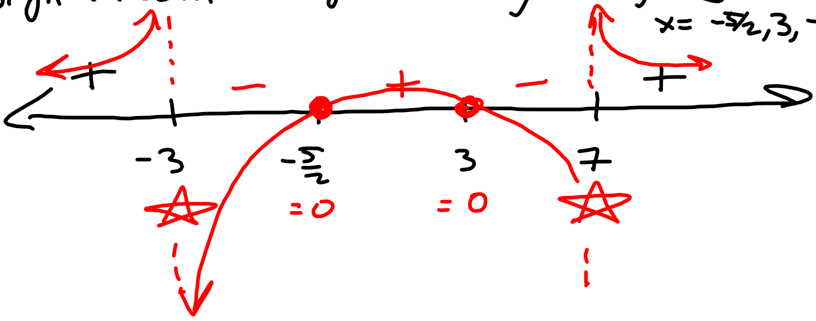
y-int: $\dfrac{-15}{-21} = \dfrac{-5}{-7} = \dfrac{5}{7} \rightsquigarrow \boxed{(0, \frac{5}{7}) \text{ y-int}}$

Horizontal Asymptote : $\boxed{y = 2}$ H.A. $\quad R(x) = \dfrac{(2x+5)(x-3)}{(x-7)(x+3)}$

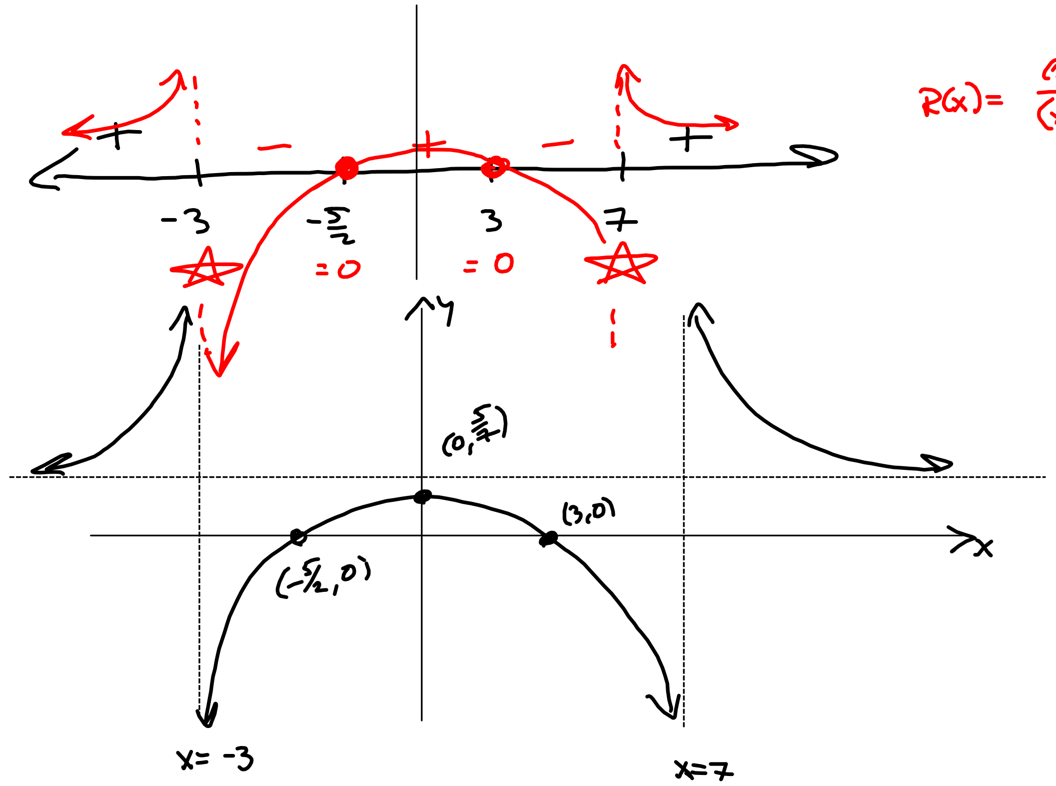
from $\dfrac{2x^2}{x^2} = 2 = y$

Sign Pattern : Sign can only change @ x-ints or V.A.

