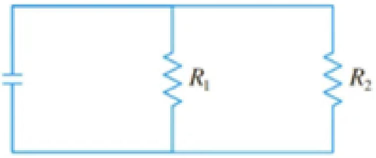
25. Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

39. If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in ohms (Ω), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 and R_2 are increasing at rates of $0.3\ \Omega/s$ and $0.2\ \Omega/s$, respectively, how fast is R changing when $R_1 = 80\ \Omega$ and $R_2 = 100\ \Omega$?



$R_1' = .3\ \Omega/s = \frac{dR_1}{dt}$

$R_2' = .2\ \Omega/s \rightsquigarrow = \frac{dR_2}{dt}$

Want

$R'\Big|_{R_1=80\Omega,\ R_2=100\Omega}$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$R^{-1} = R_1^{-1} + R_2^{-1} \Longrightarrow$

$-R^{-2}R' = -R_1^{-2}R_1' - R_2^{-2}R_2'$

$-\frac{R'}{R^2} = -\frac{R_1'}{R_1^2} - \frac{R_2'}{R_2^2}$

$\frac{R'}{R^2} = \frac{R_1'}{R_1^2} + \frac{R_2'}{R_2^2}$

$R' = R^2\left[\frac{R_1'}{R_1^2} + \frac{R_2'}{R_2^2}\right]$ $\Big|_{R_1=80,\ R_2=100}$

Scratch:

$\frac{1}{R} = \frac{1}{80} + \frac{1}{100} =$

$80 = 2\cdot2\cdot2\cdot2\cdot5$, $100 = 2\cdot2\cdot5\cdot5$

$LCD = 2^4\cdot5^2$

$= \left(\frac{1}{80}\right)\left(\frac{5}{5}\right) + \left(\frac{1}{100}\right)\left(\frac{4}{4}\right)$

$= \frac{5+4}{400} = \frac{9}{400} = \frac{1}{R} \Longrightarrow R = \frac{400}{9}$

$= \left(\frac{400}{9}\right)^2\left[\frac{.3}{80^2} + \frac{.2}{100^2}\right]$

OLD SCHOOL

$= \frac{2^8\cdot5^4}{3^4}\left[\frac{.3}{2^8\cdot5^2} + \frac{.2}{2^4\cdot5^4}\right]$

$= \frac{2^8\cdot5^4}{3^4}\cdot\frac{1}{2\cdot5}\cdot\frac{1}{2^4\cdot5^2}\left[\frac{3}{2^4} + \frac{2}{5^2}\right]$

$= \frac{2^4\cdot5^2}{3^4\cdot2\cdot5}\left[\frac{3}{2^4}\cdot\frac{5^2}{5^2} + \frac{2}{5^2}\cdot\frac{2^4}{2^4}\right]$

$= \frac{2^3\cdot5}{3^4}\left[\frac{75+32}{2^4\cdot5^2}\right]$

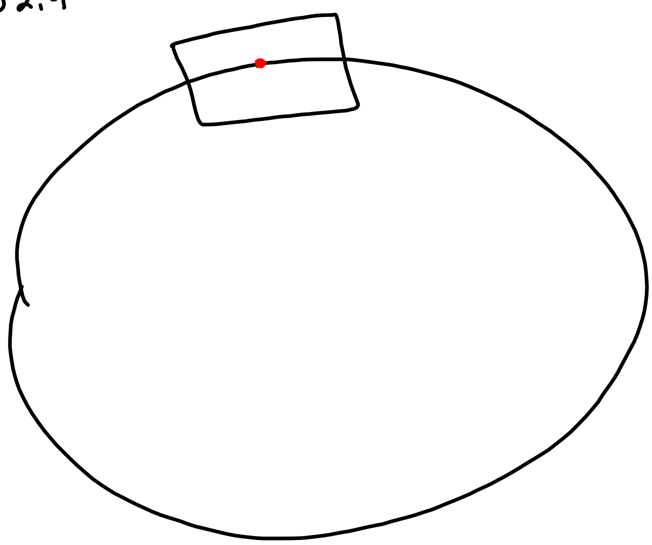
$= \frac{107}{3^4\cdot2\cdot5} = \boxed{\frac{107}{810} = R'}$

