§2.6 Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

25. $y\sin(2x) = x\cos(2y)$, $(\pi/2, \pi/4)$

$$y'\sin(2x) + y(\cos(2x)\cdot 2) = \cos(2y) + x(-\sin(2y))(2y')$$

$$\Rightarrow \sin(2x)y' + 2y\cos(2x) = \cos(2x) - 2x\sin(2y)y'$$

$$\Rightarrow \sin(2x)y' + 2x\sin(2y)y' = \cos(2x) - 2y\cos(2x)$$

$$\Rightarrow y'(\sin(2x) + 2x\sin(2y)) = \text{SAME}$$

$(x,y) = (\frac{\pi}{2}, \frac{\pi}{4})$

$$\Rightarrow y' = \frac{\cos(2y) - 2y\cos(2x)}{\sin(2x) + 2x\sin(2y)}$$

$(\frac{\pi}{2}, \frac{\pi}{4}) = (x, y)$

$(\frac{\pi}{2}, \frac{\pi}{4}) = (x, y)$

$$= \frac{\cos(2(\frac{\pi}{4})) - 2(\frac{\pi}{4})\cos(2(\frac{\pi}{2}))}{\sin(\frac{2\pi}{2}) + 2(\frac{\pi}{2})\sin(\frac{2\pi}{4})}$$

$$= \frac{\cos(\frac{\pi}{2}) + \frac{\pi}{2}\cos(\pi)}{\sin(\pi) + \pi\sin(\frac{\pi}{2})} = \frac{0 + \frac{\pi}{2}(-1)}{0 + \pi(1)} = \frac{-\frac{\pi}{2}}{\pi} = -\frac{1}{2} = m$$

$$\boxed{y = -\frac{1}{2}(x - \frac{\pi}{2}) + \frac{\pi}{4}}$$

Great for Mills

$$= -\frac{1}{2}x + \frac{\pi}{4} + \frac{\pi}{4} = \boxed{-\frac{1}{2}x + \frac{\pi}{2} = y}$$

Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$x^{2/3} + y^{2/3} = 4$

$(-3\sqrt{3}, 1)$

(astroid)

$y = $ $\left(\frac{1}{\sqrt{3}}\right)x + 4$

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}y' = 0$$

$$\Rightarrow \quad y' = \frac{-\frac{2}{3}x^{-\frac{1}{3}}}{\frac{2}{3}y^{-\frac{1}{3}}} = -\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$$

$$\rightarrow y'\Big|_{(x,y)=(-3\sqrt{3},\,1)} = -\frac{1}{(-3\sqrt{3})^{\frac{1}{3}}} = +\frac{1}{3^{\frac{1}{3}}\cdot 3^{\frac{1}{6}}} = \frac{1}{3^{\frac{2+1}{6}}} = \frac{1}{\sqrt{3}} = m$$

$$y = \frac{1}{\sqrt{3}}\left(x - (-3\sqrt{3})\right) + 1$$

$$= \frac{1}{\sqrt{3}}x + 3 + 1 = \frac{1}{\sqrt{3}}x + 4 = y$$

$(ab)^c = a^c b^c$

$a^b a^c = a^{b+c}$

$\log(xy) = \log(x) + \log(y)$

Scratch:

$\frac{1}{\cancel{\sqrt{3}}} \cdot 3\cancel{\sqrt{3}}$

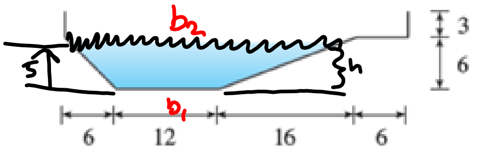
$\sqrt[3]{-1} = -1$

$-1 = (-1)^3 =$

A swimming pool is 20 ft wide, 40 ft long, 3 ft deep at the shallow end, and 9 ft deep at is deepest point. A cross-section is shown in the figure. If the pool is being filled at a rate of 0.8 ft^3/min, how fast is the water level rising when the depth at the deepest point is 5 ft? (Round your answer to five decimal places.)

0.00132 ft/min

$$\frac{dV}{dt} = \frac{.8\,ft^3}{min}$$

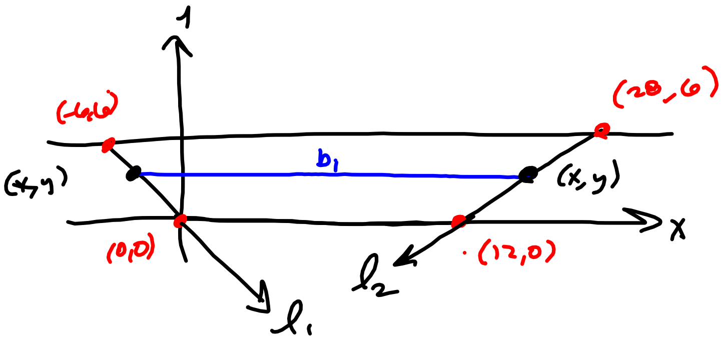


Want $\left.\frac{dh}{dt}\right|_{h=5}$

V = Volume (in ft^3) = V(h), where

h = depth of the water, for $0 \le h \le 6$

Area of trapezoid $= \frac{1}{2}(b_1+b_2)h = \frac{1}{2}(12 + \ ? \ )$



l_1:

$y = \frac{-6}{6}(x-0)+0$

$h = -x$ on l_1

$-x = h \Rightarrow$

$x_1 = x = -h$ on l_1

$y = \frac{6-0}{28-12}(x-12)+0$

$= \frac{6}{16}(x-12) = \frac{3}{8}x - \frac{9}{2} = l_2 = h$

$\frac{3\cdot\cancel{12}^{3}}{\cancel{8}_{2}}$

$\frac{3}{8}x = h + \frac{9}{2}$

$x_2 = x = \frac{8}{3}h + \frac{\cancel{8}^{4}}{\cancel{3}}\cdot\frac{\cancel{9}^{3}}{\cancel{2}_{1}} = \frac{8}{3}h + 12$ on l_2

So $x_2 - x_1 = b_1$

$= \frac{8}{3}h + 12 - (-h)$

$= \frac{11}{3}h + 12 = b_1$

So, area of trapezoid is $\frac{1}{2}\left(12 + \frac{11}{3}h + 12\right)h$

$= \frac{1}{2}\left[24 + \frac{11}{3}h\right]h = 12h + \frac{11}{6}h^2 \Rightarrow$

$12\frac{dh}{dt}$

$\frac{1}{2}$

$V = V(h) = \left(12h + \frac{11}{6}h^2\right)(20)$

$\frac{dV}{dt} = 20\left(12 + \frac{11}{3}h \cdot \frac{dh}{dt}\right) = \frac{.8\,ft^3}{min} \Rightarrow$

$$20\left(12 + \frac{11}{3}(5)\, h'\right) = .8 = \frac{8}{10} = \frac{4}{5}$$

$$12 + \frac{55}{3} h' = \frac{1}{100}$$

$$\frac{55}{3} h' = \frac{1}{100} - 12$$

$$h' = \left(\frac{1}{100} - 12\right)\left(\frac{3}{55}\right) = \text{Something wrong!}$$

$\frac{dh}{dt} > 0$; but this is < 0?!

$(12,0)$, $(28,6)$

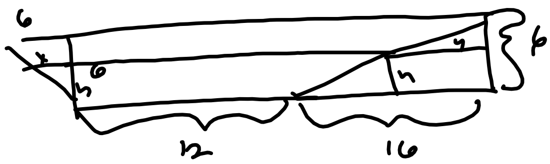
$$m = \frac{6-0}{28-12} = \frac{6}{16} = \frac{3}{8}$$

$$y = \frac{3}{8}(x-12) + 0 = \frac{3}{8}x - \frac{3}{\cancel{8}_{2}}(\cancel{12}^{3}) = \frac{3}{8}x - \frac{9}{2} = h$$

$$\frac{3}{8}x = h + \frac{9}{2}$$

$$x = \frac{8}{3}h + \frac{9}{2}\left(\frac{8}{3}\right) = \frac{8}{3}h + 12$$

$$x_2 - x_1 = \frac{8}{3}h + 12 + h = \frac{11}{3}h + 12$$



$$\frac{16}{6} = \frac{y}{h}$$

$$\frac{8}{3}h = y$$

$$\frac{6}{6} = \frac{x}{h} \Rightarrow x = h$$

$$\text{Volume} = V(h) = \frac{1}{2}\left(12 + (12+x+y)\right)h \cdot 20$$

$$= 10 \cdot \left(24 + h + \frac{8}{3}h\right)h$$

$$= 10\left(24 + \frac{11}{3}h\right)h = \left(240 + \frac{110}{3}h\right)h$$

$$= 240h + \frac{110}{3}h^2 = V$$

Chain Rule!

$$\Rightarrow \frac{dV}{dt} = 240\frac{dh}{dt} + \frac{220}{3}h \cdot \frac{dh}{dt} = .8$$

$$\Rightarrow \frac{dh}{dt}\left[240 + \frac{220}{3}h\right] = .8$$

$$\Rightarrow \left.\frac{dh}{dt}\right|_{h=5} = \left.\frac{.8}{240 + \frac{220}{3}h}\right|_{h=5}$$

The radius of a circular disk is given as 28 cm with a maximum error in measurement of 0.2 cm $\to dr!$

(a) Use differentials to estimate the maximum error in the calculated area of the disk. (Round your answer to two decimal places.)

✗ 35.19 cm^2

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

(b) What is the relative error? (Round your answer to four decimal places.)

✗ 0.0143

$dA = 2\pi r \cdot dr$

What is the percentage error? (Round your answer to two decimal places.)

✗ 1.43 %

$\Delta A \approx dA = 2\pi(28)(.2)$

$= 11.2\pi$

$$\text{Relative Error} = \frac{\text{Error}}{\text{Total}}$$

$$\approx \frac{11.2\pi}{\pi(28)^2}$$

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11.2π
            35.18583772
11.2/28²
            .0142857143
Ans*100
            1.428571429
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$$\% \text{ Error} \approx \left(\frac{11.2\pi}{28^2\pi}\right)(100\%)$$