$x = -5/2, 3, -3, 7 \rightsquigarrow -3, -\frac{5}{2}, 3, 7$



$R(x) = \dfrac{(2x+5)(x-3)}{(x-7)(x+3)}$



Bonus for College Algebra:

Find where $R(x)$ intersects its H.A.

$$R(x)= \frac{2x^2-x-15}{x^2-4x-21} \overset{SET}{=} 2\left(\frac{x^2-4x-21}{x^2-4x-21}\right) = \frac{2x^2-8x-42}{x^2-4x-21} \Rightarrow$$

$$2x^2-x-15 = 2x^2-8x-42$$

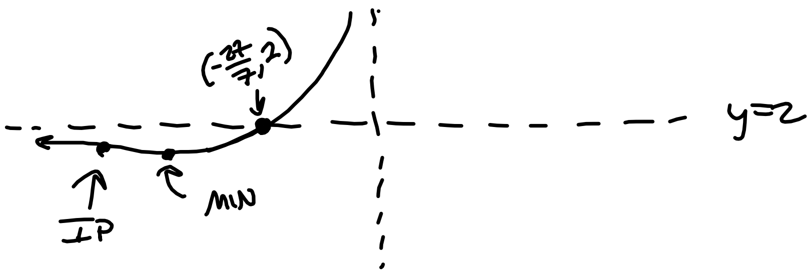
(idiot)

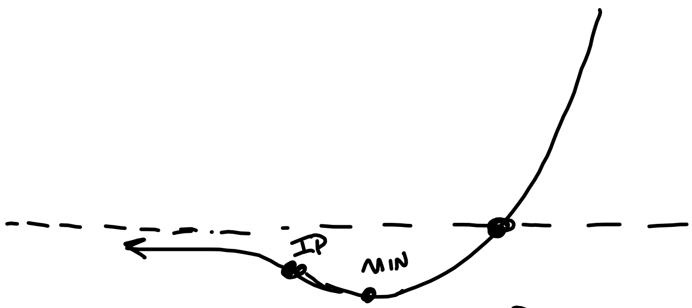
$$-x-15 = -8x-42 \Rightarrow$$

$$7x = -27 \Rightarrow$$

$x = \frac{-27}{7} \rightsquigarrow \left(-\frac{27}{7}, 2\right)$ is intersection.

So, the left part of the graph is:





Approaching $y=2$ from BELOW says it's concave Down, eventually.

Min: (-6.907736643, 1.607083130) approx

IP:

Oblique Asymptote

$$\frac{x^2-5x-14}{x-3} = R(x)$$

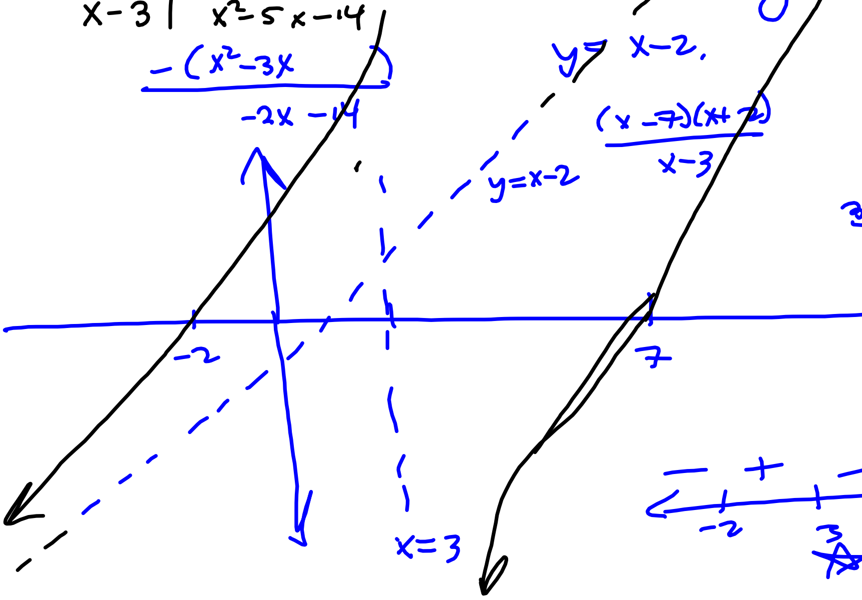
How to get it: Long Division:

$$\begin{array}{r} x-2 \\ x-3 \,\big)\, \overline{x^2-5x-14} \\ \underline{-(x^2-3x)} \\ -2x-14 \end{array}$$

This has oblique asymptote $y = x-2$.

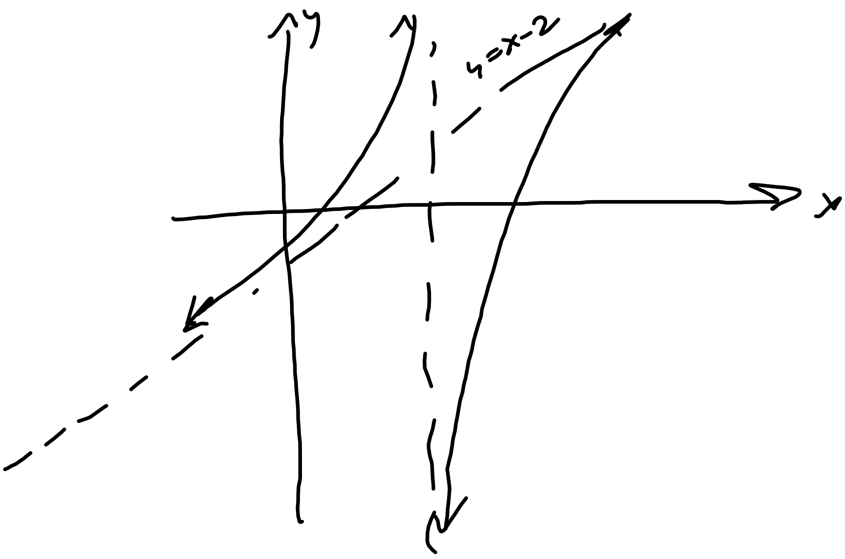
$$\frac{(x-7)(x+2)}{x-3}$$

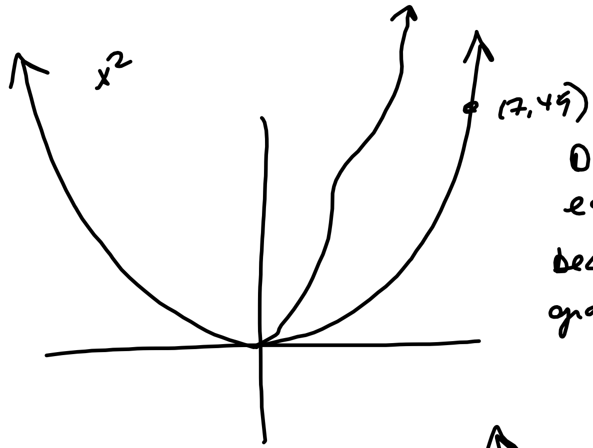
$$\begin{array}{r|rrr} 3 & 1 & -5 & -14 \\ & & 3 & -6 \\ \hline & 1 & -2 & -20 \end{array}$$



$- \quad + \quad - \quad +$

$-2 \quad 3 \quad 7$





Don't Lose the essence of the shape because of your graphing tech.

Tech loses the essence