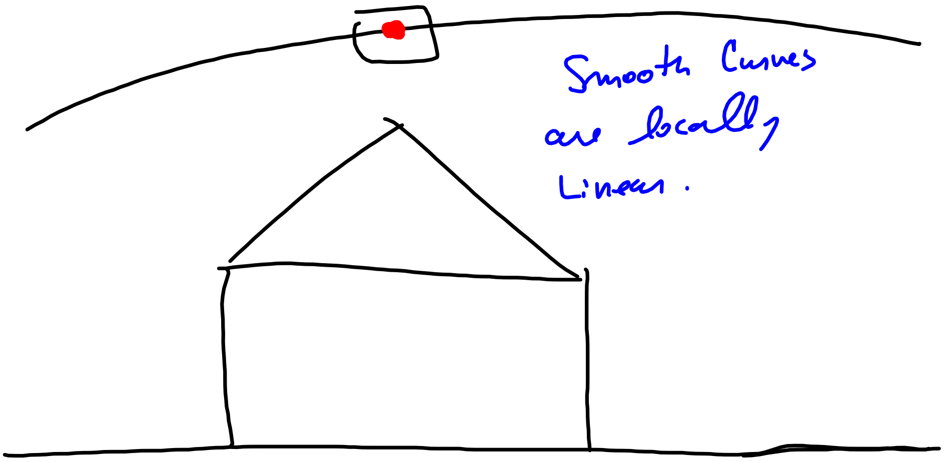
$\left(\frac{400}{9}\right)^2$

```
Ans²
          1975.308642
Ans*(.3/80²+.2/1
00²)
          .1320987654
107/810
          .1320987654
```

$3\cdot162 \quad 5 \quad \overline{810}$

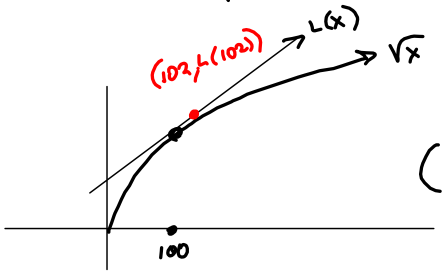
§2.9





Smooth Curves are locally Linear.

Approximate $\sqrt{102}$ without a calculator



$$L_{100}(x) = f'(100)(x-100) + f(100)$$
$$= m(x - x_1) + y_1$$
$$(x_1, y_1) = (x_1, f(x_1))$$
$$f'(x) = \frac{d}{dx}\left[\sqrt{x}\right] = \frac{d}{dx}\left[x^{\frac{1}{2}}\right]$$
$$= \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$\sqrt{100} = 10$

$$\Rightarrow f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{2\cdot 10} = \frac{1}{20} = m$$

$$\Rightarrow L_{100}(x) = \frac{1}{20}(x-100) + 10$$

$$f(102) = \sqrt{102} \approx L_{100}(102) = \frac{1}{20}(102-100) + 10$$
$$= \frac{1}{20}[2] + 10 = \frac{1}{10} + 10 = \frac{101}{10} = 10.1 \approx \sqrt{102}$$

```
102^.5
          10.09950494
```

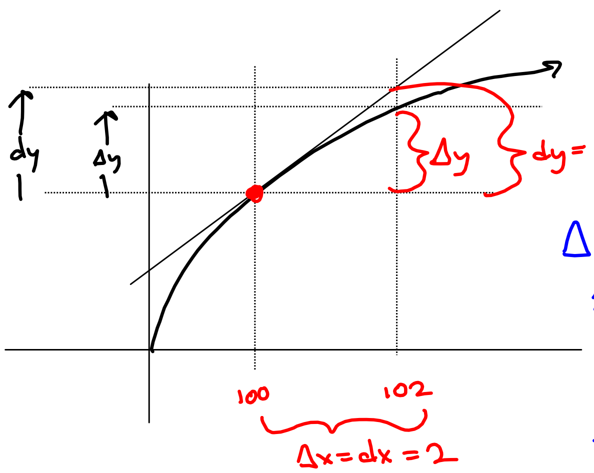
is close to 10.1 !

$$L_{100}(x) = \underbrace{f'(100)}_{\text{steepness}}\boxed{(x-100)} + f(100)$$

$(x-100)$ → How far you've moved horizontally

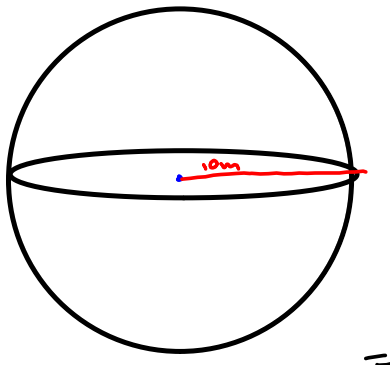
$f(100)$ → ORIGINAL HEIGHT

$\Delta x = dx$ = Differential of x,



$$dy = f'(x)\Delta x = f'(x)dx = f'(x)(x - x_1)$$

$$\Delta y = f(x_2) - f(x_1)$$
$$\approx L_{x_1}(x_2) - f(x_1)$$
$$= f'(x_1)(x_2 - x_1) + f(x_1) - f(x_1)$$
$$= f'(x_1)(x_2 - x_1)$$
$$= f'(x_1)\Delta x$$
$$= f'(x_1)dx = dy$$
$$= \text{Differential of } y$$



I want to paint this 10-meter-radius sphere with a 1-cm-thick insulating foam. How much foam will I need?

$\Delta V = \Delta$ volume

$= \frac{4}{3}\pi (10.01)^3 - \frac{4}{3}\pi (10)^3$

$(1\text{cm})\left(\frac{1\text{m}}{100\text{cm}}\right) = .01\text{m}$

$= \frac{4}{3}\pi \left[10.01^3 - 10^3\right]$

```
4/3π(10.01^3-10^
3)
      12.57894117
```

m^3

Do the same thing using differentials.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$dV = 4\pi r^2 dr$, where, $r = 10$, $dr = .01$

$dv = 4\pi (10)^2 (.01) \approx 12.56637061\ m^3$

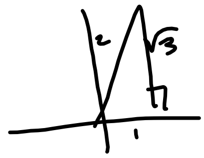
```
4/3π(10.01^3-10^
3)
      12.57894117
4π*100*.01
      12.56637061
```

$\Delta y \approx dy = f'(x_1)\,dx = f'(x)\Delta x$

$y = m(x - x_1) + y_1 = L(x)$

$dy \approx \Delta y = y - y_1 = m(x - x_1) \approx f'(x_1)\,dx \equiv dy$

Use Linearization to approximate $\sin(63°)$



$x_1 = 60° = \frac{\pi}{3}$

$\Delta x = dx = 63° - 60° = 3° = (3°)\left(\frac{\pi}{180°}\right) = \frac{\pi}{60}$

$f(x) = \sin(x)$

$f'(x) = \cos(x)$

$f'(60°) = f'(x_1) = \frac{1}{2}$

$L_{\frac{\pi}{3}}(x) = f'\left(\frac{\pi}{3}\right)\left(x - \frac{\pi}{3}\right) + f\left(\frac{\pi}{3}\right)$

$= \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$

$\Rightarrow L_{\frac{\pi}{3}}\left(\frac{\pi}{3} + \frac{\pi}{60}\right) = \frac{1}{2}\left(\frac{\pi}{3} + \frac{\pi}{60} - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$

$= \frac{1}{2}\left(\frac{\pi}{60}\right) + \frac{\sqrt{3}}{2} = \frac{\pi + 60\sqrt{3}}{120}$

```
           .0191897195
sin(π/3+π/60)
           .8910065242
sin(π/3)
           .8660254038
(π+60√(3))/120
           .8922053426
```

$L_a(x) = f'(a)(x-a) + f(a)$

§2.9 in a nutshell

$\Delta y \approx dy = f'(a)\,dx$